# Chapter 4: Continuous Random Variables

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S2 Chapter 4 Solutions

**1.** A continuous random variable $X$ has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{16}(x-1)^2 & 1 \leqslant x \leqslant 5 \\ 1 & x > 5 \end{cases}$$

(a) Find $P(X > 4)$

**(2)**

(b) Find $P(X > 3 \mid 2 < X < 4)$

**(3)**

(c) Find the exact value of $E(X)$

**(4)**

**Question 1 continued**

**(Total 9 marks)**

**7.** The continuous random variable $X$ has probability density function f($x$) given by

$$f(x) = \begin{cases} \dfrac{1}{16}x^2 & 1 \leqslant x < 3 \\ k(4 - x) & 3 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \dfrac{11}{12}$ **(3)**

(b) Sketch f($x$) for $1 \leqslant x \leqslant 4$ **(2)**

(c) Write down the mode of $X$ **(1)**

Given that $\mathrm{E}(X) = \dfrac{25}{9}$

(d) use algebraic integration to find Var($X$), giving your answer to 3 significant figures. **(4)**

The cumulative distribution function of $X$ is given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \dfrac{1}{48}(x^3 + c) & 1 \leqslant x < 3 \\ \dfrac{11}{12}(4x - \dfrac{1}{2}x^2 + d) & 3 \leqslant x \leqslant 4 \\ 1 & x > 4 \end{cases}$$

(e) (i) Find the exact value of $c$

(ii) Find the exact value of $d$ **(3)**

(f) Calculate, to 3 significant figures, the upper quartile of $X$ **(2)**

**Question 7 continued**

**6.** A random variable $X$ has probability density function given by

$$f(x) = \begin{cases} \frac{1}{4} & 0 \leqslant x < 1 \\ \frac{x^3}{5} & 1 \leqslant x \leqslant 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Use algebraic integration to find E($X$) **(3)**

(b) Use algebraic integration to find Var($X$) **(3)**

(c) Define the cumulative distribution function F($x$) for all values of $x$. **(4)**

(d) Find the median of $X$, giving your answer to 3 significant figures. **(2)**

(e) Comment on the skewness of the distribution, justifying your answer. **(2)**

**Question 6 continued**

**3.** The function $\mathrm{f}(x)$ is defined as

$$\mathrm{f}(x) = \begin{cases} \frac{1}{9}(x+5)(3-x) & 1 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

Albert believes that $\mathrm{f}(x)$ is a valid probability density function.

(a) Sketch $\mathrm{f}(x)$ and comment on Albert's belief.

**(3)**

The continuous random variable $Y$ has probability density function given by

$$\mathrm{g}(y) = \begin{cases} ky(12-y^2) & 1 \leqslant y \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where $k$ is a positive constant.

(b) Use calculus to find the mode of $Y$

**(3)**

(c) Use algebraic integration to find the value of $k$

**(3)**

(d) Find the median of $Y$ giving your answer to 3 significant figures.

**(3)**

(e) Describe the skewness of the distribution of $Y$ giving a reason for your answer.

**(2)**

Question 3 continued

**5.** The random variable $X$ has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{1}{100}(ax^3 + bx^2 + 15x) & 0 \leqslant x \leqslant 5 \\ 1 & x > 5 \end{cases}$$

Given that $E(X^2) = 6.25$

(a) show that $6a + b = 0$

**(5)**

(b) find the value of $a$ and the value of $b$

**(4)**

(c) find $P(3 \leqslant X \leqslant 7)$

**(2)**

**Question 5 continued**

**3.** Figure 1 shows an accurate graph of the cumulative distribution function, $\mathrm{F}(x)$, for the continuous random variable $X$

