Chapter 2: Poisson Distribution

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S2 Chapter 2 Solutions

5. A delivery company loses packages randomly at a mean rate of 10 per month.

The probability that the delivery company loses more than 12 packages in a randomly selected month is p

(a) Find the value of p

(1)

The probability that the delivery company loses more than k packages in a randomly selected month is at least $2p$

(b) Find the largest possible value of k

(2)

In a randomly selected month,

(c) find the probability that exactly 4 packages were lost in each half of the month.

(3)

In a randomly selected two-month period, 21 packages were lost.

(d) Find the probability that at least 10 packages were lost in each of these two months.

(4)

(e) Using a suitable approximation, find the probability that more than 27 packages are lost during a randomly selected 4-month period.

(5)

Question 5 continued

5. Cars stop at a service station randomly at a rate of 3 every 5 minutes.

(a) Calculate the probability that in a randomly selected 10 minute period,

(i) exactly 7 cars will stop at the service station,

(ii) more than 7 cars will stop at the service station.

(4)

1. Each day a restaurant opens between 11 am and 11 pm. During its opening hours, the restaurant receives calls for reservations at an average rate of 6 per hour.

(a) Find the probability that the restaurant receives exactly 1 call for a reservation between 6 pm and 7 pm.

(2)

7. Members of a conservation group record the number of sightings of a rare animal. The number of sightings follows a Poisson distribution with a rate of 1 every 2 months.

(a) Find the smallest value of n such that the probability that there are at least n sightings in 2 months is less than 0.05

(2)

(b) Find the smallest number of months, m, such that the probability of no sightings in m months is less than 0.05

(2)

(c) Find the probability that there is at least 1 sighting per month in each of 3 consecutive months.

(3)

(d) Find the probability that the number of sightings in an 8 month period is equal to the expected number of sightings for that period.

(2)

(e) Given that there were 4 sightings in a 4 month period, find the probability that there were more sightings in the last 2 months than in the first 2 months.

(3)

Question 7 continued

2. During morning hours, employees arrive randomly at an office drinks dispenser at a rate of 2 every 10 minutes.

The number of employees arriving at the drinks dispenser is assumed to follow a Poisson distribution.

(a) Find the probability that fewer than 5 employees arrive at the drinks dispenser during a 10-minute period one morning.

(2)

During a 30-minute period one morning, the probability that n employees arrive at the drinks dispenser is the same as the probability that $n + 1$ employees arrive at the drinks dispenser.

(b) Find the value of n

(3)

During a 45-minute period one morning, the probability that between c and 12, inclusive, employees arrive at the drinks dispenser is 0.8546

(c) Find the value of c

(3)

(d) Find the probability that exactly 2 employees arrive at the drinks dispenser in exactly 4 of the 6 non-overlapping 10-minute intervals between 10 am and 11am one morning.

(4)

Question 2 continued

Answer ALL questions. Write your answers in the spaces provided.

1. A company produces NuttyChok bars that are made of chocolate and whole hazelnuts.

A 40 kg batch of the chocolate and hazelnut mixture used to make NuttyChok bars will contain 4750 hazelnuts.

(a) Show that the mean number of hazelnuts in an 80 g bar of NuttyChok is 9.5

(1)

A Poisson model is suggested for the number of hazelnuts in each 80 g bar of NuttyChok.

(b) State two assumptions that need to be made for a Poisson model to be suitable in this case.

(2)

(c) Find the probability that an 80 g bar of NuttyChok

(i) contains exactly 12 hazelnuts,

(ii) contains fewer than 8 hazelnuts.

(4)

(d) Find the probability that a pack of three 80 g bars of NuttyChok includes exactly one bar with fewer than 8 hazelnuts.

(3)

Question 1 continued

3. For a particular website, hacking attempts occur randomly at a rate of 0.3 per hour.

(a) Show that the probability of at least one hacking attempt taking place in a randomly selected one-hour period is 0.259 to 3 significant figures.

(2)

(b) Find the probability that exactly 6 hacking attempts take place in a randomly selected 24-hour period.

(3)

1. *Flogcar* is a car hire company that only allows cars to be hired for single days. The numbers of cars hired on different days are statistically independent. The number of cars hired on a randomly selected day from *Flogcar* is modelled by a Poisson distribution with mean 4

(a) Calculate the probability that exactly 6 cars are hired from *Flogcar* on a randomly selected day.

