

January 2018

Question	Scheme	Marks
3. (a)	$[f(d) =] \begin{cases} \frac{1}{5} & -2.5 \leq d \leq 2.5 \\ 0 & \text{otherwise} \end{cases}$	B1 B1 (2)
(b)	$\sqrt{\frac{(2.5 - (-2.5))^2}{12}} = 1.4433...$ awrt 1.44	M1 A1 (2)
(c)	0	B1 (1)
(d)	$\left[\frac{1 - (-1)}{5} \right] = \frac{2}{5}$	B1 (1)
(e)	$X \sim B(10, '0.4')$ $[P(X \geq 6) =] 1 - P(X \leq 5) = 1 - 0.8338 = 0.1662$ awrt 0.166	M1 M1 A1 (3) Total 9

June 2018

Question Number	Scheme	Marks
3.(a)	$(102 - 100)p = \frac{1}{3} \quad \text{or} \quad \frac{102 - 100}{k - 100} = \frac{1}{3} \quad \text{or} \quad \frac{k - 102}{k - 100} = \frac{2}{3} \quad \text{or} \quad (k - 100)p = 1 \quad (\text{o.e.})$ $p = \frac{1}{6} \quad \text{so} \quad (k - 100)\frac{1}{6} = 1$ $k - 100 = 6 \quad \text{therefore} \quad k = 106 \quad *$	M1 dM1 A1 also (3)
(b)(i)	$\frac{5}{6}$ (or exact equivalent)	B1
(ii)	0	B1 (2)
(c)	103	B1 (1)
(d)	$\frac{r - 100}{6} = 0.15$ $= \mathbf{100.9}$	M1 A1 (2)
(e)	$3P(X \leq x - 1.5) = P(X \geq x + 1.5) \quad \text{so} \quad \frac{3}{6}(x - 1.5 - 100) = \frac{1}{6}(106 - x - 1.5)$ $[3(x - 1.5 - 100) = (106 - x - 1.5)] \text{ implies } 4x - 304.5 = 104.5 \quad (\text{o.e.})$ $x = \mathbf{102.25} \quad (\text{o.e.})$	M1 dM1 A1 (3)

Question	Scheme	Marks
6.(a)	$a = 4 \times (-0.5)$ and $b = 4 \times 0.5$	B1 (1)
(b)(i)	$\sqrt{\frac{(2 - (-2))^2}{12}} = \frac{2\sqrt{3}}{3} = 1.1547...$ awrt <u>1.15</u>	M1 A1
(ii)	$[P(-\frac{2\sqrt{3}}{3} < W < \frac{2\sqrt{3}}{3}) =] \frac{\frac{2\sqrt{3}}{3} - (-\frac{2\sqrt{3}}{3})}{2 - (-2)} = \frac{\sqrt{3}}{3} = 0.57735...$ awrt <u>0.577</u>	M1 A1 (4)
(c)	$P(W > 1.9) = \frac{2-1.9}{2-(-2)} [=0.025]$ $X \sim B(100, 0.025)$ $\rightarrow \text{Po}(2.5) \quad P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.8912 = 0.1088$ awrt <u>0.109</u>	M1 M1 M1 A1 (4) Total 9

January 2019

Question Number	Scheme	Marks
5. (a)(i)	$\frac{(b-2a)-a}{b-a} = \frac{1}{3}$ $3(b-3a) = (b-a)$ $b = 4a$ $E(X) = \frac{a+4a}{2} = \frac{5a}{2} *$ or $\frac{b-(b-2a)}{b-a} = \frac{2}{3}$ $6a = 2(b-a)$ $b = 4a$ $E(X) = \frac{a+4a}{2} = \frac{5a}{2} *$ oe	M1 M1A1cso (3)
(ii)	$[P(X > b-4a) = P(X > 0) =] \quad 1$	B1 (1)
(b)(i)	$\frac{(c-3)^2}{12} = 3c-9$ $c^2-42c+117=0 \rightarrow (c-3)(c-39)=0$ $\underline{c=39}$	M1 M1 A1 (3)
(ii)	$P(2Y-7 < 20-Y) = P(Y < 9)$ $\frac{9-3}{'39'-3} = \frac{1}{6}$ awrt 0.167	M1 A1ft A1 (3)
(iii)	$E(Y^2) = \text{Var}(Y) + [E(Y)]^2 = (3('39')-9) + \left(\frac{'39'+3}{2}\right)^2$ $E(Y^2) = \underline{549}$	M1 A1ft A1 (3) [13 marks]

