Chapter 6: Sampling and Sampling Distribution

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S2 Chapter 6 Solutions

2. A farmer sells boxes of eggs.

The eggs are sold in boxes of 6 eggs and boxes of 12 eggs in the ratio $n:1$

A random sample of three boxes is taken.

The number of eggs in the first box is denoted by X_1
The number of eggs in the second box is denoted by X_2
The number of eggs in the third box is denoted by X_3
The random variable $T = X_1 + X_2 + X_3$

Given that $\mathrm{P}(T = 18) = 0.729$

(a) show that $n = 9$

(2)

(b) find the sampling distribution of T

(4)

The random variable R is the range of X_1, X_2, X_3

(c) Using your answer to part (b), or otherwise, find the sampling distribution of R

(2)

Question 2 continued

4. The volume of milk, M litres, in cartons produced by a dairy, has distribution $N(\mu, \sigma^2)$, where μ and σ are unknown. A random sample of 12 cartons is taken and the volume of milk in each carton is measured ($M_1, M_2, \ldots, M_{12}$). A statistic X is based on this sample.

(a) Explain what is meant by "a random sample" in this case.

(1)

(b) State the population in this case.

(1)

(c) Write down the distribution of $\dfrac{M_{12} - \mu}{\sigma}$

(1)

(d) Explain what you understand by the sampling distribution of X.

(1)

(e) State, giving a reason, which of the following is not a statistic based on this sample.

(I) $3M_1 + \dfrac{2M_{11}}{6}$ (II) $\displaystyle\sum_{i=1}^{12}\left(\frac{M_i - \mu}{\sigma}\right)^2$ (III) $\displaystyle\sum_{i=1}^{12}(2M_i - 3)$

(2)

Question 4 continued

(Total 6 marks)

4. A bag contains a large number of marbles, each of which is blue or red.

A random sample of 3 marbles is taken from the bag.

The random variable D represents the number of blue marbles taken minus the number of red marbles taken.

Given that 20% of the marbles in the bag are blue,

(a) show that $P(D = -1) = 0.384$

(2)

(b) find the sampling distribution of D

(3)

(c) write down the mode of D

(1)

Takashi claims that the true proportion of blue marbles is greater than 20% and tests his claim by selecting a random sample of 12 marbles from the bag.

(d) Find the critical region for this test at the 10% level of significance.

(2)

(e) State the actual significance level of this test.

(1)

Question 4 continued

2. The random variable X has probability distribution

x	-1	0	1
$P(X = x)$	0.2	0.4	0.4

Samples of size 2 are selected at random from the distribution and the random variable T is the sum of the 2 values of X.

(a) Show that $P(T = 0) = 0.32$

(2)

(b) Find the sampling distribution of T.

(4)

5. A bag contains a large number of 1p coins and 2p coins in the ratio $2:3$ respectively.

A jar contains a large number of 2p coins and 5p coins in the ratio $1:4$ respectively.

Two coins are selected at random from the bag and one coin is selected at random from the jar. The random variable T represents the total of the values of the 3 coins selected.

(a) Find the sampling distribution of T.

(7)

(b) Find the sampling distribution of the median of the values of the 3 selected coins.

(4)

Question 5 continued

6. (a) Explain what you understand by the sampling distribution of a statistic.

(1)

A factory produces beads in bags for craft shops. A small bag contains 40 beads, a medium bag contains 80 beads and a large bag contains 150 beads. The factory produces small, medium and large bags in the ratio 5:3:2 respectively.

A random sample of 3 bags is taken from the factory.

(b) Find the sampling distribution for the range of the number of beads in the 3 bags in the sample.

(7)

A random sample of n sets of 3 bags is taken. The random variable Y represents the number of these n sets of 3 bags that have a range of 70

(c) Calculate the minimum value of n such that $P(Y = 0) < 0.2$

(3)

Question 6 continued

6. The owner of a very large youth club has designed a new method for allocating people to teams. Before introducing the method he decided to find out how the members of the youth club might react.

(a) Explain why the owner decided to take a random sample of the youth club members rather than ask all the youth club members.

(1)

(b) Suggest a suitable sampling frame.

(1)

(c) Identify the sampling units.

(1)

The new method uses a bag containing a large number of balls. Each ball is numbered either 20, 50 or 70
When a ball is selected at random, the random variable X represents the number on the ball where

$$P(X = 20) = p \qquad P(X = 50) = q \qquad P(X = 70) = r$$

A youth club member takes a ball from the bag, records its number and replaces it in the bag.
He then takes a second ball from the bag, records its number and replaces it in the bag.

The random variable M is the mean of the 2 numbers recorded.

Given that

$$P(M = 20) = \frac{25}{64} \qquad P(M = 60) = \frac{1}{16} \qquad \text{and} \quad q > r$$

(d) show that $P(M = 50) = \dfrac{1}{16}$

(7)

Question 6 continued

4. A bag contains a large number of balls, each with one of the numbers 1, 2 or 5 written on it in the ratio $2:3:4$ respectively.

A random sample of 3 balls is taken from the bag.

The random variable B represents the range of the numbers written on the balls in the sample.

(i) Find $\mathrm{P}(B = 4)$

(ii) Find the sampling distribution of B.

(10)

Question 4 continued

5. A bag contains a large number of counters.

40% of the counters are numbered 1
35% of the counters are numbered 2
25% of the counters are numbered 3

In a game Alif draws two counters at random from the bag. His score is 4 times the number on the first counter minus 2 times the number on the second counter.