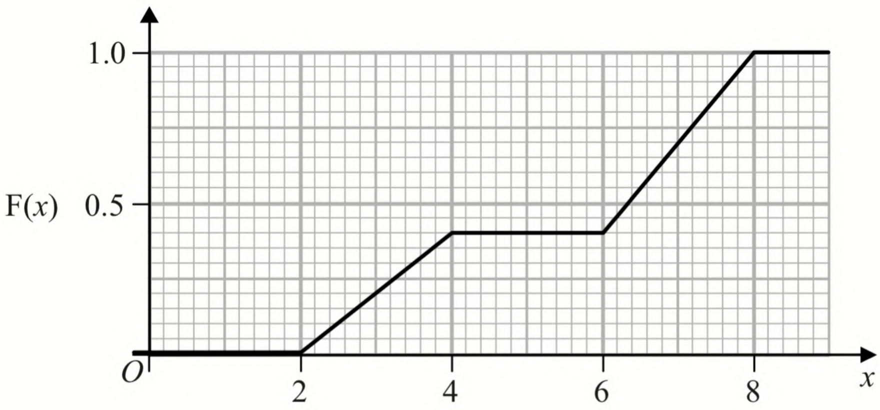


**Figure 1**

(a) Find $\mathrm{P}(3 < X < 7)$

**(2)**

The probability density function of $X$ is given by

$$\mathrm{f}(x) = \begin{cases} a & 2 \leqslant x < 4 \\ b & 4 \leqslant x < 6 \\ c & 6 \leqslant x \leqslant 8 \\ 0 & \text{otherwise} \end{cases}$$

where $a$, $b$ and $c$ are constants.

(b) Find the value of $a$, the value of $b$ and the value of $c$

**(3)**

(c) Find $\mathrm{E}(X)$

**(3)**

Question 3 continued

**(Total 8 marks)**

**7.** The continuous random variable $X$ has probability density function

$$f(x) = \begin{cases} c(x+3) & -3 \leqslant x < 0 \\ \dfrac{5}{36}(3-x) & 0 \leqslant x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where $c$ is a positive constant.

(a) Show that $c = \dfrac{1}{12}$

**(3)**

(b) (i) Sketch the probability density function.

(ii) Explain why the mode of $X = 0$

**(3)**

(c) Find the cumulative distribution function of $X$, for all values of $x$

**(4)**

(d) Find, to 3 significant figures, the value of $d$ such that $P(X > d \mid X > 0) = \dfrac{2}{5}$

**(4)**

**Question 7 continued**

**5.** A continuous random variable $X$ has probability density function

$$f(x) = \begin{cases} k\{(x-2)^2 + 1\} & 0 \leqslant x \leqslant a \\ 0 & \text{otherwise} \end{cases}$$

where $a$ and $b$ are constants.
Given that the mode of $X$ is 0

(a) find the range of possible values for $a$.

**(2)**

Given that $a = 3$

(b) (i) sketch the probability density function for all values of $x$.

(ii) Give the coordinates of the turning point.

**(2)**

(c) Use algebraic integration to show that $k = \frac{1}{6}$

**(4)**

Given that $E(X^2) = 2.1$

(d) use algebraic integration to find $\text{Var}(X)$.

**(6)**

(e) Show that $P(2 < X < 3) = \frac{2}{9}$

**(2)**

(f) State, giving a reason, whether the median of $X$ is less than or greater than 1

**(3)**

**Question 5 continued**

**7.** A continuous random variable $X$ has probability density function

$$f(x) = \begin{cases} \dfrac{3}{4}(x-1) & 1 \leqslant x < 2 \\ \dfrac{3}{32}x(x-4)^2 & 2 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the cumulative distribution function for all values of $x$.

**(7)**

(b) Verify that the median of $X$ is 2.17 to 2 decimal places.

**(2)**

**Question 7 continued**

**4.** The continuous random variable $X$ has probability density function given by

$$f(x) = \begin{cases} \dfrac{1}{15}x^2(3 - x) & 1 \leqslant x < 3 \\ \dfrac{3}{10}(x - 3) & 3 \leqslant x \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Use algebraic integration to find $E(X^2)$

**(4)**

(b) Define fully the cumulative distribution function $F(x)$

**(5)**

(c) Find $P(2 < X < 4)$

**(2)**

(d) Find, to 3 significant figures, the value of $k$ such that $P(X > k) = 0.2$

**(3)**

**Question 4 continued**

**7.** Jonathan believes that the time, in minutes, people spend queuing for lunch at a particular fast food restaurant can be modelled using the continuous random variable $X$, which has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{1}{3125}(25x^4 - 4x^5) & 0 \leqslant x \leqslant 5 \\ 1 & x > 5 \end{cases}$$

(a) Find the mode of $X$.

