Chapter 5: Continuous Unifom Distribution

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S2 Chapter 5 Solutions

3. Albert uses scales in his kitchen to weigh some fruit.

The random variable D represents, in grams, the weight of the fruit given by the scales **minus** the true weight of the fruit. The random variable D is uniformly distributed over the interval $[-2.5, 2.5]$

(a) Specify the probability density function of D

(2)

(b) Find the standard deviation of D

(2)

Albert weighs a banana on the scales.

(c) Write down the probability that the weight given by the scales equals the true weight of the banana.

(1)

(d) Find the probability that the weight given by the scales is within 1 gram of the banana's true weight.

(1)

Albert weighs 10 bananas on the scales, one at a time.

(e) Find the probability that the weight given by the scales is within 1 gram of the true weight for at least 6 of the bananas.

(3)

Question 3 continued

3. A machine pours oil into bottles. It is electronically controlled to cut off the flow of oil randomly between 100 ml and k ml, where $k > 100$. It is equally likely to cut off the flow at any point in this range. The random variable X is the volume of oil poured into a bottle.

Given that $P(102 \leqslant X \leqslant k) = \frac{2}{3}$

(a) show that $k = 106$

(3)

(b) Find the probability that the volume of oil poured into a bottle is
(i) less than 105 ml,
(ii) exactly 105 ml.

(2)

(c) Write down the value of $E(X)$

(1)

(d) Find the 15th percentile of this distribution.

(2)

(e) Determine the value of x such that $3P(X \leqslant x - 1.5) = P(X \geqslant x + 1.5)$

(3)

Question 3 continued

6. One side of a square is measured to the nearest centimetre and this measurement is multiplied by 4 to estimate the perimeter of the square. The random variable, W cm, represents the estimated perimeter of the square minus the true perimeter of the square.

W is uniformly distributed over the interval $[a, b]$

(a) Explain why $a = -2$ and $b = 2$

(1)

The standard deviation of W is σ

(b) (i) Find σ

(ii) Find the probability that the estimated perimeter of the square is within σ of the true perimeter of the square.

(4)

One side of each of 100 squares are now measured. Using a suitable approximation,

(c) find the probability that W is greater than 1.9 for at least 5 of these squares.

(4)

Question 6 continued

5. The continuous random variable X is uniformly distributed over the interval $[a, b]$ where $0 < a < b$

Given that $P(X < b - 2a) = \frac{1}{3}$

(a) (i) show that $E(X) = \frac{5a}{2}$

(3)

(ii) find $P(X > b - 4a)$

(1)

The continuous random variable Y is uniformly distributed over the interval $[3, c]$ where $c > 3$

Given that $\text{Var}(Y) = 3c - 9$, find

(b) (i) the value of c

(3)

(ii) $P(2Y - 7 < 20 - Y)$

(3)

(iii) $E(Y^2)$

(3)

Question 5 continued

3. The continuous random variable Y is such that $Y \sim \mathrm{U}[a, b]$, where a and b are constants.

The mean of Y is 6 and the variance of Y is 3

(a) Find the value of a and the value of b.

(3)

(b) Find the probability that Y is more than 1 standard deviation above the mean.

(2)

(c) Define fully the cumulative distribution function of Y.

(2)

Question 3 continued

(Total 7 marks)

2. The continuous random variable X is uniformly distributed over the interval $[-3, 12]$

(a) Find the upper quartile of X.

(2)

(b) Find $P(5 \leqslant X < 14)$

(2)

The continuous random variable Y is uniformly distributed over the interval $[a, b]$ where $0 < a < b$

Given that $E(X) = E(Y - 3)$ and $Var(X) = 4Var(Y)$

(c) find the value of a and the value of b.

(5)

Question 2 continued

(Total 9 marks)

6. A machine is designed to cut bolts. The length, in millimetres, of each bolt is uniformly distributed over the interval [39.5, 40.5]

Bolts longer than 40.45 millimetres cannot be used.

One bolt is selected at random.

(a) Find the probability that the bolt **can** be used.

(1)

In a random sample of 10 bolts,

(b) find the probability that at least 9 bolts can be used.

(3)

In a random sample of 120 bolts,

(c) find, using a suitable approximation, the probability that more than 117 bolts can be used.

(4)

Question 6 continued

3. A task was designed to take a person 25 minutes to complete. The random variable T represents the time, in minutes, it takes to complete the task and is uniformly distributed over the interval $[25 - k, 25 + 3k]$ where $k > 0$

(a) Find $P(25 - 0.5k < T < 25 + 2.5k)$

(1)

Given that $E(T^2) = 918.76$

(b) find the value of k.

(6)

The task is to be completed by 50 people. Assuming that people complete the task independently,

(c) calculate the probability that at least 20 of these people complete the task in less than 25 minutes.

