

Chapter 7: Hypothesis Testing

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4. A sweet shop produces different coloured sweets and sells them in bags.

The shop claims that 10% of its customers buy more than two bags of sweets. A random sample of 40 customers is taken and 1 customer buys more than two bags of sweets.

- (c) Test, at the 5% level of significance, whether or not there is evidence that the proportion of customers who buy more than two bags of sweets is less than the shop's claim. State your hypotheses clearly.

(4)

6. In a local council, 60% of households recycle at least half of their waste. A random sample of 80 households is taken.

The number of bags recycled per family per week was known to follow a Poisson distribution with mean 1.5

Following a recycling campaign, the council believes the mean number of bags recycled per family per week has increased. To test this belief, 6 families are selected at random and the total number of bags they recycle the following week is recorded.

The council wishes to test, at the 5% level of significance, whether or not there is evidence that the mean number of bags recycled per family per week has increased.

- (b) Find the critical region for the total number of bags recycled by the 6 families.

(2)

7. A manufacturer produces packets of sweets. Each packet contains 25 sweets. The manufacturer claims that, on average, 40% of the sweets in each packet are red.

A packet is selected at random.

- (a) Using a 1% level of significance, find the critical region for a two-tailed test that the proportion of red sweets is 0.40

You should state the probability in each tail, which should be as close as possible to 0.005

(5)

- (b) Find the actual significance level of this test. (1)

The manufacturer changes the production process to try to reduce the number of red sweets. She chooses 2 packets at random and finds that 8 of the sweets are red.

- (c) Test, at the 1% level of significance, whether or not there is evidence that the manufacturer's changes to the production process have been successful. State your hypotheses clearly. (4)

Question 7 continued

7.4

1. Each day a restaurant opens between 11 am and 11 pm. During its opening hours, the restaurant receives calls for reservations at an average rate of 6 per hour.

The restaurant distributes leaflets to local residents to try and increase the number of calls for reservations. After distributing the leaflets, it records the number of calls for reservations it receives over a 90 minute period.

Given that it receives 14 calls for reservations during the 90 minute period,

- (b) test, at the 5% level of significance, whether the rate of calls for reservations has increased from 6 per hour. State your hypotheses clearly.

(5)

- Takashi claims that the true proportion of blue marbles is greater than 20% and tests his claim by selecting a random sample of 12 marbles from the bag.

- (e) State the actual significance level of this test. (1)

4. At a shop, past figures show that 35% of customers pay by credit card. Following the shop's decision to no longer charge a fee for using a credit card, a random sample of 20 customers is taken and 11 are found to have paid by credit card.

Hadi believes that the proportion of customers paying by credit card is now greater than 35%

- (a) Test Hadi's belief at the 5% level of significance. State your hypotheses clearly.

(5)

4. A new supermarket opens with its entrance on a busy road.

In the 4 years before the supermarket opened there were 10 traffic accidents on the road.
In the 12 months after opening there were 6 traffic accidents on the road.

The supermarket manager claims that there has been no change in the rate of traffic accidents on the road since the supermarket opened.

- (a) Test the manager's claim at the 5% level of significance. State your hypotheses clearly.

(5)

A local resident claims that the rate of traffic accidents on the road has increased.

- (b) Explain how a hypothesis test, with a 5% level of significance, to assess the resident's claim would be different from your hypothesis test in part (a). State the conclusion you would reach.

(3)

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Question 4 continued

7.9

6. Sarah makes clay pots in batches of size n . The clay pots need to be fired in a kiln. Sarah finds that 30% of the clay pots crack when they are fired. The remainder do not crack.

Sarah attends a course to improve her pottery skills. She claims that the course will reduce the proportion of clay pots that crack when fired.

After completing the course Sarah makes a batch of 20 clay pots to assess her claim.

- (e) Stating your hypotheses clearly, determine the critical region for the hypothesis test she must use to assess her claim using a 5% significance level.

(3)

- (f) Write down the actual significance level of this test.

(1)

3. For a particular website, hacking attempts occur randomly at a rate of 0.3 per hour.

Following a security update, Saira believes that the rate of hacking attempts on the website has decreased. She records 38 hacking attempts in a 7-day period following the update.

- (c) Comment on Saira's belief by using an appropriate approximation and a suitable test at the 5% level of significance. State your hypotheses clearly.

(7)

7. Jonathan believes that the time, in minutes, people spend queuing for lunch at a particular fast food restaurant can be modelled using the continuous random variable X , which has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{3125}(25x^4 - 4x^5) & 0 \leq x \leq 5 \\ 1 & x > 5 \end{cases}$$

Olivia believes that this model overestimates the proportion of queuing times that are more than 4.0 minutes. She takes a random sample of 25 queuing times and finds that 3 of them are more than 4.0 minutes.