**(4)**

(b) Show that the upper quartile of $X$ is 4.0 to one decimal place.

**(3)**

**Question 7 continued**

4. The random variable $T$ has probability density function $\mathrm{f}(t)$ where

$$\mathrm{f}(t) = \begin{cases} \dfrac{1}{3} & 1 \leqslant t < 2 \\ k(4t^2 - t^3) & 2 \leqslant t \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \dfrac{1}{22}$
**(3)**

(b) Sketch the probability density function.
**(2)**

(c) Find the mode of $T$.
**(3)**

(d) Find the cumulative distribution function $\mathrm{F}(t)$ for all values of $t$.
**(5)**

(e) Find the probability that $T$ is greater than 3
**(2)**

**Question 4 continued**

**6.** The random variable $X$ has probability density function f($x$) where

$$f(x) = \begin{cases} \frac{1}{8}(x^2 + 2x + 1) & -1 \leqslant x < 1 \\ \frac{1}{4} & 1 \leqslant x \leqslant \frac{11}{3} \\ 0 & \text{otherwise} \end{cases}$$

Given that $E(X) = \frac{31}{18}$

(a) use algebraic integration to find Var($X$)

**(4)**

(b) Calculate $P\left(X < -\frac{1}{2}\right) + P\left(X > \frac{1}{2}\right)$

**(4)**

**Question 6 continued**

**1.**

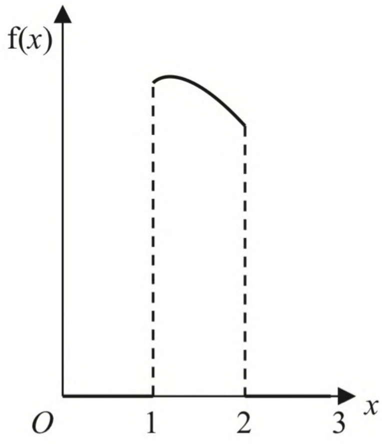


**Figure 1**

Figure 1 shows a sketch of the probability density function f($x$) of the random variable $X$.

For $1 \leqslant x \leqslant 2$, f($x$) is represented by a curve with equation $f(x) = k\left(\frac{1}{2}x^3 - 3x^2 + ax + 1\right)$ where $k$ and $a$ are constants.

For all other values of $x$, $f(x) = 0$

(a) Use algebraic integration to show that $k(12a - 33) = 8$

**(4)**

Given that $a = 5$

(b) calculate the mode of $X$.

**(4)**

**Question 1 continued**

**5.** The waiting time, $T$ minutes, of a customer to be served in a local post office has probability density function

$$f(t) = \begin{cases} \dfrac{1}{50}(18 - 2t) & 0 \leqslant t \leqslant 3 \\ \dfrac{1}{20} & 3 < t \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

Given that the mean number of minutes a customer waits to be served is 1.66

(a) use algebraic integration to find Var($T$), giving your answer to 3 significant figures. **(5)**

(b) Find the cumulative distribution function F($t$) for all values of $t$. **(4)**

(c) Calculate the probability that a randomly chosen customer's waiting time will be more than 2 minutes. **(2)**

(d) Calculate $\mathrm{P}([\mathrm{E}(T) - 2] < T < [\mathrm{E}(T) + 2])$ **(2)**

Question 5 continued

**2.** The distance, in metres, a novice tightrope artist, walking on a wire, walks before falling is modelled by the random variable $W$ with cumulative distribution function

$$\mathrm{F}(w) = \begin{cases} 0 & w < 0 \\ \dfrac{1}{3}\left(w - \dfrac{w^4}{256}\right) & 0 \leqslant w \leqslant 4 \\ 1 & w > 4 \end{cases}$$

(a) Find the probability that a novice tightrope artist, walking on the wire, walks at least 3.5 metres before falling.