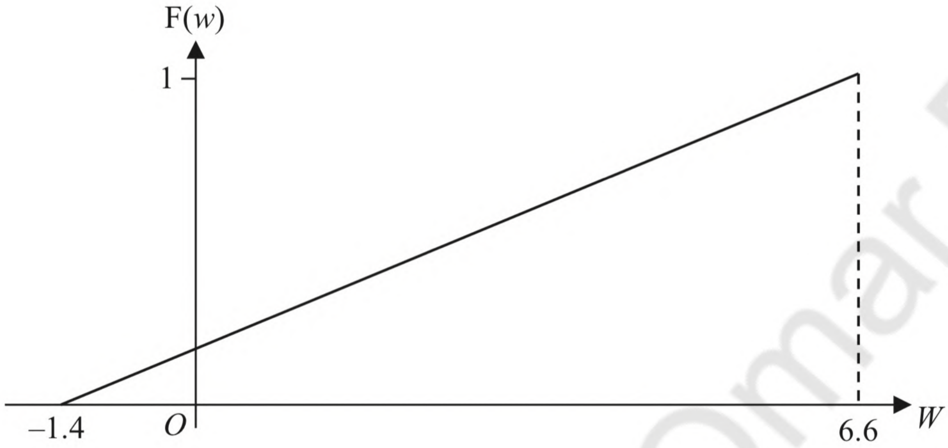
(3)

Question 3 continued

2. In the summer Kylie catches a local steam train to work each day. The published arrival time for the train is 10 am.

The random variable W is the train's actual arrival time minus the published arrival time, in minutes. When the value of W is positive, the train is late.

The cumulative distribution function $F(w)$ is shown in the sketch below.



(a) Specify fully the probability density function $f(w)$ of W.

(2)

(b) Write down the value of $E(W)$

(1)

(c) Calculate α such that $P(\alpha \leqslant W \leqslant 1.6) = 0.35$

(2)

A day is selected at random.

(d) Calculate the probability that on this day the train arrives between 1.2 minutes late and 2.4 minutes late.

(2)

Given that on this day the train was between 1.2 minutes late and 2.4 minutes late,

(e) calculate the probability that it was more than 2 minutes late.

(2)

A random sample of 40 days is taken.

(f) Calculate the probability that for at least 10 of these days the train is between 1.2 minutes late and 2.4 minutes late.

(3)

Question 2 continued

5. A piece of wood AB is 3 metres long. The wood is cut at random at a point C and the random variable W represents the length of the piece of wood AC.

(a) Find the probability that the length of the piece of wood AC is more than 1.8 metres.
(2)

The two pieces of wood AC and CB form the two shortest sides of a right-angled triangle. The random variable X represents the length of the longest side of the right-angled triangle.

(b) Show that $X^2 = 2W^2 - 6W + 9$
(2)

[You may assume for random variables S, T and U and for constants a and b that if $S = aT + bU$ then $E(S) = aE(T) + bE(U)$]

(c) Find $E(X^2)$
(6)

(d) Find $P(X^2 > 5)$
(4)

Question 5 continued

2. (i) The continuous random variable X is uniformly distributed over the interval $[a, b]$

Given that $P(8 < X < 14) = \dfrac{1}{5}$ and $E(X) = 11$

(a) write down $P(X > 14)$

(1)

(b) find $P(6X > a + b)$

(4)

(ii) Susie makes a strip of pasta 45 cm long. She then cuts the strip of pasta, at a randomly chosen point, into two pieces. The random variable S is the length of the shortest piece of pasta.

(a) Write down the distribution of S

(1)

(b) Calculate the probability that the shortest piece of pasta is less than 12 cm long.

(2)

Susie makes 20 strips of pasta, all 45 cm long, and separately cuts each strip of pasta, at a randomly chosen point, into two pieces.

(c) Calculate the probability that exactly 6 of the pieces of pasta are less than 12 cm long.

(3)

Question 2 continued

7 The sides of a square are each of length L cm and its area is A cm^2

Given that A is uniformly distributed on the interval [10, 30]

(a) find $P(L \geqslant 4.5)$

(2)

(b) find $\text{Var}(L)$

(6)

3. A point is to be randomly plotted on the x-axis, where the units are measured in cm.

The random variable R represents the x coordinate of the point on the x-axis and R is uniformly distributed over the interval $[-5, 19]$

A negative value indicates that the point is to the left of the origin and a positive value indicates that the point is to the right of the origin.

(a) Find the exact probability that the point is plotted to the right of the origin.

(1)

(b) Find the exact probability that the point is plotted more than 3.5 cm away from the origin.

(2)

(c) Sketch the cumulative distribution function of R

(2)

Three independent points with x coordinates R_1, R_2 and R_3 are plotted on the x-axis.