(2)

(b) Calculate the probability that on 2 randomly selected days, the total number of cars hired from *Flogcar* is between 3 and 7 inclusive.

(3)

(d) (i) Explain why the distribution of the number of cars returned in good condition would not be well approximated by the Poisson distribution.

(1)

(ii) By considering the number of cars hired out that are **not** returned in good condition, use a Poisson approximation to estimate the probability that more than 95 of the cars hired out in this period are returned in good condition. Show your working clearly.

(3)

Question 1 continued

4. In a peat bog, Common Spotted-orchids occur at a mean rate of 4.5 per m^2

(a) Give an assumption, not already stated, that is required for the number of Common Spotted-orchids per m^2 of the peat bog to follow a Poisson distribution.

(1)

Given that the number of Common Spotted-orchids in 1 m^2 of the peat bog can be modelled by a Poisson distribution,

(b) find the probability that in a randomly selected 1 m^2 of the peat bog

(i) there are exactly 6 Common Spotted-orchids,

(ii) there are fewer than 10 but more than 4 Common Spotted-orchids.

(4)

Question 4 continued

3. The number of water fleas, in 100 ml of pond water, has a Poisson distribution with mean 7

(a) Find the probability that a sample of 100 ml of the pond water does **not** contain exactly 4 water fleas.

(2)

Aja collects 5 separate samples, each of 100 ml, of the pond water.

(b) Find the probability that exactly 1 of these samples contains exactly 4 water fleas.

(3)

Question 3 continued

2. Luis makes and sells rugs. He knows that faults occur randomly in his rugs at a rate of 3 every $4\,m^2$

(a) Find the probability of there being exactly 5 faults in one of his rugs that is $4\,m^2$ in size.

(2)

(b) Find the probability that there are more than 5 faults in one of his rugs that is $6\,m^2$ in size.

(2)

Luis makes a rug that is $4\,m^2$ in size and finds it has exactly 5 faults in it.

(c) Write down the probability that the next rug that Luis makes, which is $4\,m^2$ in size, will have exactly 5 faults. Give a reason for your answer.

(2)

A small rug has dimensions 80 cm by 150 cm. Faults still occur randomly at a rate of 3 every $4\,m^2$
Luis makes a profit of £80 on each small rug he sells that contains no faults but a profit of £60 on any small rug he sells that contains faults.

Luis sells n small rugs and expects to make a profit of at least £4000

(d) Calculate the minimum value of n

(4)

Question 2 continued

4. The number of cars entering a safari park per 10-minute period can be modelled by a Poisson distribution with mean 6

(a) Find the probability that in a given 10-minute period exactly 8 cars will enter the safari park.

(2)

(b) Find the smallest value of n such that the probability that at least n cars enter the safari park in 10 minutes is less than 0.05

(2)

The probability that no cars enter the safari park in m minutes, where m is an integer, is less than 0.05

(c) Find the smallest value of m

(2)

A car enters the safari park.

(d) Find the probability that there is less than 5 minutes before the next car enters the safari park.

(3)

Given that exactly 15 cars entered the safari park in a 30-minute period,

(e) find the probability that exactly 1 car entered the safari park in the first 5 minutes of the 30-minute period.

(4)

Question 4 continued

1 A local pottery makes cups. The number of faulty cups made by the pottery in a week follows a Poisson distribution with a mean of 6

In a randomly chosen week, the probability that there will be at least x faulty cups made is 0.1528

(a) Find the value of x

(3)

1. The independent random variables W and X have the following distributions.

$$W \sim \text{Po}(4) \qquad X \sim \text{B}(3, 0.8)$$

(a) Write down the value of the variance of W

(1)

(b) Determine the mode of X
Show your working clearly.

(2)

One observation from each distribution is recorded as W_1 and X_1 respectively.

(c) Find $\text{P}(W_1 = 2 \text{ and } X_1 = 2)$

(3)

(d) Find $\text{P}(X_1 < W_1)$

(4)

Question 1 continued

5. The number of particles per millilitre in a solution is modelled by a Poisson distribution with mean 0.15

A randomly selected 50 millilitre sample of the solution is taken.

(a) Find the probability that

(i) exactly 10 particles are found,

(ii) between 6 and 11 particles (inclusive) are found.

(4)

Petra takes 12 independent samples of m millilitres of the solution.