Assuming that the model gives a probability of 0.25 that the queuing time is more than 4.0 minutes,

- (c) use a suitable test, at the 5% level of significance, to assess Olivia's belief. State your hypotheses clearly.

(5)

- (b) Use a suitable test, at the 5% level of significance, to determine whether or not there is evidence to support Kiyoshi's belief. State your hypotheses clearly.

(5)

- Chris runs boat trips between two islands during the summer. During her trips, Chris claims that whales are seen randomly at a mean rate of 2 in every 9 trips.

To investigate Chris's claim, a random sample of 18 boat trips run by Chris was taken.

- (a) Using a 5% level of significance, find the critical region, for a two-tailed test, to enable Chris to test her claim.

(3)

- (b) State the actual level of significance of this test.

(1)

The total number of whales seen in the 18 boat trips was 6

- (c) State what this suggests about Chris's claim. Give a reason for your answer.

(1)

The following summer, Chris decides to run n trips so that the probability of her seeing at least one whale is more than 0.9

- (d) Assuming Chris's claim is true, find the minimum value of n .

(4)

Chris believes that she sees fewer whales during the winter. A random sample of 45 trips run in the winter was taken and the total number of whales seen was 5

- (e) Test, at the 5% level of significance, whether or not the mean rate of whales seen in winter is less than 2 in every 9 trips. State your hypotheses clearly.

(4)

Question 5 continued

7.15

- A manufacturer produces plates. The proportion of plates that are flawed is 45%, with flawed plates occurring independently.

The manufacturer claims that, after a change to the production process, the proportion of flawed plates has decreased. A random sample of 30 plates, taken after the change to the production process, contains 8 flawed plates.

- (c) Use a suitable hypothesis test, at the 5% level of significance, to assess the manufacturer's claim. State your hypotheses clearly.

(4)

- Juan believes that by introducing a new management scheme the number of Common Spotted-orchids in the peat bog will increase. After three years under the new management scheme, a randomly selected 2 m² of the peat bog contains 11 Common Spotted-orchids.

(c) Using a 5% significance level assess Juan's belief. State your hypotheses clearly.

(5)

- (5)

3. The number of water fleas, in 100ml of pond water, has a Poisson distribution with mean 7

After the pond has been cleaned, the number of water fleas in a 100ml random sample of the pond water is 15

- (d) Using a suitable test, at the 1% level of significance, assess whether or not there is evidence that the number of water fleas per 100ml of the pond water has increased. State your hypotheses clearly.

(5)

1. *Spany* sells seeds and claims that 5% of its pansy seeds do not germinate. A packet of pansy seeds contains 20 seeds. Each seed germinates independently of the other seeds.

Jem planted all of the 100 seeds she bought from *Spany* and found that 8 did not germinate.

- (d) Using a suitable approximation, carry out the test using a 5% level of significance.

(6)

2. Luis makes and sells rugs. He knows that faults occur randomly in his rugs at a rate of 3 every 4m^2

Luis wishes to increase the productivity of his business and employs Rhiannon. Faults also occur randomly in Rhiannon's rugs and independently to faults made by Luis. Luis randomly selects 10 small rugs made by Rhiannon and finds 13 faults.

- (e) Test, at the 5% level of significance, whether or not there is evidence to support the suggestion that the rate at which faults occur is higher for Rhiannon than for Luis. State your hypotheses clearly.

(5)

1. A research project into food purchases found that 35% of people who buy eggs do **not** buy free range eggs.

The manager of the farm shop believes that the proportion of customers who buy eggs but do **not** buy free range eggs is more than 35%

In a survey of 200 customers who buy eggs, 86 do **not** buy free range eggs.

Using a suitable test and a normal approximation,

- (d) determine, at the 5% level of significance, whether there is evidence to support the manager's belief. State your hypotheses clearly.

(7)

4. The number of cars entering a safari park per 10-minute period can be modelled by a Poisson distribution with mean 6

Aston claims that the mean number of cars entering the safari park per 10-minute period is more than 6

He selects a 15-minute period at random in order to test whether there is evidence to support his claim.

- (f) Determine the critical region for the test at the 5% level of significance.

(2)

- (a) Give a reason why a Poisson distribution could be used to model the number of breakdowns.

(b) Write down suitable null and alternative hypotheses that the headteacher should use.

(1)

(c) Using a 5% level of significance, find the critical region to test whether the rate of breakdowns has now changed.

(3)

- (d) Find the actual significance level of a test based on the critical region from part (c).

(2)

(e) Comment on this finding in the light of the critical region found in part (c).

(2)

- 5 Applicants for a pilot training programme with a passenger airline are screened for colour blindness. Past records show that the proportion of applicants identified as colour blind is 0.045

Jaymini claims that 75% of all applicants for this training programme go on to become pilots.

From a random sample of 96 applicants for this training programme 67 go on to become pilots.

- (e) Using a suitable approximation, test Jaymini's claim at the 5% level of significance.

State your hypotheses clearly.

