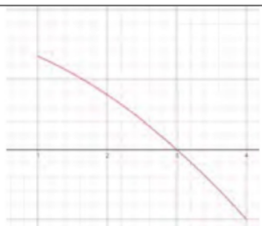
January 2018

Question	Scheme	Marks
1.(a)	$1 - \mathrm{F}(4) = 1 - \frac{1}{16}(4-1)^2 = \frac{7}{16}$	M1 A1
		(2)
(b)	$[\mathrm{P}(X>3 \mid 2<X<4) =]\ \dfrac{\mathrm{F}(4)-\mathrm{F}(3)}{\mathrm{F}(4)-\mathrm{F}(2)} = \dfrac{\frac{9}{16}-\frac{4}{16}}{\frac{9}{16}-\frac{1}{16}} = \dfrac{5}{8}$	M1 dM1 A1
		(3)
(c)	$\mathrm{f}(x) = \frac{\mathrm{d}}{\mathrm{d}x}\mathrm{F}(x) = \frac{1}{8}(x-1)$	M1
	$\mathrm{E}(X) = \int_1^5 \frac{1}{8}x(x-1)\mathrm{d}x$	dM1
	$\mathrm{E}(X) = \left[\frac{1}{24}x^3 - \frac{1}{16}x^2\right]_1^5 = \left(\dfrac{5^3}{24} - \dfrac{5^2}{16}\right) - \left(\dfrac{1}{24} - \dfrac{1}{16}\right) = \dfrac{11}{3}$	dM1 A1
		(4)
		Total 9

January 2018

Question	Scheme	Marks
7. (a)	$\int_1^3 \frac{1}{16}x^2\mathrm{d}x + \int_3^4 k(4-x)\mathrm{d}x = 1$	M1
	$\left[\dfrac{x^3}{48}\right]_1^3 + \left[k(4x - \dfrac{x^2}{2})\right]_3^4 = 1$	
	$\left(\frac{27}{48} - \frac{1}{48}\right) + k\left((16-8) - (12 - \frac{9}{2})\right) = 1$	M1
	$k = \frac{11}{12}$ **	A1cso
		(3)
(b)	Correct shape for $1 \le x < 3$	B1
	Correct shape for $3 \le x \le 4$	B1
		(2)
(c)	$[X =]\ 3$	B1
		(1)
(d)	$\mathrm{E}(X^2) = \int_1^3 \frac{1}{16}x^4\mathrm{d}x + \int_3^4 \frac{11}{12}(4x^2 - x^3)\mathrm{d}x$	M1
	$\mathrm{E}(X^2) = \left[\frac{1}{80}x^5\right]_1^3 + \left[\frac{11}{12}(\frac{4}{3}x^3 - \frac{1}{4}x^4)\right]_3^4$	dM1
	$\mathrm{Var}(X) = \frac{5863}{720} - (\frac{25}{9})^2 = \frac{2767}{6480}, = 0.427...$ **awrt 0.427**	M1, A1
		(4)
(e)(i)	$c = -1$	B1cao
(ii)	F(4)=1	
	$\frac{11}{12}(4(4) - \frac{1}{2}(4^2) + d) = 1$	M1
	$d = -\frac{76}{11}$	A1cao
		(3)
(f)	$\frac{11}{12}(4x - \frac{1}{2}x^2 - '\frac{76}{11}') = 0.75$	
	$11x^2 - 88x + 170 = 0$	M1
	$x = 3.26$ only	A1
		(2)
		Total 15

Question Number	Scheme	Marks
6(a)	$\mathrm{E}(X)=\int_0^1 \frac{1}{4}x\ \mathrm{d}x+\int_1^2 \frac{x^4}{5}\ \mathrm{d}x$	M1
	$=\left[\frac{x^2}{8}\right]_0^1+\left[\frac{x^5}{25}\right]_1^2$; $\left[=\frac{1}{8}+\frac{32}{25}-\frac{1}{25}\right]=$ $\underline{\frac{273}{200}}$ or $\underline{1.365}$ (oe) or awrt $\underline{\mathbf{1.37}}$	dM1; A1cso
		(3)
(b)	$\mathrm{E}(X^2)=\int_0^1\left(\frac{1}{4}x^2\right)\mathrm{d}x+\int_1^2\left(\frac{1}{5}x^5\right)\mathrm{d}x$	M1
	$\mathrm{Var}(X)=\left[\frac{x^3}{12}\right]_0^1+\left[\frac{x^6}{30}\right]_1^2-[1.365]^2$; $\left[=\frac{1}{12}+\frac{64}{30}-\frac{1}{30}-[1.365]^2\right]=0.32010..$ $\underline{\mathbf{0.320}}$	dM1; A1cso
		(3)
(c)	$\int\left(\frac{t^3}{5}\right)\mathrm{d}t=\left[\frac{t^4}{20}\right]+D$ and use of F(2) = 1 or F(1) = "$\frac{1}{4}$" <u>or</u> "$\frac{1}{4}$"$+\int_1^x\left(\frac{t^3}{5}\right)\mathrm{d}t$	M1
	$\mathrm{F}(x)=\begin{cases}0 & x<0\\ \frac{1}{4}x & 0\leqslant x<1\\ \frac{x^4}{20}+\frac{1}{5} & 1\leqslant x\leqslant 2\\ 1 & x>2\end{cases}$ Correct 2nd row with range; Correct 3rd row with range; Correct 1st and 4th rows with ranges	B1; A1; B1
		(4)
(d)	$\frac{x^4}{20}+\frac{1}{5}=0.5$	M1
	$x=\left[\sqrt[4]{6}\right]=1.565\ldots$ awrt $\underline{\mathbf{1.57}}$	A1
		(2)
(e)	Mean < median <u>or</u> mode = 2 and mean or median < mode [or sketch]	M1
	$\therefore$ negative skew	A1ft (2)