**(2)**

A random sample of 30 novice tightrope artists is taken.

(b) Find the probability that more than 1 of these novice tightrope artists, walking on the wire, walks at least 3.5 metres before falling.

**(3)**

Given $\mathrm{E}(W) = 1.6$

(c) use algebraic integration to find $\mathrm{Var}(W)$

**(5)**

**Question 2 continued**

**4.** A continuous random variable $X$ has probability density function

$$f(x) = \begin{cases} k(a-x)^2 & 0 \leqslant x \leqslant a \\ 0 & \text{otherwise} \end{cases}$$

where $k$ and $a$ are constants.

(a) Show that $ka^3 = 3$

**(3)**

Given that $E(X) = 1.5$

(b) use algebraic integration to show that $a = 6$

**(4)**

(c) Verify that the median of $X$ is 1.2 to one decimal place.

**(3)**

**Question 4 continued**

**3.** The continuous random variable $Y$ has the following probability density function

$$f(y) = \begin{cases} \dfrac{6}{25}(y-1) & 1 \leqslant y < 2 \\ \dfrac{3}{50}(4y^2 - y^3) & 2 \leqslant y < 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch $f(y)$

**(2)**

(b) Find the mode of $Y$

**(3)**

(c) Use algebraic integration to calculate $E(Y^2)$

**(4)**

Given that $E(Y) = 2.696$

(d) find $\text{Var}(Y)$

**(2)**

(e) Find the value of $y$ for which $P(Y \geqslant y) = 0.9$
Give your answer to 3 significant figures.

**(4)**

Question 3 continued

**5.** A game uses two turntables, one red and one yellow. Each turntable has a point marked on the circumference that is lined up with an arrow at the start of the game. Jim spins both turntables and measures the distance, in metres, each point is from the arrow, around the circumference in an anticlockwise direction when the turntables stop spinning.

The continuous random variable $Y$ represents the distance, in metres, the point is from the arrow for the yellow turntable. The cumulative distribution function of $Y$ is given by $\mathrm{F}(y)$ where

$$\mathrm{F}(y) = \begin{cases} 0 & y < 0 \\ 1 - (\alpha + \beta y^2) & 0 \leqslant y \leqslant 5 \\ 1 & y > 5 \end{cases}$$

(a) Explain why (i) $\alpha = 1$

(ii) $\beta = -\dfrac{1}{25}$

**(2)**

(b) Find the probability density function of $Y$

**(2)**

The continuous random variable $R$ represents the distance, in metres, the point is from the arrow for the red turntable. The distribution of $R$ is modelled by a continuous uniform distribution over the interval $[d, 3d]$

Given that $\mathrm{P}\left(R > \dfrac{11}{5}\right) = \mathrm{P}\left(Y > \dfrac{5}{3}\right)$

(c) find the value of $d$

**(3)**

In the game each turntable is spun 3 times. The distance between the point and the arrow is determined for each spin. To win a prize, at least 5 of the distances the point is from the arrow when a turntable is spun must be less than $\dfrac{11}{5}$ m

Jo plays the game once.

(d) Calculate the probability of Jo winning a prize.

**(4)**

**Question 5 continued**

**3.** A continuous random variable $X$ has cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0 \\ 4ax^2 & 0 \leqslant x \leqslant 1 \\ a\left(bx^3 - x^4 + 1\right) & 1 < x \leqslant 3 \\ 1 & x > 3 \end{cases}$$

where $a$ and $b$ are positive constants.

