Edexcel International A Level (IAL) Maths: Statistics 2

Your notes

Hypothesis Testing (Binomial & Poisson Distributions)

Contents

Your notes

Binomial Hypothesis Testing

How is a hypothesis test carried out with the binomial distribution?

- The **population parameter** being tested will be the **probability, p** in a **binomial distribution** $B(n, p)$
- A hypothesis test is used when the assumed probability is questioned
- The **null hypothesis**, H_0 and **alternative hypothesis**, H_1 will always be given in terms of p
  - Make sure you clearly define p before writing the hypotheses
  - The null hypothesis will always be $\mathbf{H_0 : p = \ldots}$
  - The alternative hypothesis will depend on if it is a one-tailed or two-tailed test
  - A one-tailed test would test to see if the **value of p** has **either increased or decreased**
    - The **alternative hypothesis, H_1 will be $H_1 : p > \ldots$ or $H_1 : p < \ldots$**
  - A two-tailed test would test to see if the **value of p** has **changed**
    - The **alternative hypothesis, H_1 will be $H_1 : p \neq \ldots$**
- To carry out a hypothesis test with the binomial distribution, the **test statistic** will be the number of **successes** in a defined number of **trials**
- When defining the test statistic, remember that the value of p is being tested, so this should be written as p in the original definition, followed by the null hypothesis stating the assumed value of p
- The binomial distribution will be used to **calculate the probability** of the test statistic taking the observed value **or a more extreme value**
- The hypothesis test can be carried out by
  - either calculating the probability of the test statistic taking the observed or a more extreme value and comparing this with the significance level
    - You may have to use a normal approximation to calculate the probability
  - or by finding the critical region and seeing whether the observed value of the test statistic lies within it
    - Finding the critical region can be more useful for considering more than one observed value or for further testing

How is the critical value found in a hypothesis test with the binomial distribution?

Your notes

- The **critical value** will be the **first value** to fall within the **critical region**
  - The binomial distribution is a **discrete** distribution so the probability of the observed value being within the critical region, given a true null hypothesis may be less than the **significance level**
  - This is the **actual significance level** and is the probability of **incorrectly** rejecting the null hypothesis
- For a **one-tailed test** use your calculator to find the first value for which the probability of that **or a more extreme value** is less than the given significance level
  - Check that the next value would cause this probability to be greater than the significance level
    - For $\mathbf{H_1}: p < ...$ if $\mathrm{P}(X \le c) \le \alpha\%$ and $\mathrm{P}(X \le c+1) > \alpha\%$ then c is the critical value
    - For $\mathbf{H_1}: p > ...$ if $\mathrm{P}(X \ge c) \le \alpha\%$ and $\mathrm{P}(X \ge c-1) > \alpha\%$ then c is the critical value
- For a **two-tailed test** you will need to find **both critical values,** one at each end of the distribution
  - Find the first value for which the probability of that **or a more extreme value** is less than **half** of the given significance level in both the upper and lower tails
    - Often one of the critical regions will be bigger than the other

What steps should I follow when carrying out a hypothesis test with the binomial distribution?

- **Step 1.** Define the probability, *p*
- **Step 2.** Write the null and alternative hypotheses clearly using the form

$$\mathrm{H_0}: p = ...$$

$$\mathrm{H_1}: p = ...$$

- **Step 3.** Define the test statistic, usually $X \sim B(n,p)$ where *n* is a defined number of trials and *p* is the population parameter to be tested
- **Step 4.** Calculate either the **critical value(s)** or the **necessary probability** for the test
- **Step 5.** Compare the observed value of the test statistic with the critical value(s) or the probability with the significance level
- **Step 6.** Decide whether there is enough evidence to reject $\mathrm{H_0}$ or whether it has to be accepted
- **Step 7.** Write a conclusion in context

Worked Example

Jacques, a breadmaker, claims that fewer than 40% of people that shop in a particular supermarket buy his brand of bread. Jacques takes a random sample of 12 customers that have purchased bread and asks them which brand of bread they have purchased. He records that 2 of them had purchased his brand of bread. Test, at the 10% level of significance, whether Jacques' claim is justified.

Step 1: Define the probability, p
Let p be the proportion of people who purchase Jacques' brand of bread.

Step 2: Write the hypotheses
$H_0: p = 0.4$
$H_1: p < 0.4$

Step 3: Define the distribution
For $X \sim B(12, p)$

$n = 12$

p is being tested so do not use 0.4 here

Step 4: Calculate the p-value
Assuming H_0 is true, then $P(X \leqslant 2) = 0.0834$

Using the tables

Step 5: Compare the p-value with the given significance level
$0.0834... < 10\%$

Step 6: Decide whether there is enough evidence to reject H_0
The probability calculated is less than the significance level so there is enough evidence to reject H_0.

Step 7: Write a conclusion in context
There is sufficient evidence to reject the null hypothesis at the 10% level of significance.

The test supports Jacques' claim that fewer than 40% of people who shop in the supermarket buy his brand of bread.

* Note: If finding the critical region, steps 4 and 5 would be:

Using the tables

Step 4: Calculate the critical value
Assuming H_0 is true,
then $P(X \leqslant 2) = 0.0834 < 0.10$ and $P(X \leqslant 3) = 0.2253... > 0.10$
So the critical value is 10 and the critical region is $X \geqslant 10$

Step 5: Compare the observed value of the test statistic with the critical region:
10 is inside the critical region.