(a) Show that Alif gets a score of 8 with probability 0.0875 **(1)**

(b) Find the sampling distribution of Alif's score. **(5)**

(c) Calculate Alif's expected score. **(2)**

Question 5 continued

(Total 8 marks)

6 (a) Explain what you understand by the sampling distribution of a statistic.

(1)

At Sam's cafe a standard breakfast consists of 6 breakfast items. Customers can then choose to upgrade to a medium breakfast by adding 1 extra breakfast item or they can upgrade to a large breakfast by adding 2 extra breakfast items. Standard, medium and large breakfasts are sold in the ratio $6:3:2$ respectively.

A random sample of 2 customers is taken from customers who have bought a breakfast from Sam's cafe on a particular day.

(b) Find the sampling distribution for the total number, T, of breakfast items bought by these 2 customers. Show your working clearly.

(7)

(c) Find $\mathrm{E}(T)$

(2)

Question 6 continued

7. A bag contains 10 counters each with exactly one number written on it.

There are 6 counters with the number 7 on them
There are 3 counters with the number 8 on them
There is 1 counter with the number 9 on it

A random sample of 3 counters is taken from the bag (without replacement).
These counters are then put back in the bag.

This process is then repeated until 20 samples have been taken.

The random variable Y represents the number of these 20 samples that contain the counter with the number 9 on it.

(a) (i) Find the mean of Y

(ii) Find the variance of Y

(5)

A random sample of 3 counters is chosen from the bag (without replacement).

(b) List all possible samples where the median of the numbers on the 3 counters is 7

(2)

(c) Find the sampling distribution of the median of the numbers on the 3 counters.

(5)

Question 7 continued

6. A bag contains a large number of counters with one of the numbers 5, 10 or 20 written on each of them in the ratio $5:2:a$

A jar contains a large number of counters with one of the numbers 5 or 10 written on each of them in the ratio $1:3$

One counter is selected at random from the bag and then two counters are selected at random from the jar.
The random variable R represents the range of the numbers on the 3 counters.

Given that $\mathrm{P}(R = 15) = \dfrac{63}{256}$

(a) by forming and solving an equation in a, show that $a = 9$

(3)

(b) find the sampling distribution of R

(6)

Question 6 continued

2. A bag contains a large number of coins. It only contains 20p and 50p coins. A random sample of 3 coins is taken from the bag.

(a) List all the possible combinations of 3 coins that might be taken.

(2)

Let $\overline{X}$ represent the mean value of the 3 coins taken.

Part of the sampling distribution of $\overline{X}$ is given below.

$\bar{x}$	20	a	b	50
$\mathrm{P}(\overline{X} = \bar{x})$	$\frac{4913}{8000}$	c	d	$\frac{27}{8000}$

(b) Write down the value of a and the value of b

(1)

The probability of taking a 20p coin at random from the bag is p

The probability of taking a 50p coin at random from the bag is q

(c) Find the value of p and the value of q

(2)

(d) Hence, find the value of c and the value of d

(3)

Let M represent the mode of the 3 coins taken at random from the bag.

(e) Find the sampling distribution of M

(3)

Question 2 continued

2. (a) State one characteristic of a population that would make a census a practical alternative to sampling.

(1)

A leisure centre has 2500 members.

It asks a sample of 300 members for their opinions on the fees it charges for using the centre.

For the sample,

(b) (i) identify a suitable sampling frame,

(ii) identify a sampling unit.

(2)

The leisure centre has the following pieces of information.

A is the list of the different types of membership that can be paid for by members.

B is the mean of the membership fees paid by **all** 2500 members.

C is the number in the **sample** of 300 members who are satisfied with the fees they pay.

(c) State the piece of information that is a statistic.
Give a reason for your answer.

(1)

Question 2 continued

(Total for Question 2 is 4 marks)

4. A manufacturer makes t-shirts in 3 sizes, small, medium and large.

20% of the t-shirts made by the manufacturer are small and sell for £10

30% of the t-shirts made by the manufacturer are medium and sell for £12

The rest of the t-shirts made by the manufacturer are large and sell for £15

(a) Find the mean value of the t-shirts made by the manufacturer.

(2)

A random sample of 3 t-shirts made by the manufacturer is taken.

(b) List all the possible combinations of the individual selling prices of these 3 t-shirts.

(2)

(c) Find the sampling distribution of the **median** selling price of these 3 t-shirts.

(6)

Question 4 continued

6. A bag contains a large number of counters with an odd number or an even number written on each.

Odd and even numbered counters occur in the ratio 4 : 1

In a game a player takes a random sample of 4 counters from the bag.

The player scores

5 points for each counter taken that has an even number written on it

2 points for each counter taken that has an odd number written on it

The random variable X represents the total score, in points, from the 4 counters.

(a) Find the sampling distribution of X

(6)

A random sample of n sets of 4 counters is taken. The random variable Y represents the number of these n sets that have a total score of exactly 14

(b) Calculate the minimum value of n such that $P(Y \geqslant 1) > 0.95$

(3)

Question 6 continued

4 A bag contains 50 counters, each with one of the numbers 4, 7 or 10 written on it in the ratio $2:3:5$ respectively.

A random sample of 2 counters is taken from the bag. The numbers on the 2 counters are recorded as D_1 and D_2

The random variable M represents the mean of D_1 and D_2

(a) Show that $P(M = 4) = \dfrac{9}{245}$

(1)

(b) Find the sampling distribution of M

(6)

A random sample of n sets of 2 counters is taken. The random variable T represents the number of these n sets of 2 counters that have a mean of 4

Given that each set of 2 counters is replaced after it is drawn,

(c) calculate the minimum value of n such that $P(T = 0) < 0.15$

(3)

Question 4 continued