(d) Find the exact probability that

(i) all three points are more than 10 cm from the origin

(3)

(ii) the point furthest from the origin is more than 10 cm from the origin.

(2)

Question 3 continued

7. (i) The continuous random variable X is uniformly distributed over the interval $[a, b]$

Given that $P(5 < X < 13) = \frac{1}{5}$ and $E(X) = 9$, find $P(3X > a + b)$

(3)

(ii) The continuous random variable Y is uniformly distributed over the interval $[1, c]$

Given that $\text{Var}(Y) = 0.48$, find the exact value of $E(Y^2)$

(4)

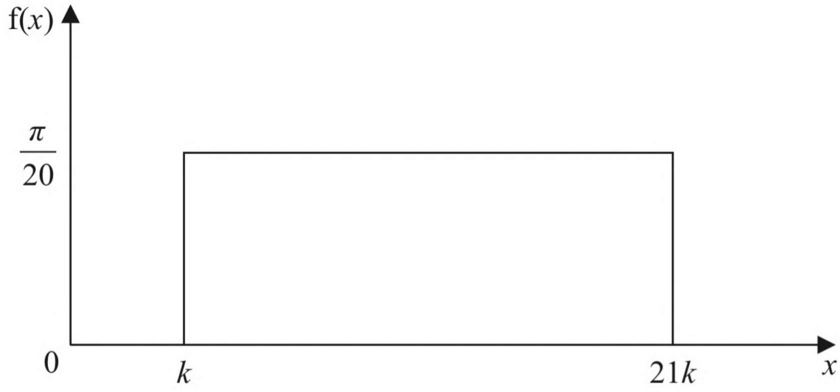
(iii) A wire of length 20 cm is cut into 2 pieces at a random point.
The longest piece of wire is then cut into 2 pieces, equal in length, giving 3 pieces of wire altogether.

Find the probability that the length of the shortest piece of wire is less than 6 cm.

(5)

Question 7 continued

4. The continuous random variable X has probability density function $\mathrm{f}(x)$, shown in the diagram, where k is a constant.



(a) Find $\mathrm{P}(X < 10k)$ **(1)**

(b) Show that $k = \frac{1}{\pi}$ **(2)**

(c) Find, in terms of π, the values of

(i) $\mathrm{E}(X)$

(ii) $\mathrm{Var}(X)$ **(3)**

Circles are drawn with area A, where

$$A = \pi\left(X + \frac{2}{\pi}\right)^2$$

(d) Find $\mathrm{E}(A)$ **(4)**

Question 4 continued

6. Akia selects at random a value from the continuous random variable W, which is uniformly distributed over the interval $[a\ ,\ b]$

The probability that Akia selects a value greater than 17 is $\frac{1}{5}$

The probability that Akia selects a value less than k is $\frac{53}{60}$

(a) Find the probability that Akia selects a value between 17 and k

(2)

It is known that $\mathrm{Var}(W) = 75$

(b) (i) Find the value of a and the value of b

(4)

(ii) Find the value of k

(2)

(c) Find $\mathrm{P}(-5 < W < 5)$

(2)

(d) Find $\mathrm{E}(W^2)$

(2)

Question 6 continued

3. Every morning Navtej travels from home to work. Navtej leaves home at a random time between 08:00 and 08:15

- It always takes Navtej 3 minutes to walk to the bus stop
- Buses run every 15 minutes and Navtej catches the first bus that arrives
- Once Navtej has caught the bus it always takes a further 29 minutes for Navtej to reach work

The total time, T minutes, for Navtej's journey from home to work is modelled by a continuous uniform distribution over the interval $[\alpha, \beta]$

(a) (i) Show that $\alpha = 32$

(ii) Show that $\beta = 47$

(2)

(b) State fully the probability density function for this distribution.

(2)

(c) Find the value of

(i) $\mathrm{E}(T)$

(ii) $\mathrm{Var}(T)$

(3)

(d) Find the probability that the time for Navtej's journey is within 5 minutes of 35 minutes.

(2)

Question 3 continued

(Total for Question 3 is 9 marks)

5. The random variable W has a continuous uniform distribution over the interval $[-6, a]$ where a is a constant.

Given that $\mathrm{Var}(W) = 27$

(a) show that $a = 12$

(2)

Given that $\mathrm{P}(W > b) = \frac{3}{5}$

(b) (i) find the value of b

(2)

(ii) find $\mathrm{P}\left(-12 < W < \frac{b}{2}\right)$

(2)

A piece of wood AB has length 160 cm. The wood is cut at random into 2 pieces. Each of the pieces is then cut in half. The four pieces are used to form the sides of a rectangle.

(c) Calculate the probability that the area of the rectangle is greater than $975\,\mathrm{cm}^2$

(4)

Question 5 continued