Examiner Tips and Tricks

Your notes

- Most of the time the values of n and p will be in the tables
- if not, you will have to use the formula to calculate the probabilities or use a normal approximation

Your notes

Poisson Hypothesis Testing

How is a hypothesis test carried out for the mean of a Poisson distribution?

- The **population parameter** being tested will be the **mean, λ ,** in a **Poisson distribution Po(λ)**
  - As it is the population mean, sometimes μ will be used instead
- A hypothesis test is used when the mean is questioned
- The **null hypothesis**, H_0 and **alternative hypothesis**, H_1 will be given in terms of λ (or μ)
  - Make sure you clearly define λ before writing the hypotheses
  - The null hypothesis will always be $\mathbf{H_0}$: λ = ...
  - The alternative hypothesis will depend on if it is a one-tailed or two-tailed test
  - A one-tailed test would test to see if the **value of λ** has **either increased or decreased**
    - The **alternative hypothesis, will be H_1 will be $H_1 : \lambda > \ldots$ or $H_1 : \lambda < \ldots$**
  - A two-tailed test would test to see if the **value of λ** has **changed**
    - The **alternative hypothesis, H_1 will be $H_1 : \lambda \neq \ldots$**
- To carry out a hypothesis test with the Poisson distribution, the **random variable** will be the mean number of occurrences of the event within the given time/space interval
  - Remember you may need to change the mean to fit the interval of time or space for your observed value
- When defining the distribution, remember that the value of λ is being tested, so this should be written as λ in the original definition, followed by the null hypothesis stating the assumed value of λ
- The Poisson distribution will be used to **calculate the probability** of the random variable taking the observed value **or a more extreme value**
- The hypothesis test can be carried out by
  - either calculating the probability of the random variable taking the observed or a more extreme value and comparing this with the significance level
  - or by finding the critical region and seeing whether the observed value of the test statistic lies within it
    - Finding the critical region can be more useful for considering more than one observed value or for further testing

Your notes

How is the critical value found in a hypothesis test with the Poisson distribution?

- The **critical value** will be the **first value** to fall within the **critical region**
  - The Poisson distribution is a **discrete** distribution so the probability of the observed value being within the critical region, given a true null hypothesis may be less than the **significance level**
  - This is the **actual significance level** and is the probability of **incorrectly** rejecting the null hypothesis (a Type I error)
- For a **one-tailed test** use the formula to find the first value for which the probability of that **or a more extreme value** is less than the given significance level
  - Check that the next value would cause this probability to be greater than the significance level
    - For $H_1 : \lambda < \ldots$ if $P(X \le c) \le \alpha\%$ and $P(X \le c+1) > \alpha\%$ then c is the critical value
    - For $H_1 : \lambda > \ldots$ if $P(X \ge c) \le \alpha\%$ and $P(X \ge c-1) > \alpha\%$ then c is the critical value
- For a **two-tailed test** you will need to find **both critical values,** one at each end of the distribution

What steps should I follow when carrying out a hypothesis test with the Poisson distribution?

Step 1. Define the mean, λ

Step 2. Write the null and alternative hypotheses clearly using the form

$$H_0 : \lambda = \ldots$$

$$H_1 : \lambda = \ldots$$

Step 3. Define the distribution, usually $X \sim \mathrm{Po}(\lambda)$ where λ is the mean to be tested

Step 4. Calculate the probability of the random variable being at least as extreme as the observed value

- Or if told to find the critical region

Step 5. Compare this probability with the significance level

- Or compare the observed value with the critical region

Step 6. Decide whether there is enough evidence to reject H_0 or whether it has to be accepted

Step 7. Write a conclusion in context

Worked Example

Mr Viajo believes that his travel blog receives an average of 8 likes per day (24 hour period). He tries a new advertising campaign and carries out a hypothesis test at the 5% level of significance to see if there is a change in the number of likes he gets. Over a 12-hour period chosen at random Mr Viajo's travel blog receives 7 likes.

(i) State null and alternative hypotheses for Mr Viajo's test.

(ii) Find the critical regions for the test.

(iii) Find the actual level of significance.

(iv) Carry out the hypothesis test, writing your conclusion clearly.

(i) Let λ be the average number of likes per day

$H_0: \lambda = 8$

$H_1: \lambda \neq 8$ Change so use $\neq$

(ii) 8 likes per 24 hours = 4 likes per 12 hours

For $X \sim P_o(4)$ ← The observed value is for 12 hours so change the mean to match

Two-tailed test so 2.5% in each tail

Using the tables:

$P(X = 0) = 0.0183 < 2.5\%$

$P(X \leq 1) = 0.0916 > 2.5\%$

Critical region for lower tail is $X = 0$

$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9786 = 0.0214 < 2.5\%$

$P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9489 = 0.0511 > 2.5\%$

Critical region for upper tail is $X \geq 9$

Critical region $X = 0$ or $X \geq 9$

(iii) Actual significance level is probability of being in critical region

$P(X = 0) + P(X \geq 9)$

$0.0183 + 0.0214 = 0.0397$

$X = 3.97\%$

(iv) $0 < 7 < 9$ so 7 is not in critical region.

Do not reject H_0

At the 5% significance level there is insufficient evidence to reject the null hypothesis.

The results of the test do not support Mr Viajo's belief that the number of likes has changed.

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Examiner Tips and Tricks

- You might have to change the value of λ based on the length of the interval
  - Always make it clear to the examiner which value of λ you are using when you calculate probabilities