(a) Show that $b = 4$

**(1)**

(b) Find the exact value of $a$

**(2)**

(c) Find $\mathrm{P}(X > 2.25)$

**(2)**

(d) Showing your working clearly,

(i) sketch the probability density function of $X$

(ii) calculate the mode of $X$

**(5)**

**Question 3 continued**

**6.** The continuous random variable $Y$ has probability density function f($y$) given by

$$\mathrm{f}(y) = \begin{cases} \dfrac{1}{14}(y+2) & -1 < y \leqslant 1 \\ \dfrac{3}{14} & 1 < y \leqslant 3 \\ \dfrac{1}{14}(6-y) & 3 < y \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the probability density function f($y$)

**(2)**

Given that $\mathrm{E}(Y^2) = \dfrac{131}{21}$

(b) find $\mathrm{Var}(2Y - 3)$

**(4)**

The cumulative distribution function of $Y$ is F($y$)

(c) Show that $\mathrm{F}(y) = \dfrac{1}{14}\left(\dfrac{y^2}{2} + 2y + \dfrac{3}{2}\right)$ for $-1 < y \leqslant 1$

**(2)**

(d) Find F($y$) for all values of $y$

**(5)**

(e) Find the exact value of the 30th percentile of $Y$

**(2)**

(f) Find $\mathrm{P}(4Y \leqslant 5 \mid Y \leqslant 3)$

**(2)**

**Question 6 continued**

**2** The continuous random variable $X$ has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < -k \\ \dfrac{x+k}{4k} & -k \leqslant x \leqslant 3k \\ 1 & x > 3k \end{cases}$$

where $k$ is a positive constant.

(a) Specify fully, in terms of $k$, the probability density function of $X$

**(2)**

(b) Write down, in terms of $k$, the value of $E(X)$

**(1)**

(c) Show that $\mathrm{Var}(X) = \dfrac{4}{3}k^2$

**(2)**

(d) Find, in terms of $k$, the value of $E(3X^2)$

**(3)**

**Question 2 continued**

**4** The continuous random variable $X$ has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}k(x-1) & 1 \leqslant x \leqslant 3 \\ k & 3 < x \leqslant 6 \\ \frac{1}{4}k(10-x) & 6 < x \leqslant 10 \\ 0 & \text{otherwise} \end{cases}$$

where $k$ is a positive constant.

(a) Sketch $f(x)$ for all values of $x$

**(2)**

(b) Show that $k = \frac{1}{6}$

**(2)**

(c) Specify fully the cumulative distribution function $F(x)$ of $X$

**(7)**

Given that $E(X) = \frac{61}{12}$

(d) find $P(X > E(X))$

**(2)**

(e) Describe the skewness of the distribution, giving a reason for your answer.

**(2)**

Question 4 continued

**2.** The time, in minutes, spent waiting for a call to a call centre to be answered is modelled by the random variable $T$ with probability density function

$$f(t) = \begin{cases} \frac{1}{192}\left(t^3 - 48t + 128\right) & 0 \leqslant t \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Use algebraic integration to find, in minutes and seconds, the mean waiting time.

**(3)**

(b) Show that $P(1 < T < 3) = \frac{7}{16}$

**(3)**

A supervisor randomly selects 256 calls to the call centre.

(c) Use a suitable approximation to find the probability that more than 125 of these calls take between 1 and 3 minutes to be answered.

**(5)**

**Question 2 continued**

**6.** The continuous random variable $X$ has probability density function

$$f(x) = \begin{cases} 0.1x & 0 \leqslant x < 2 \\ kx(8-x) & 2 \leqslant x < 4 \\ a & 4 \leqslant x < 6 \\ 0 & \text{otherwise} \end{cases}$$

where $k$ and $a$ are constants.

It is known that $P(X < 4) = \dfrac{31}{45}$

(a) Find the exact value of $k$

**(4)**

(b) (i) Find the exact value of $a$

(ii) Find the exact value of $P(0 \leqslant X \leqslant 5.5)$

**(3)**

(c) Specify fully the cumulative distribution function of $X$

**(6)**

**Question 6 continued**

**2.** A random variable $X$ has probability density function given by

$$f(x) = \begin{cases} \frac{1}{4} & -\frac{1}{2} \leqslant x < \frac{1}{2} \\ 2x - \frac{3}{4} & \frac{1}{2} \leqslant x \leqslant k \\ 0 & \text{otherwise} \end{cases}$$

where $k$ is a positive constant.