The probability that at least 2 of these samples contain no particles is 0.1184

(b) Using the Statistical Tables provided, find the value of m

(6)

Question 5 continued

1. Bhavna produces rolls of cloth. She knows that faults occur randomly in her cloth at a mean rate of 1.5 every 15 metres.

(a) Find the probability that in 15 metres of her cloth there are

(i) less than 3 faults,

(ii) at least 6 faults.

(3)

Each roll contains 100 metres of Bhavna's cloth.
She selects 15 rolls at random.

(b) Find the probability that exactly 10 of these rolls each have fewer than 13 faults.

(4)

Bhavna decides to sell her cloth in pieces.

Each piece of her cloth is 4 metres long.
The cost to make each piece is £5.00
She sells each piece of her cloth that contains no faults for £7.40
She sells each piece of her cloth that contains faults for £2.00

(c) Find the expected profit that Bhavna will make on each piece of her cloth that she sells.

(4)

Question 1 continued

1. A shop sells shoes at a mean rate of 4 pairs of shoes per hour on a weekday.

(a) Suggest a suitable distribution for modelling the number of sales of pairs of shoes made per hour on a weekday.

(1)

(b) State one assumption necessary for this distribution to be a suitable model of this situation.

(1)

(c) Find the probability that on a weekday the shop sells

(i) more than 4 pairs of shoes in a one-hour period,

(ii) more than 4 pairs of shoes in each of 3 consecutive one-hour periods.

(4)

5. A company produces steel cable.

Defects in the steel cable produced by this company occur at random, at a constant rate of 1 defect per 16 metres.

On one day the company produces a piece of steel cable 80 metres long.

(a) Find the probability that there are at most 5 defects in this piece of steel cable.

(2)

7. A bakery sells muffins individually at an average rate of 8 muffins per hour.

(a) Find the probability that, in a randomly selected one-hour period, the bakery sells at least 4 but not more than 8 muffins.

(3)

A sample of 5 non-overlapping **half-hour** periods is selected at random.

(b) Find the probability that the bakery sells fewer than 3 muffins in exactly 2 of these periods.

(5)

Given that 4 muffins were sold in a one-hour period,

(c) find the probability that more muffins were sold in the first 15 minutes than in the last 45 minutes.

(4)

Question 7 continued

5. A supermarket receives complaints at a mean rate of 6 per week.

(a) State one assumption necessary, in order for a Poisson distribution to be used to model the number of complaints received by the supermarket.

(1)

(b) Find the probability that, in a given week, there are

(i) fewer than 3 complaints received by the supermarket,

(ii) at least 6 complaints received by the supermarket.

(3)

1. The manager of a supermarket is investigating the number of complaints per day received from customers.

A random sample of 180 days is taken and the results are shown in the table below.

Number of complaints per day	0	1	2	3	4	5	6	$\geqslant 7$
Frequency	12	28	37	38	29	17	19	0

(a) Calculate the mean and the variance of these data.

(3)

(b) Explain why the results in part (a) suggest that a Poisson distribution may be a suitable model for the number of complaints per day.

(1)

The manager uses a Poisson distribution with mean 3 to model the number of complaints per day.

(c) For a randomly selected day find, using the manager's model, the probability that there are

(i) at least 3 complaints,

(ii) more than 4 complaints but less than 8 complaints.

(4)

Question 1 continued

1 A garage sells tyres. The number of customers arriving at the garage to buy tyres in a 10-minute period is modelled by a Poisson distribution with mean 2

(a) Find the probability that

(i) fewer than 4 customers arrive to buy tyres in the next 10 minutes,

(ii) more than 5 customers arrive to buy tyres in the next 10 minutes.

(3)

The manager randomly selects 20 non-overlapping, 30-minute periods.

(b) Find the probability that there are between 4 and 7 (inclusive) customers arriving to buy tyres in exactly 15 of these 30-minute periods.

(4)

The manager believes that placing an advert in the local paper will lead to a significant increase in the number of customers arriving at the garage.
A week after the advert is placed, the manager randomly selects a 25-minute period and finds that 10 customers arrive at the garage to buy tyres.

(c) Test, at the 5% level of significance, whether or not there is evidence to support the manager's belief.
State your hypotheses clearly.

(5)

(d) Explain why the Poisson distribution is unlikely to be valid for the number of tyres sold during a 10-minute period.

(1)

Question 1 continued