Question	Scheme	Marks
3.(a)		B1 (shape) B1 (domain)
	This is not a valid probability density function since $f(x) < 0$ for $x > 3$	B1 (3)
(b)	$[g(y) = k(12y - y^3)]$	
	$[g'(y)] = k(12 - 3y^2)$	M1
	$12 - 3y^2 = 0$	M1
	$y = 2$	A1 (3)
(c)	$\int k(12y - y^3)\mathrm{d}y = k\left[6y^2 - \frac{y^4}{4}\right]_1^3$	M1
	$k\left(6\times 3^2 - \frac{3^4}{4} - 6 + \frac{1}{4}\right) = 1$	M1
	$k = \frac{1}{28}$	A1 (3)
(d)	$\int_1^m \frac{1}{28}(12y - y^3)\mathrm{d}y = \frac{1}{28}\left(6\times m^2 - \frac{m^4}{4} - \left(6\times 1^2 - \frac{1^4}{4}\right)\right) = 0.5$	M1
	$m^4 - 24m^2 + 79 = 0 \rightarrow m = 1.98437...$ awrt **<u>1.98</u>**	M1 A1 (3)
(e)	Median ≈ Mode, therefore there is no skew.	M1A1ft (2)
		Total 14

October 2018

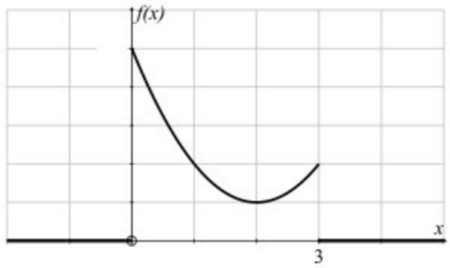
Question	Scheme	Marks
5.(a)	$f(x) = \frac{1}{100}(3ax^2 + 2bx + 15)$	M1 A1
	$E(X^2) = \int x^2 f(x)dx$	
	$\frac{1}{100}[\frac{3}{5}ax^5 + \frac{1}{2}bx^4 + 5x^3]_0^5 = 6.25$	M1
	$1875a + 312.5b + 625 = 625$	dM1
	$6a + b = 0$ *	A1cso
		(5)
(b)	$F(5) = 1$	
	$\frac{1}{100}(125a + 25b + 75) = 1$	M1
	Solving simultaneously $\begin{cases} 5a + b = 1 \\ 6a + b = 0 \end{cases}$	M1
	$\underline{a = -1}$ and $\underline{b = 6}$	A1 A1
		(4)
(c)	[P(3 ≤ X ≤ 7) =] F(5) – F(3) or 1 – F(3)	M1
	$1 - \frac{1}{100}(27a + 9b + 45)$	
	<u>0.28</u>	A1
		(2)
		Total 11

January 2019

Question Number	Scheme	Marks
3. (a)	$P(3 < X < 7) = F(7) - F(3)$ [$= 0.7 - 0.2$]	M1
	<u>0.5</u>	A1
		(2)
(b)	$a = \dfrac{0.4 - 0}{4 - 2}$ $\quad b = \dfrac{0}{6 - 4}$ $\quad c = \dfrac{1 - 0.4}{8 - 6}$	M1
	$\underline{a = 0.2}$ $\underline{b = 0}$ $\underline{c = 0.3}$	A1 A1
		(3)
(c)	$E(X) = \int_2^4 \text{"}0.2\text{"}x\ dx \left[+ \int_4^6 \text{"}0\text{"}x\ dx \right] + \int_6^8 \text{"}0.3\text{"}x\ dx$	M1
	$E(X) = \left[\dfrac{0.2x^2}{2}\right]_2^4 + \left[\dfrac{0.3x^2}{2}\right]_6^8$	A1ft
	$= \underline{\mathbf{5.4}}$	A1
		(3)
	Alternative	**Alternative**
	$E(X) = 3 \times p + 7 \times 2 \times \text{"their } c\text{"}$ or $3 \times 2 \times \text{"their } a\text{"} + 7 \times p$ where $0 < p < 1$	M1
	$= 3 \times \text{"}0.4\text{"} + 7 \times \text{"}0.6\text{"}$	A1ft
	$= \underline{\mathbf{5.4}}$	A1
		[8 marks]