(a) Sketch the graph of f($x$)

**(2)**

(b) By forming and solving an equation in $k$, show that $k = 1.25$

**(4)**

(c) Use calculus to find E($X$)

**(4)**

(d) Calculate the interquartile range of $X$

**(5)**

**Question 2 continued**

**5.** The continuous random variable $X$ has cumulative distribution function given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 3 \\ \dfrac{1}{6}(x-3)^2 & 3 \leqslant x < 4 \\ \dfrac{x}{3} - \dfrac{7}{6} & 4 \leqslant x < c \\ 1 - \dfrac{1}{6}(d-x)^2 & c \leqslant x < 7 \\ 1 & x \geqslant 7 \end{cases}$$

where $c$ and $d$ are constants.

(a) Show that $c = 6$

**(4)**

(b) Find $\mathrm{P}(X > 3.5)$

**(2)**

(c) Find $\mathrm{P}(X > 4.5 \mid 3.5 < X < 5.5)$

**(3)**

**Question 5 continued**

**6.** The continuous random variable $X$ has cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0 \\ ax + bx^2 & 0 \leqslant x \leqslant k \\ 1 & x > k \end{cases}$$

where $a$, $b$ and $k$ are positive constants.

(a) Show that $ak = 1 - bk^2$

**(1)**

Using part (a) and given that $\mathrm{E}(X) = \dfrac{6}{5}$

(b) show that $5bk^3 = 36 - 15k$

**(6)**

Using part (a) and given that $\mathrm{E}(X) = \dfrac{6}{5}$ and $\mathrm{Var}(X) = \dfrac{22}{75}$

(c) show that $5bk^4 = 52 - 10k^2$

**(5)**

Given that $k < 3$

(d) find the value of $k$

**(4)**

(e) Hence find the value of $a$ and the value of $b$

**(2)**

**Question 6 continued**

**3.** The continuous random variable $X$ has probability density function given by

$$f(x) = \begin{cases} \frac{1}{48}\left(x^2 - 8x + c\right) & 2 \leqslant x \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $c = 31$

**(3)**

(b) Find $P(2 < X < 3)$

**(2)**

(c) State whether the lower quartile of $X$ is less than 3, equal to 3 or greater than 3
Give a reason for your answer.

**(1)**

Kei does the following to work out the mode of $X$

$$f'(x) = \frac{1}{48}(2x - 8)$$

$$0 = \frac{1}{48}(2x - 8)$$

$$x = 4$$

Hence the mode of $X$ is 4

Kei's answer for the mode is incorrect.

(d) Explain why Kei's method does not give the correct value for the mode.

**(1)**

(e) Find the mode of $X$
Give a reason for your answer.

**(2)**

**Question 3 continued**

**5.** A continuous random variable $Y$ has cumulative distribution function given by

$$\mathrm{F}(y) = \begin{cases} 0 & y < 3 \\ \dfrac{1}{16}\left(y^2 - 6y + a\right) & 3 \leqslant y \leqslant 5 \\ \dfrac{1}{12}\left(y + b\right) & 5 < y \leqslant 9 \\ \dfrac{1}{12}\left(100y - 5y^2 + c\right) & 9 < y \leqslant 10 \\ 1 & y > 10 \end{cases}$$

where $a$, $b$ and $c$ are constants.

(a) Find the value of $a$ and the value of $c$

**(4)**

(b) Find the value of $b$

**(2)**

(c) Find $\mathrm{P}(6 < Y \leqslant 9)$
Show your working clearly.

**(3)**

(d) Specify the probability density function, $\mathrm{f}(y)$, for $5 < y \leqslant 9$

**(1)**

Using the information

$$\int_3^5 (6y - 5)\mathrm{f}(y)\,\mathrm{d}y + \int_9^{10} (6y - 5)\mathrm{f}(y)\,\mathrm{d}y = 26.5$$

(e) find $\mathrm{E}(6Y - 5)$
You should make your method clear.