Question Number	Scheme	Marks
3(a)	$\frac{a+b}{2} = 6 \quad \frac{1}{12}(b-a)^2 = 3$ $a+b=12$ and $b-a = \pm 6$ or $a^2 - 12a + 27 = 0$ or $b^2 - 12b + 27 = 0$ $a = 3, b = 9$	B1 dM1 A1 (3)
(b)	$[P(Y > 6 + \sqrt{3}) =] \frac{"9" - (6 + \sqrt{3})}{"9" - "3"}$ $= \frac{3 - \sqrt{3}}{6}$ or 0.211(32...) awrt 0.211	M1 A1 (2)
(c)	$[F(y) =] \begin{cases} 0 & y < 3 \\ \frac{y-3}{6} & 3 \leq y \leq 9 \\ 1 & y > 9 \end{cases}$	B1 B1 (2)

Question	Scheme	Marks
2(a)	$\frac{x - (-3)}{12 - (-3)} = 0.75$ $x = 8.25$ 8.25 oe	M1 A1 (2)
(b)	$P(5 \leq X < 14) = P(5 \leq X < 12) = \frac{12 - (5)}{12 - (-3)} = \frac{7}{15}$	M1 A1 (2)
(c)	$E(X) = 4.5 \rightarrow E(Y) = 7.5$ $\text{Var}(X) = \frac{(12 - (-3))^2}{12} [= \frac{75}{4} \text{ or } 18.75]$ or $\text{Var}(Y) = \frac{(12 - (-3))^2}{48} [= \frac{75}{16} \text{ or } 4.6875]$ $\frac{a+b}{2} = '7.5'$ and $\frac{(b-a)^2}{12} = '4.6875'$ or $(b-a)^2 = \frac{225}{4}$ $(b - (15 - b))^2 = 56.25$ oe $a = 3.75$ $b = 11.25$	B1 B1 M1 dM1 A1 (5) Total [9]

Question	Scheme	Marks
6(a)	0.95	B1 (1)
(b)	$X \sim B(10, '0.95')$ $P(X \geq 9) = P(X = 9) + P(X = 10) = 10(0.95)^9(0.05) + 0.95^{10}$ 0.91386... awrt <u>0.914</u>	B1 M1 A1 (3)
(c)	$Y = \text{Number of bolts that cannot be used}$ $Y \sim B(120, 0.05)$ can be approximated by Po(6) $P(\text{more than 117 bolts can be used}) = P(Y \leq 2)$ $P(Y \leq 2) = 0.06196...$ awrt <u>0.062</u>	M1A1 dM1 A1 (4) Total [8]

January 2020

Question Number	Scheme	Marks
3(a)	$\frac{3}{4}$	B1 (1)
(b)	$E(T) = \frac{50 + 2k}{2} [= 25 + k]$ $\text{Var}(T) = \frac{(4k)^2}{12} \left[= \frac{4k^2}{3} \right]$ $E(T^2) = \frac{4k^2}{3} + (25 + k)^2$ $\frac{7k^2}{3} + 625 + 50k = 918.76$ $7k^2 + 150k - 881.28 = 0$ $k = \frac{-150 \pm \sqrt{150^2 + 4 \times 7 \times 881.28}}{14}$ $k = 4.8$ oe only	B1 B1 M1 dM1 dM1 A1 (6)
(c)	$P(T < 25) = \frac{1}{4}$ $B(50, 0.25)$ $P(X \geq 20) = 1 - P(X \leq 19)$ $= 1 - 0.9861$ $= 0.0139$ awrt 0.0139	B1 M1 A1 (3)

Question Number	Scheme	Marks
2(a)	$f(w) = \begin{cases} \frac{1}{8} & -1.4 < w < 6.6 \\ 0 & \text{otherwise} \end{cases}$	M1 A1
		(2)
(b)	$E(W) = 2.6$ oe	B1
		(1)
(c)	$(1.6 - \alpha) \times \frac{1}{8} = 0.35$	M1
	$\alpha = -1.2$ oe	A1 cso
		(2)
(d)	$P(1.2 < W < 2.4) = (2.4 - 1.2) \times \frac{1}{8}$	M1
	$= \frac{3}{20}$ or 0.15 oe	A1 ft
		(2)
(e)	$P(W > 2 \mid 1.2 < W < 2.4) = \frac{0.4 \times \frac{1}{8}}{0.15}$	M1
	$= \frac{1}{3}$ awrt 0.333	A1
		(2)
(f)	The random variable Y is the number of days the train is between 1.2 minutes and 2.4 minutes late $Y \sim B(40, 0.15)$	M1
	$P(Y \geq 10) = 1 - P(Y \leq 9)$ or $1 - 0.9328$	M1
	$= 0.0672$ awrt 0.0672	A1 (3)