Question Number	Scheme	Marks
7. (a)	$\int_{-3}^{0} c(x+3)\,\mathrm{d}x + \int_{0}^{3} \frac{5}{36}(3-x)\,\mathrm{d}x$ **or** area of triangle $= \dfrac{3\times 3c}{2} + \dfrac{3\times \frac{5}{36}(3)}{2}$	M1
	$c(-\frac{9}{2}+9)+\frac{5}{8}=1$ oe $\qquad \dfrac{3\times 3c}{2}+\frac{5}{8}=1$ oe	dM1
	$c = \dfrac{1}{12}$ *	A1cso (3)
	Alternative	
	$\left[\int c(x+3)\right] = c\left(\dfrac{x^2}{2}+3x\right)+d$ and $\left[\int \dfrac{5}{36}(3-x)\right] = \dfrac{5}{36}\left(3x-\dfrac{x^2}{2}\right)+e$	M1
	Using $\mathrm{F}(-3)=0$ and $\mathrm{F}(3)=1$ and $d=e$	dM1
	$c = \dfrac{1}{12}$ *	A1 cso
(b)(i)	A correct sketch of f(x) with straight line positive gradient from $-3 \le x < 0$ and straight line negative gradient from $0 \le x \le 3$, LHS below RHS at y axis	B1
	Correct labels: -3 and 3 on x-axis and $\frac{1}{4}$ and $\frac{5}{12}$ oe on y-axis	B1
(ii)	**Highest point** [at f(0), so $X = 0$ is the mode] oe	B1 (3)
(c)	$\int_{-3}^{x} \frac{1}{12}(t+3)\,\mathrm{d}t$ or $\int \frac{1}{12}(x+3)\,\mathrm{d}x$ with $+c$ and $\mathrm{F}(-3)=0$ $\int_{-3}^{0} \frac{1}{12}(t+3)\,\mathrm{d}t + \int_{0}^{x} \frac{5}{36}(3-t)\,\mathrm{d}t$ or $\int \frac{5}{36}(3-x)\,\mathrm{d}x$ with $+d$ and $\mathrm{F}(3)=1$	M1
	$\mathrm{F}(x) = \begin{cases} 0 & x < -3 \\ \frac{1}{24}x^2+\frac{1}{4}x+\frac{3}{8} & -3 \le x < 0 \\ \frac{5}{12}x-\frac{5}{72}x^2+\frac{3}{8} & 0 \le x \le 3 \\ 1 & x > 3 \end{cases}$ **or** $\mathrm{F}(x) = \begin{cases} 0 \\ \frac{1}{24}(x+3)^2 \\ 1-\frac{5}{72}(3-x)^2 \\ 1 \end{cases}$	B1 A1 A1 (4)
(d)	$[\mathrm{P}(X > d \mid X > 0) =]\ \dfrac{1-\mathrm{F}(d)}{1-\mathrm{F}(0)} = \dfrac{2}{5}$ oe	M1
	$1-\mathrm{F}(d) = \dfrac{2}{5}\times\dfrac{5}{8}\left[=\dfrac{1}{4}\right]$ oe	A1
	Using cdf **or** area of triangle $1-\left(\text{"}\dfrac{5}{12}d-\dfrac{5}{72}d^2+\dfrac{3}{8}\text{"}\right) = \text{"}\dfrac{2}{5}\times\dfrac{5}{8}\text{"}$ **or** $\dfrac{(3-d)\,\frac{5}{36}(3-d)}{2} = \text{"}\dfrac{2}{5}\times\dfrac{5}{8}\text{"}$ oe	dM1
	$d =$ awrt <u>**1.10**</u>	A1 (4) **[14 marks]**

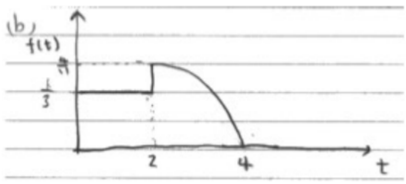
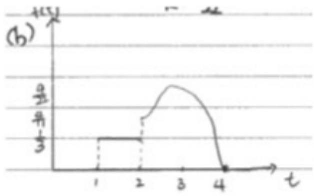
June 2019