(7)

- This season it is believed that the proportion of pears that are unfit for sale has decreased. To test this belief a random sample of n pears is taken. The random variable Y represents the number of pears in the sample that are unfit for sale.

- (3)

(b) Test, using a suitable approximation, whether there is evidence of a change in the proportion of pears weighing more than 180 g.

- (6)

Question 4 continued

7.27

- A packet is selected at random.

- (4)

- (1)

A random sample of 250 of the company's sunflower seeds is taken and 11 of them grow to a height of more than 3 metres.

- (5)

Question 3 continued

7.29

1. A shop sells shoes at a mean rate of 4 pairs of shoes per hour on a weekday.

The area manager visits the shop on a weekday, the day after an advert for the shop appears in a local paper.

In a one-hour period during the manager's visit, the shop sells 7 pairs of shoes.

This leads the manager to believe that the advert has increased the shop's sales of pairs of shoes.

- (d) Stating your hypotheses clearly, test at the 5% level of significance whether or not there is evidence of an increase in sales of pairs of shoes following the appearance of the advert.

(5)

3. *Superbounce* is a manufacturer of tennis balls.

It knows from past records that 10% of its tennis balls fail a bounce test.

The managing director makes changes to the production process and claims that these changes will reduce the probability of its tennis balls failing the bounce test.

After the changes were made a random sample of 50 of the tennis balls were tested and it was found that 2 failed the bounce test.

- (b) Test, at the 5% significance level, whether or not this result supports the managing director's claim.

In a second random sample of n tennis balls it was found that none failed the bounce test. As a result of this sample, the managing director's claim is supported at the 1% significance level. (4)

- (c) Find the smallest possible value of n (3)

Question 3 continued

7.32

(2)

Question 4 continued

7.34

5. A supermarket receives complaints at a mean rate of 6 per week.

In a randomly selected week, the supermarket received 12 complaints.

- (c) Test, at the 5% level of significance, whether or not there is evidence that the mean number of complaints is greater than 6 per week.

State your hypotheses clearly.

(5)

Following changes made by the supermarket, it received 26 complaints over a 6-week period.

- (d) Use a suitable approximation to test whether or not there is evidence that, following the changes, the mean number of complaints received is less than 6 per week.

You should state your hypotheses clearly and use a 5% significance level.

(7)

3. Rowan believes that 35% of type A vacuum tubes shatter when exposed to alternating high and low temperatures.

Rowan takes a random sample of 15 of these type *A* vacuum tubes and uses a two-tailed test, at the 5% level of significance, to test his belief.

- (a) Give **two** assumptions, in context, that Rowan needs to make for a binomial distribution to be a suitable model for the number of these type *A* vacuum tubes that shatter when exposed to alternating high and low temperatures.

(2)

- (b) Using a binomial distribution, find the critical region for the test.
You should state the probability of rejection in each tail, which should be as close as possible to 0.025

(3)

- (c) Find the actual level of significance of the test based on your critical region from part (b)

(1)

Rowan records that in the latest batch of 15 type A vacuum tubes exposed to alternating high and low temperatures, 4 of them shattered.

- (d) With reference to part (b), comment on Rowan's belief. Give a reason for your answer.

(1)

Rowan changes to type B vacuum tubes. He takes a random sample of 40 type B vacuum tubes and finds that 8 of them shatter when exposed to alternating high and low temperatures.

- (e) Test, at the 5% level of significance, whether or not there is evidence that the proportion of type *B* vacuum tubes that shatter when exposed to alternating high and low temperatures is lower than 35%.
You should state your hypotheses clearly.

(5)

Question 3 continued

7.37

- 3 Jian owns a large group of shops. She decides to visit a random sample of the shops to check if the stocktaking system is being used incorrectly.

(a) Suggest a suitable sampling frame for Jian to use.

(1)

(b) Identify the sampling units.

(1)

(c) Give one advantage and one disadvantage of taking a sample rather than a census.

(2)

Jian believes that the stocktaking system is being used incorrectly in 40% of the shops.

To investigate her belief, a random sample of 30 of the shops is taken.

- (d) Using a 5% level of significance, find the critical region for a two-tailed test of Jian's belief.

You should state the probability in each tail, which should each be as close as possible to 2.5%

(3)

The total number of shops, in the sample of 30, where the stocktaking system is being used incorrectly is 20

- (e) Using the critical region from part (d), state what this suggests about Jian's belief. Give a reason for your answer.

(1)

Jian introduces a new, simpler, stocktaking system to all the shops.

She takes a random sample of 150 shops and finds that in 47 of these shops the new stocktaking system is being used incorrectly.

- (f) Using a suitable approximation, test, at the 5% level of significance, whether or not there is evidence that the proportion of shops where the stocktaking system is being used incorrectly is now **less than** 0.4

You should state your hypotheses and show your working clearly.

(7)

Question 3 continued

7.38