January 2021

Question Number	Scheme	Marks
5(a)	$U[0, 3]$	M1
	$\frac{3 - 1.8}{3} = 0.4$	A1
		(2)
(b)	$X^2 = W^2 + (3 - W)^2$	M1
	$X^2 = W^2 + 9 + W^2 - 6W \Rightarrow X^2 = 2W^2 - 6W + 9$	A1
		(2)
(c)	$E(W) = 1.5$	B1
	$\text{Var}(W) = \frac{9}{12} = \frac{3}{4}$	B1
	$E(W^2) = \frac{3}{4} + 1.5^2$	M1
	$E(W^2) = 3$	A1
	So $E(X^2) = 2 \times 3 - 6 \times 1.5 + 9 = 6$	M1A1
		(6)
(d)	$P(X^2 > 5) = P(2W^2 - 6W + 4 > 0)$	M1
	$= P((2W - 2)(W - 2) > 0)$	M1
	$= P(W > 2) + P(W < 1)$	dM1
	$= \frac{2}{3}$ oe	A1
		(4)

Question Number	Scheme	Marks
2(i)(a)	$P(X > 14) = \frac{2}{5}$ oe	B1
		(1)
	(b) $a = 8 - 2(14 - 8) [= -4]$	M1
	$b = 14 + 2(14 - 8) [= 26]$	M1
	$P(6X > a + b) = \left(\frac{26 - \frac{26 - 4}{6}}{26 + 4} \right)$ oe	M1
	$= \frac{67}{90}$ oe awrt 0.744	A1
		(4)
	(ii)(a) $S \sim U[0, 22.5]$ or $f(s) = \begin{cases} \frac{2}{45} & 0 \leq s \leq 22.5 \\ 0 & \text{otherwise} \end{cases}$	B1
		(1)
	(b) $P(S < 12) = \frac{12}{22.5}$	M1
(c)	$= \frac{8}{15}$ awrt 0.533	A1
		(2)
	$P(T = 6) = {}^{20}C_6 \left(\frac{8}{15} \right)^6 \left(1 - \frac{8}{15} \right)^{14}$	M1M1
	$= 0.02072 \dots$ awrt 0.0207	A1
		(3)

Question Number	Scheme		Marks
7 (a)	$P(L \geq 4.5) \Rightarrow P(A \geq 20.25)$		
	$P(A \geq 20.25) = (30 - 20.25) \times \frac{1}{20}$		M1
	$= 0.4875$		A1
			(2)
(b)	$\text{Var}(L) = E(L^2) - E(L)^2$		
	$[E(L^2) = E(A)] = 20$		B1
		$g(L) = \begin{cases} \frac{L}{10} & \sqrt{10} \leq L \leq \sqrt{30} \\ 0 & \text{otherwise} \end{cases}$	
	$E(L) = E(\sqrt{A}) = \frac{1}{20} \int_{10}^{30} \sqrt{a} \, da$	$E(L) = \frac{1}{10} \int_{\sqrt{10}}^{\sqrt{30}} L^2 \, dL$	M1
	$= \frac{1}{20} \left[\frac{2}{3} a^{\frac{3}{2}} \right]_{10}^{30}$	$\frac{1}{10} \left[\frac{L^3}{3} \right]_{\sqrt{10}}^{\sqrt{30}}$	A1
	$= 4.4231 \dots$		A1
	$\text{Var}(L) = "20" - ("4.4231 \dots")^2$		M1
	$= 0.4358 \dots$	awrt 0.436	A1
			(6)

Question	Scheme	Marks
3. (a)	$\frac{19}{24}$	B1
		(1)
(b)	$P(R > 3.5) = \frac{-3.5 - (-5)}{19 - (-5)} + \frac{19 - 3.5}{19 - (-5)} = \frac{17}{24}$	M1, A1
		(2)
(c)		M1 A1
		(2)
(d)(i)	$P(R_1 > 10) = \frac{19 - 10}{19 - (-5)} \left[= \frac{9}{24} = 0.375 \right]$	M1
	$[P(R > 10)]^3 = \left(\frac{9}{24} \right)^3 = \frac{27}{512}$	M1 A1
		(3)
(ii)	$1 - [P(R < 10)]^3 = \frac{387}{512}$	M1 A1
		(2)