**(4)**

**Question 5 continued**

**2.** The continuous random variable $X$ has probability density function $f(x)$ given by

$$f(x) = \begin{cases} ax^3 & 0 \leqslant x \leqslant 4 \\ bx + c & 4 < x \leqslant d \\ 0 & \text{otherwise} \end{cases}$$

where $a$, $b$, $c$ and $d$ are constants such that

- $bx + c = ax^3$ at $x = 4$
- $bx + c$ is a straight line segment with end coordinates $(4, 64a)$ and $(d, 0)$

(a) State the mode of $X$

**(1)**

Given that the mode of $X$ is equal to the median of $X$

(b) use algebraic integration to show that $a = \dfrac{1}{128}$

**(2)**

(c) Find the value of $d$

**(2)**

(d) Hence find the value of $b$ and the value of $c$

**(3)**

**Question 2 continued**

**6.** The continuous random variable $Y$ has cumulative distribution function given by

$$\mathrm{F}(y) = \begin{cases} 0 & y < 0 \\ \dfrac{1}{21}y^2 & 0 \leqslant y \leqslant k \\ \dfrac{2}{15}\left(6y - \dfrac{y^2}{2}\right) - \dfrac{7}{5} & k < y \leqslant 6 \\ 1 & y > 6 \end{cases}$$

(a) Find $\mathrm{P}\left(Y < \frac{1}{4}k \mid Y < k\right)$

**(2)**

(b) Find the value of $k$

**(4)**

(c) Use algebraic calculus to find $\mathrm{E}(Y)$

**(6)**

**Question 6 continued**

**4.** The continuous random variable $G$ has probability density function $f(g)$ given by

$$f(g) = \begin{cases} \frac{1}{15}(g+3) & -1 < g \leqslant 2 \\ \frac{3}{20} & 2 < g \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of $f(g)$

**(2)**

(b) Find $P\left((1 \leqslant 2G \leqslant 6) \mid G \leqslant 2\right)$

**(4)**

The continuous random variable $H$ is such that $E(H) = 12$ and $\text{Var}(H) = 2.4$

(c) Find $E\left(2H^2 + 3G + 3\right)$
Show your working clearly.
(*Solutions relying on calculator technology are not acceptable.*)

**(6)**

**Question 4 continued**

**7.** A continuous random variable $X$ has cumulative distribution function $\mathrm{F}(x)$ given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 1 \\ k(ax + bx^3 - x^4 - 4) & 1 \leqslant x \leqslant 2 \\ 1 & x > 2 \end{cases}$$

where $a$, $b$ and $k$ are non-zero constants.

Given that the mode of $X$ is 1.5

(a) show that $b = 3$

**(3)**

(b) Hence show that $a = 2$

**(1)**

(c) Show that the median of $X$ lies between 1.4 and 1.5

**(4)**

**Question 7 continued**

**2** The continuous random variable $H$ has cumulative distribution function given by

$$\mathrm{F}(h) = \begin{cases} 0 & h \leqslant 0 \\ \dfrac{h^2}{48} & 0 < h \leqslant 4 \\ \dfrac{h}{6} - \dfrac{1}{3} & 4 < h \leqslant 5 \\ \dfrac{3}{10}h - \dfrac{h^2}{75} - \dfrac{2}{3} & 5 < h \leqslant d \\ 1 & h > d \end{cases}$$

where $d$ is a constant.

(a) Show that $2d^2 - 45d + 250 = 0$

**(2)**

(b) Find $\mathrm{P}(H < 1.5 \mid 1 < H < 4.5)$

**(4)**

(c) Find the probability density function $\mathrm{f}(h)$
You may leave the limits of $h$ in terms of $d$ where necessary.

**(3)**

Question 2 continued

**6** **In this question solutions relying entirely on calculator technology are not acceptable.**

The continuous random variable $X$ has the following probability density function

$$f(x) = \begin{cases} a + bx & -1 \leqslant x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where $a$ and $b$ are constants.

(a) Show that $4a + 4b = 1$

**(3)**

Given that $E(X^2) = \dfrac{17}{5}$

(b) (i) find an equation in terms of $a$ only

**(5)**

(ii) hence show that $b = 0.1$

**(2)**

(c) Sketch the probability density function f($x$) of $X$

**(2)**

(d) Find the value of $k$ for which $P(X \geqslant k) = 0.8$

**(4)**

**Question 6 continued**