Question Number	Scheme	Marks
5(a)	$\mathrm{f}(0) = 5k$	
	$k\left\{(a-2)^2+1\right\} < 5k$ condone $k\left\{(a-2)^2+1\right\} = 5k$	M1
	$(a-2)^2 < 4 \Rightarrow$ $0 < a < 4$	A1
		(2)
(b)(i)(ii)		
	Shape for $0 < x < 3$	B1
	$(2, k)$ or labels 2 and k	B1
		(2)
(c)	$\int_0^3 \mathrm{f}(x)\,\mathrm{d}x = 1$	
	$k\left[\frac{x^3}{3} - 2x^2 + 5x\right]_0^3 = 1$ or $k\left[\frac{(x-2)^3}{3} + x\right]_0^3 = 1$	M1A1
	$k\left(\frac{27}{3} - 2\times 9 + 15\right) - 0k = 1$ or $k\left(\frac{1}{3} + 3\right) - \frac{-8}{3}k = 1$	dM1
	$6k = 1 \Rightarrow k = \frac{1}{6}$ *	A1*
		(4)
(d)	$\mathrm{E}(X) = \int_0^3 x \times \frac{1}{6}(x^2 - 4x + 5)\,\mathrm{d}x$	M1
	$= \int_0^3 \frac{1}{6}(x^3 - 4x^2 + 5x)\,\mathrm{d}x = \frac{1}{6}\left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{5x^2}{2}\right]_0^3$	dM1A1
	$= \frac{1}{6}\left(\frac{81}{4} - \frac{108}{3} + \frac{45}{2} - 0\right)$ or $\frac{9}{8}$	dM1
	$\mathrm{Var}(X) = 2.1 - \left(\frac{9}{8}\right)^2 = \frac{267}{320}$	dM1
	$= \frac{267}{320}$ awrt 0.834	dA1
		(6)
(e)	$\frac{1}{6}\left[\frac{x^3}{3} - 2x^2 + 5x\right]_2^3$ oe or $1 - \frac{1}{6}\left[\frac{x^3}{3} - 2x^2 + 5x\right]_0^2$ oe	M1
	$\frac{1}{6}\left[9 - 18 + 15 - \left(\frac{8}{3} - 8 + 10\right)\right] = \frac{2}{9}$ * or $1 - \frac{1}{6}\left[\left(\frac{8}{3} - 8 + 10\right)\right] = \frac{2}{9}$ *	A1* cso
		(2)
(f)	P $1 < X < 2 = \frac{2}{9}$ or use of symmetry | $\mathrm{P}(X < 1) = \left[\frac{1}{6}\left(\frac{x^3}{3} - 2x^2 + 5x\right)\right]_0^1$	M1
	P $X > 1 = \frac{4}{9}$ | P $X < 1 = \frac{5}{9}$	A1
	Therefore median < 1	A1cso
		(3)
		Total 19

Question Number	Scheme	Marks
7(a)	$1 \leqslant x < 2$:	
	$\int \frac{3}{4}(x-1)\,\mathrm{d}x = \frac{3x^2}{8} - \frac{3x}{4}$ $[+c]$ $\left[\text{OR } \ldots = \frac{3}{8}(x-1)^2\,[+c]\right]$	M1
	$\mathrm{F}(1) = 0 \Rightarrow c = \frac{3}{8}$ [OR …. $[c=0]$	dM1
	$2 \leqslant x \leqslant 4$:	
	$\int \frac{3}{32}x(x-4)^2\,\mathrm{d}x = \frac{3x^4}{128} - \frac{x^3}{4} + \frac{3x^2}{4}$ $[+k]$	M1
	$\mathrm{F}(4) = 1$ or $\mathrm{F}(2) = \frac{3}{8} \Rightarrow k = -1$	dM1
	$$\mathrm{F}(x) = \begin{cases} 0 & x < 1 \\ \frac{3x^2}{8} - \frac{3x}{4} + \frac{3}{8} \text{ o.e. e.g. } \frac{3}{8}(x-1)^2 & 1 \leqslant x < 2 \\ \frac{3x^4}{128} - \frac{x^3}{4} + \frac{3x^2}{4} - 1 \text{ o.e. e.g. } \frac{(x-4)^3(3x+4)}{128} + 1 & 2 \leqslant x \leqslant 4 \\ 1 & x > 4 \end{cases}$$	A1 A1 A1
		(7)
(b)	$\mathrm{F}(m) = 0.5$	
	$\mathrm{F}(2.165) = 0.493\ldots$	M1
	$\mathrm{F}(2.175) = 0.5001\ldots$	
	$\Rightarrow \mathrm{F}(2.165) < 0.5 < \mathrm{F}(2.175)$ $\therefore m = 2.17$ (2 dp)	A1
		(2)
	Notes	**Total 9**

Question	Scheme	Marks
4(a)	$\mathrm{E}(X^2) = \int_1^3 \frac{1}{15}(3x^4 - x^5)\ \mathrm{d}x + \int_3^5 \frac{3}{10}(x^3 - 3x^2)\ \mathrm{d}x$	M1
	$= \left[\frac{1}{15}\left(\frac{3x^5}{5} - \frac{x^6}{6}\right)\right]_1^3 + \left[\frac{3}{10}\left(\frac{x^4}{4} - x^3\right)\right]_3^5$	M1 A1
	$= \frac{2923}{225} = 12.99....$ awrt **13.0**	A1 (4)
(b)	$\int_1^x \frac{1}{15}(3t^2 - t^3)\ \mathrm{d}t$ or $\int \frac{1}{15}(3x^2 - x^3)\ \mathrm{d}x$ with $+ c$ and $\mathrm{F}(1) = 0$	M1
	$\int_1^3 \frac{1}{15}(3t^2 - t^3)\ \mathrm{d}t + \int_3^x \frac{3}{10}(t-3)\ \mathrm{d}t$ or $\int \frac{3}{10}(x-3)\ \mathrm{d}x$ with $+ d$ and $\mathrm{F}(5) = 1$	M1
	$[\mathrm{F}(x) =] \begin{cases} 0 & x < 1 \\ \frac{1}{15}x^3 - \frac{1}{60}x^4 - \frac{1}{20} & 1 \le x < 3 \\ \frac{3}{20}x^2 - \frac{9}{10}x + \frac{7}{4} \text{ or } \frac{3}{20}(x-3)^2 + 0.4 & 3 \le x \le 5 \\ 1 & x > 5 \end{cases}$	B1 A1 A1 (5)
(c)	$\mathrm{P}(2 < X < 4) = \mathrm{F}(4) - \mathrm{F}(2)$ or $\int_2^3 \frac{1}{15}(3x^2 - x^3)\ \mathrm{d}x + \int_3^4 \frac{3}{10}(x-3)\ \mathrm{d}x$	M1
	$\frac{3}{20}(4^2) - \frac{9}{10}(4) + \frac{7}{4} - \left(\frac{1}{15}(2^3) - \frac{1}{60}(2^4) - \frac{1}{20}\right) = \underline{\frac{1}{3}}$	A1 (2)
(d)	$1 - \mathrm{F}(k) = 0.2$ or $\int_k^5 \frac{3}{10}(x-3)\ \mathrm{d}x = 0.2$	M1
	$\frac{3}{20}k^2 - \frac{9}{10}k + \frac{7}{4} = 0.8$ or $\frac{3}{20}(k-3)^2 = 0.4 \rightarrow k = 4.63299...$ awrt **4.63**	dM1 A1 (3)