Question Number	Scheme	Marks
7(i)	$\frac{a+b}{2} = 9$ and $\frac{13-5}{b-a} = \frac{1}{5}$ or $a = -11$ and $b = 29$	M1
	$P\left(X > \frac{29 - (-11)}{3}\right) [= P(X > 6)]$ or $P\left(X > \frac{9 \times 2}{3}\right) [= P(X > 6)] = \frac{23}{40}$	M1A1
		(3)
(ii)	$\frac{1}{12}(c-1)^2 = 0.48 \Rightarrow c = 3.4$	M1
	$E(Y) = \frac{1 + 3.4}{2}$	M1
	$E(Y^2) = 0.48 + \left(\frac{1+3.4}{2}\right)^2 = 5.32$	M1A1
		(4)
(iii)	$W \sim U[0,20]$ $X \sim U[10,20]$ $Y \sim U[0,10]$	
	$W < 6$ or $W > 14$ or $X < 12$ or $Y > 8$ any letter, ignore distribution	M1
	$2 \times P(W < 6) = 2 \times \frac{(6-0)}{20}$ or $2 \times P(W > 14) = 2 \times \frac{(20-14)}{20}$ or	M1
	$P(W < 6) = \frac{6-0}{20}$ and $P(W > 14) = \frac{20-14}{20}$ or $P(X > 14) = \frac{20-14}{10}$ or $P(Y < 6) = \frac{6-0}{10}$	
	$P(8 < W < 12) = \frac{12-8}{20}$ or $P(X < 12) = \frac{12-10}{10}$ or $P(Y > 8) = \frac{10-8}{10}$	M1
	$P(\text{shortest side} < 6) = \frac{3}{10} + \frac{3}{10} + \frac{1}{5}$ or $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$	dM1A1
		(5)
	Alternative 1	
	$6 < W < 8$ or $12 < W < 14$ any letter, ignore distribution	M1
	$P(6 < W < 8) = \frac{8-6}{20}$ or $P(12 < W < 14) = \frac{14-12}{20}$	M2 for $P(12 < X < 14) = \frac{14-12}{10}$ or $P(6 < Y < 8) = \frac{8-6}{10}$
	$P(6 < W < 8) = \frac{1}{10}$ and $P(12 < W < 14) = \frac{1}{10}$ or	
	$2 \times P(6 < W < 8) = 2 \times \frac{1}{10}$ or $2 \times P(12 < W < 14) = 2 \times \frac{1}{10}$	M1
	$P(\text{shortest side} < 6) = 1 - \frac{1}{10} - \frac{1}{10}$ or $1 - \frac{2}{10} = \frac{4}{5}$	dM1A1
	Alternative 2	(5)
	$W > 10$ or $12 < W < 14$ or $6 < W < 8$ or $W < 10$ any letter, ignore distribution	M1
	$P(12 < W < 14) = \frac{14-12}{20}$ and $P(W > 10) = \frac{20-10}{20}$ or	M1
	$P(6 < W < 8) = \frac{8-6}{20}$ and $P(W < 10) = \frac{10-0}{20}$	
	$P(12 < W < 14 W > 10) = \frac{\frac{1}{10}}{\frac{1}{2}} \left[= \frac{1}{5} \right]$ or $P(6 < W < 8 W < 10) = \frac{\frac{1}{10}}{\frac{1}{2}} \left[= \frac{1}{5} \right]$	M1
	$P(\text{shortest side} < 6) = 1 - \frac{1}{5} = \frac{4}{5}$	dM1A1
		(5)
	NB Any answer of $\frac{4}{5}$ scores 5/5 provided it has not come from incorrect working	