7(a)	$\mathrm{f}(x) = \frac{1}{3125}(100x^3 - 20x^4)$ or $\frac{4}{125}x^3 - \frac{4}{625}x^4$	M1	
	$\mathrm{f}'(x) = \frac{1}{3125}(300x^2 - 80x^3) = 0$ or $\frac{12}{125}x^2 - \frac{16}{625}x^3$	M1A1	
	$x = 3.75$	A1	
			(4)
(b)	F(3.95) = 0.7166… F(4.05) = 0.7576…	M1A1	
	F(3.95) < 0.75 < F(4.05), therefore the **upper quartile** (oe) is **4.0** to 1 decimal place.	A1	
			(3)
(c)	$\mathrm{H}_0 : p = 0.25$ $\mathrm{H}_1 : p < 0.25$	B1	
	$Y \sim \mathrm{B}(25, 0.25)$ **and** $\mathrm{P}(Y \le 3) =$ 0.0962 or $\mathrm{P}(Y \le 2) =$ awrt 0.0321 CR $Y \le 2$	M1 A1	
	Do not reject H_0 / not significant	dM1	
	There is <u>not</u> enough evidence to suggest that the model <u>overestimates</u> the <u>proportion</u> of <u>queuing</u> more than 4 minutes/<u>Olivia's belief</u> is <u>not supported</u>.	A1cso	
	SC If H_1 written using > 0.25 and they then go on to use $\mathrm{P}(Y \ge 3) = 0.9679$ allow B0M1A0dM0A0. If they go on to use $\mathrm{P}(Y \le 3)$ or $\mathrm{P}(Y \ge 4)$ mark as original scheme		

		Notes
4(a)	**M1**	Adding the two integrals together with correct limits and setting = 1 (may be done in stages) Allow $\frac{1}{3}$ instead of first integral
	A1	Correct integration (again allow $\frac{1}{3}$ instead of first integration)
	A1cso	Must have at least one line of working before the given answer and no errors
(b)	**B1**	Correct shape with correct curvature Horizontal line, then quadratic (increasing then decreasing as t increases) starting above horizontal line and finishing on horizontal axis. The sketch is not continuous. There should be no **solid** vertical lines.
	dB1	Fully correct with 1, 2 and 4 each labelled at appropriate place on horizontal axis (Ignore vertical labelling). e.g.  B0B0 (solid vertical line)  B1B1Condone curvature
(c)	**B1**	For $k\left(8t-3t^2\right)$
	M1	Putting their differential = 0 ignore missing k
	A1	Allow awrt 2.67 only
(d)	**M1**	For $\int_1^t \frac{1}{3}\,dx$ with attempt to integrate. Must have correct limits. Or for integration with +C and use of F(1) = 0
	M1	For $F(2)+\int_2^t \frac{1}{22}\left(4x^2-x^3\right)dx$ and attempt to integrate or $\int \frac{1}{22}\left(4t^2-t^3\right)dt=\frac{4t^3}{66}-\frac{t^4}{88}+C$ and using F(4) =1 or F(2) = $\frac{1}{3}$ – must attempt to integrate, have + C
	A1	For 2nd line of cdf oe (allow $<$ instead of $\le$ and vice versa ditto $>$ and $\ge$) (allow any letter to be used for this A1 mark)
	A1	For 3rd line of cdf oe (allow $<$ instead of $\le$ and vice versa ditto $>$ and $\ge$) (allow any letter to be used for this A1 mark)
	A1	All correct and in terms of t including F(t). Allow the otherwise to be for any of the parts but there must be only one. (allow $<$ instead of $\le$ and vice versa ditto $>$ and $\ge$)
(e)	**M1**	Attempting to find 1 – F(3) with attempt to use 3rd line of their F(t) or $\int_3^4 k(4t^2-t^3)dt$
	A1	$\frac{67}{264}$ oe or awrt 0.254

January 2020

Question Number	Scheme			Marks
6(a)	$\text{E}(X^2)=\int_{-1}^{1}\frac{1}{8}\left(x^4+2x^3+x^2\right)\,\text{d}x+\int_{1}^{\frac{11}{3}}\frac{1}{4}x^2\text{d}x$			M1
	$=\left[\frac{1}{8}\left(\frac{x^5}{5}+\frac{2x^4}{4}+\frac{x^3}{3}\right)\right]_{-1}^{1}+\left[\frac{x^3}{12}\right]_{1}^{\frac{11}{3}}$			A1
	$=\frac{1684}{405}$			
	$\text{Var}(X)=\text{"}\frac{1684}{405}\text{"}-\left(\frac{31}{18}\right)^2$			dM1
	$=\frac{1931}{1620}$ or 1.1919…		awrt **<u>1.19</u>**	A1
				(4)
(b)	$\text{P}\left(X<-\frac{1}{2}\right)=\int_{-1}^{-0.5}\frac{1}{8}\left(x^2+2x+1\right)\text{d}x$	**or** $1-\int_{-0.5}^{0.5}\frac{1}{8}\left(x^2+2x+1\right)\text{d}x$ (gets M2)		M1
	$\text{P}\left(X>\frac{1}{2}\right)=\frac{2}{3}+\int_{0.5}^{1}\frac{1}{8}\left(x^2+2x+1\right)\text{d}x$			M1
	$\text{P}\left(X<-\frac{1}{2}\right)=\left[\frac{x^3}{24}+\frac{x^2}{8}+\frac{x}{8}\right]_{-1}^{-0.5}$ **or** $\text{P}\left(X>\frac{1}{2}\right)=\frac{2}{3}+\left[\frac{x^3}{24}+\frac{x^2}{8}+\frac{x}{8}\right]_{0.5}^{1}$ **or** $1-\left[\frac{x^3}{24}+\frac{x^2}{8}+\frac{x}{8}\right]_{-0.5}^{0.5}$			A1
	$=\frac{83}{96}$ or 0.8645…		awrt **<u>0.865</u>**	A1
				(4)
				Total 8

June 2020

Question Number	Scheme		Marks
1 (a)	$\int_{1}^{2}k\left(\frac{1}{2}x^3-3x^2+ax+1\right)\text{d}x\,[=1]$		M1
	$k\left[\frac{1}{8}x^4-x^3+\frac{1}{2}ax^2+x\right]_{1}^{2}\,[=1]$		A1
	$k(2-8+2a+2)-k\left(\frac{1}{8}-1+\frac{1}{2}a+1\right)=1$ **or** $k(2a-4)-k\left(\frac{1}{8}+\frac{1}{2}a\right)=1$		dM1
	$-\frac{33}{8}k+\frac{3}{2}ka=1\quad\therefore k(12a-33)=8$ *		A1 *
			(4)
(b)	$\frac{\text{d}f(x)}{\text{d}x}=k\left(\frac{3}{2}x^2-6x+a\right)$		M1
	$\frac{3}{2}x^2-6x+5=0$ **or** $\frac{4}{9}x^2-\frac{16}{9}x+\frac{40}{27}=0$		dM1
	$x=\frac{6\pm\sqrt{6^2-4\times1.5\times5}}{3}$		M1
	$x=2-\frac{\sqrt{6}}{3}$ oe or 1.183…	awrt 1.18	A1
			(4)

June 2020

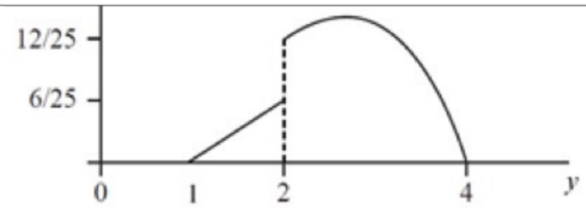
Question Number	Scheme	Marks
5(a)	$\mathrm{E}(T^2) = \int_0^3 \frac{1}{50}\left(18t^2 - 2t^3\right)\,\mathrm{d}t + \int_3^5 \frac{1}{20}t^2\mathrm{d}t$	M1
	$= \left[\frac{1}{50}\left(6t^3 - \frac{t^4}{2}\right)\right]_0^3 + \left[\frac{t^3}{60}\right]_3^5$ or $= \left[\frac{3}{25}t^3 - \frac{t^4}{100}\right]_0^3 + \left[\frac{t^3}{60}\right]_3^5$ oe	A1
	$= \frac{1}{50}\left(6\times 3^3 - \frac{3^4}{2}\right) + \left(\frac{125}{60} - \frac{27}{60}\right)$ or $= \frac{1}{50}\left(162 - \frac{81}{2}\right) + \left(\frac{25}{12} - \frac{9}{20}\right)$ oe	M1d
	$= \frac{1219}{300} = 4.063...$	
	$\mathrm{Var}(T) = \text{"}4.063...\text{"} - (1.66)^2$	M1
	$= 1.3077\ldots$ awrt 1.31	A1
		(5)
(b)	$\int_3^t \frac{1}{20}\mathrm{d}x + C$ where $C = 0.9$ or $\int_0^3 \frac{1}{50}(18 - 2t)\,\mathrm{d}t$ **or** using F(5) =1 to find C	M1
	$[\mathrm{F}(t) =] \begin{cases} 0 & t < 0 \\ \frac{1}{50}\left(18t - t^2\right) \text{or } 1.62 - \frac{(18-2t)^2}{200} & 0 \le t \le 3 \\ \frac{1}{20}t + 0.75 & 3 < t \le 5 \\ 1 & t > 5 \end{cases}$	B1 A1 A1
		(4)
(c)	$\mathrm{P}(T > 2) = 1 - \text{"}\frac{1}{50}\left(18\times 2 - 2^2\right)\text{"}$ or $1 - \int_0^2 \frac{1}{50}(18 - 2t)\,\mathrm{d}t$	M1
	$= \frac{9}{25}$ or 0.36	A1
		(2)
(d)	$\mathrm{P}(0 < T < 3.66) = \mathrm{F}(3.66)$	M1
	$= 0.933$	A1
		(2)