Question Number	Scheme		Marks
4 (a)	$\frac{9}{20}$		B1
			(1)
(b)	$(21k - k) \times \frac{\pi}{20} = 1$		M1
	$k = \frac{1}{\pi} *$		A1*
			(2)
(c) (i)	$\left[E(X) = \frac{1}{2}(k + 21k) \right] = \frac{11}{\pi}$		B1
(ii)	$\text{Var}(X) = \frac{1}{12}(21k - k)^2$	or $\text{Var}(X) = \int_{\frac{1}{\pi}}^{\frac{21}{\pi}} \frac{\pi}{20} x^2 dx - \left(\frac{11}{\pi} \right)^2$	M1
	$= \frac{100}{3\pi^2}$		A1
			(3)
(d)	$E(A) = \pi E(X^2) + 4E(X) + \frac{4}{\pi}$	$E(A) = \int_k^{21k} f(x)(A) dx = \int_k^{21k} \frac{\pi}{20} (\pi) \left(x^2 + \frac{4}{\pi} x + \frac{4}{\pi^2} \right) dx$	M1
	$E(X^2) = \frac{100}{3\pi^2} + \left(\frac{11}{\pi} \right)^2 = \frac{463}{3\pi^2}$	$E(A) = \frac{\pi}{20} (\pi) \left(\frac{x^3}{3} + \left(\frac{4}{\pi} \right) \frac{x^2}{2} + \frac{4}{\pi^2} x \right)$	M1
	$E(A) = \frac{463}{3\pi} + \frac{44}{\pi} + \frac{4}{\pi}$	sub limits $\frac{21}{\pi}$ and $\frac{1}{\pi}$	M1
	$= \frac{607}{3\pi}$	$= \text{awrt } 64.4$	A1
			(4)

Question Number	Scheme	Marks
6. (a)	$P(17 < W < k) = P(W < k) - P(W < 17) = \frac{53}{60} - \left(1 - \frac{1}{5}\right) = \frac{1}{12}$	M1 A1 (2)
(b)(i)	$\frac{(b-a)^2}{12} = 75$, $\frac{b-17}{b-a} = \frac{1}{5}$ or $\frac{17-a}{b-a} = \frac{4}{5}$ $\frac{(b-a)^2}{12} = 75 \rightarrow (b-a) = 30$ $\frac{b-17}{30} = \frac{1}{5}$ $b = 23$ and $a = -7$	B1, B1 M1 A1 (4)
(ii)	$P(W < k) = \frac{k - (-7)}{23 - (-7)} = \frac{53}{60}$ or $P(17 < W < k) = \frac{k-17}{30} = \frac{1}{12}$ or $P(W > k) = \frac{23-k}{23 - (-7)} = \frac{7}{60}$ $k = 19.5$	M1 A1 (2)
(c)	$P(-5 < W < 5) = \frac{5 - (-5)}{23 - (-7)} = \frac{1}{3}$	M1A1ft (2)
(d)	$E(W^2) = \text{Var}(W) + E(W)^2 = 75 + \left(\frac{23 + (-7)}{2}\right)^2 = 139$	M1 A1 (2)
		[Total 12]

Question Number	Scheme	Marks
3 (a)(i)	$3 + [0] + 29 = 32^*$	B1*
(ii)	$3 + 15 + 29 = 47^*$	B1*
		(2)
(b)	$f(t) = \begin{cases} \frac{1}{15} & 32 \leq t \leq 47 \\ 0 & \text{otherwise} \end{cases}$	M1 A1
		(2)
(c) (i)	$[E(T) =] 39.5$ oe	B1
(ii)	$[\text{Var}(T) =] \frac{(47-32)^2}{12}$	M1
	$\frac{75}{4} = 18.75$	A1
		(3)
(d)	$(40-32) \times \frac{1}{15}$	M1
	$= \frac{8}{15}$	A1
		(2)

Question Number	Scheme		Marks
5(a)	$\frac{(a+6)^2}{12} = 27$		M1
	$a = \sqrt{27 \times 12} - 6 \Rightarrow 12^*$ or $a^2 + 12a - 288 = 0 \Rightarrow a = 12^*$		A1*
			(2)
(b)(i)	$\frac{12-b}{18} = \frac{3}{5}$ or $\frac{b+6}{18} = \frac{2}{5}$		M1
	$b = 1.2$		A1
			(2)
(ii)	$P(-6 < W < "0.6") = \frac{"0.6"+6}{18}$		M1
	$= \frac{11}{30}$ or 0.3666....		A1ft
			(2)
(c)	Let C be the point where the wood is cut and x is the distance AC		
	$\frac{x}{2}$ and $\left(\frac{160-x}{2}\right)$	$L+W=80$ and $LW=975$	M1
	$\frac{x}{2} \times \left(\frac{160-x}{2}\right) = 975 \Rightarrow x = 30$ or 130	$L(80-L) = 975 \Rightarrow L = 15$ or 65	M1
	$P("30" < x < "130") = \frac{"130"- "30"}{160} \left[= \frac{5}{8} \right]$ oe	$P("15" < x < "65") = \frac{"65"- "15"}{80} \left[= \frac{5}{8} \right]$ oe	dM1
	$= \frac{5}{8}$ oe		A1
		(4)	