January 2021

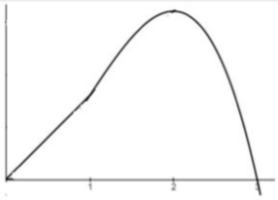
Question Number	Scheme		Marks
2(a)	$1-\text{F}(3.5) = 1 - 0.97127\ldots$		M1
	$= 0.028727\ldots$	awrt 0.0287	A1
			(2)
(b)	$W \sim \text{B}(30,\text{"}0.0287\text{"})$		M1
	$1-\text{P}(W \le 1) = 1-\left(\left(1-\text{"}0.0287\text{"}\right)^{30} + {}^{30}C_1\left(\text{"}0.0287\text{"}\right)^1\left(1-\text{"}0.0287\text{"}\right)^{29}\right)$ oe		M1
	$= 1 - 0.78748\ldots = 0.2125\ldots$ awrt 0.213 to awrt 0.216		A1
			(3)
(c)	$\frac{\text{dF}(w)}{\text{d}w} = \frac{1}{3}\left(1-\frac{w^3}{64}\right)$		M1
	$\text{E}(W^2) = \int_0^4 \frac{1}{3}\left(w^2 - \frac{w^5}{64}\right)\text{d}w = \frac{1}{3}\left[\frac{w^3}{3} - \frac{w^6}{384}\right]_0^4$		dM1
	$= \frac{32}{9}$		A1
	$\text{Var}(W) = \frac{32}{9} - 1.6^2$		M1
	$= \frac{224}{225}$		A1
			(5)

January 2021

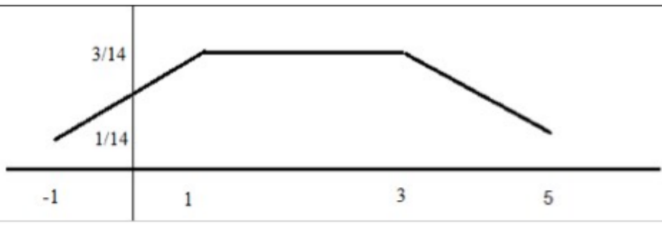
Question Number	Scheme		Marks
4(a)	$\int_0^a k(a-x)^2\text{d}x = \left[k\left(a^2x - ax^2 + \frac{x^3}{3}\right)\right]_0^a$ or $\left[\frac{-k(a-x)^3}{3}\right]_0^a$		M1 A1
	$k\left(a^3 - a^3 + \frac{a^3}{3}\right) = 1$ or $\frac{ka^3}{3} = 1 \quad \Rightarrow ka^3 = 3$		A1 cso
			(3)
(b)	$\int_0^a kx(a-x)^2\text{d}x = \left[k\left(\frac{a^2x^2}{2} - \frac{2ax^3}{3} + \frac{x^4}{4}\right)\right]_0^a$ or $\left[\frac{-kx(a-x)^3}{3} + \frac{k(a-x)^4}{12}\right]_0^a$		M1A1
	$k\left(\frac{a^2a^2}{2} - \frac{2aa^3}{3} + \frac{a^4}{4}\right) = 1.5$ or $\left[\frac{ka(a)^3}{3} - \frac{k(a)^4}{12}\right]_0^a = 1.5$ or $ka^4 = 18$ oe		dM1
	$\frac{ka^4}{ka^3} = 6$ or $\frac{18}{3} = 6 \quad [\therefore a = 6]$		A1cso
			(4)
(c)	$\text{F}(x) = \frac{1}{72}\left(36x - 6x^2 + \frac{x^3}{3}\right)$	$\frac{1}{72}\left(36x - 6x^2 + \frac{x^3}{3}\right) = 0.5$ oe	M1
	F(1.15)(= 0.47…) <u>and</u> F(1.25) (= 0.5038…)	1.2377…	M1
	F(1.15) = awrt 0.47, F(1.25) = awrt 0.504 (0.47(18…) < 0.5 < 0.503(8…)) therefore the median is **1.2** to 1 decimal place.	therefore the median is **1.2** to 1 decimal place.	A1
			(3)

Question Number	Scheme	Marks
3. (a)		M1 A1 (2)
(b)	$\dfrac{\mathrm{d}\left(\frac{3}{50}\left(4y^2-y^3\right)\right)}{\mathrm{d}y}=\frac{3}{50}\left(8y-3y^2\right)$	M1
	$\frac{3}{50}\left(8y-3y^2\right)=0$; $\underline{y=\frac{8}{3}}$ oe	M1; A1 (3)
(c)	$\mathrm{E}(Y^2)=\int_1^2\left(\frac{6}{25}y^3-\frac{6}{25}y^2\right)\mathrm{d}y+\int_2^4\left(\frac{12}{50}y^4-\frac{3}{50}y^5\right)\mathrm{d}y$	M1
	$=\left[\frac{6}{100}y^4-\frac{6}{75}y^3\right]_1^2+\left[\frac{12}{250}y^5-\frac{3}{300}y^6\right]_2^4$	A1
	$=\left[\left(\frac{8}{25}\right)-\left(-\frac{1}{50}\right)\right]+\left[\left(\frac{1024}{125}\right)-\left(\frac{112}{125}\right)\right]$; $=\underline{\frac{1909}{250}}$ or **7.636** or **7.64**	dM1; A1 (4)
(d)	$\mathrm{Var}(Y)=\text{"}\frac{1909}{250}\text{"}-2.696^2$	M1
	$=0.367584$ awrt **0.368**	A1 (2)
(e)	$\frac{1}{2}(y-1)\times\frac{6}{25}(y-1)=0.1$ or $\int_1^x\frac{6}{25}(y-1)\,\mathrm{d}y=0.1$	M1
	$\frac{1}{2}(y-1)\times\frac{6}{25}(y-1)=0.1$ or $\frac{6}{25}\left[\left(\frac{x^2}{2}-x\right)+\frac{1}{2}\right]=0.1$ or $\frac{6}{50}(x-1)^2=0.1$	A1
	$(y-1)^2=\frac{5}{6}$ or $y=1\pm\sqrt{\frac{5}{6}}$; $y=1.9128...$ awrt **1.91**	dM1; A1
		(4) **Total 15**

Question Number	Scheme	Marks
5 (a)(i)	If $y=0$ then $1-(\alpha+\beta y^2)=0 \quad \therefore\ \alpha=1$ *	B1cso
(ii)	If $y=5$ then $1-(\alpha+\beta y^2)=1$	
	$1+25\beta=0 \quad \therefore \beta=-\dfrac{1}{25}$ *	B1cso
		(2)
(b)	$\mathrm{F}(y)=\dfrac{1}{25}y^2$ so $\mathrm{f}(y)=\dfrac{\mathrm{dF}(y)}{\mathrm{d}y}=\dfrac{2}{25}y$	M1
	$\therefore [\mathrm{f}(y)=]\begin{cases}\dfrac{2}{25}y & 0\leqslant y\leqslant 5\\ 0 & \text{otherwise}\end{cases}$	A1
		(2)
(c)	$\left[\mathrm{P}\left(R>\dfrac{11}{5}\right)=\mathrm{P}\left(Y>\dfrac{5}{3}\right)=1-\dfrac{1}{25}\times\left(\dfrac{5}{3}\right)^2=\right]\ \underline{\underline{\dfrac{8}{9}}}$ oe	B1
	$\dfrac{3d-\frac{11}{5}}{3d-d}=\dfrac{8}{9}$ oe or $\dfrac{\frac{11}{5}-d}{3d-d}=\dfrac{1}{9}$ oe	M1
	$d=\underline{\underline{\dfrac{9}{5}}}$ oe	A1
		(3)
(d)	$\mathrm{P}\left(Y<\dfrac{11}{5}\right)=\dfrac{121}{625}$ or 0.1936	B1
	[Let G = the number of spins with distance < 2.2 m]	
	$[\mathrm{P}(G\geqslant 5)=]$	
	$\left(\text{"}\dfrac{1}{9}\text{"}\right)^3\times\left(\text{"}\dfrac{121}{625}\text{"}\right)^3+3\times\left(\text{"}\dfrac{1}{9}\text{"}\right)^2\times\left(\text{"}\dfrac{8}{9}\text{"}\right)\times\left(\text{"}\dfrac{121}{625}\text{"}\right)^3+3\times\left(\text{"}\dfrac{1}{9}\text{"}\right)^3\times\left(\text{"}\dfrac{121}{625}\text{"}\right)^2\times\left(\text{"}\dfrac{504}{625}\text{"}\right)$	M1, M1
	$=0.000\,373226$ awrt **<u>0.000 373</u>**	A1
		(4)
		Total 11

Question Number	Scheme	Marks
3(a)	$4a = a(b) \Rightarrow b = 4$ *	B1*cso
		(1)
(b)	$a(27b - 81 + 1) = 1$	M1
	$a = \dfrac{1}{28}$	A1
		(2)
(c)	$\mathrm{P}(X > 2.25) = 1 - \mathrm{F}(2.25)$	M1
	$= 0.25237\ldots$ awrt 0.252	A1
		(2)
(d)(i)	$\mathrm{f}(x) = \frac{3}{7}x^2 - \frac{1}{7}x^3$ or $\frac{2}{7}x$	M1
	 Sketch	B1
(ii)	$\mathrm{f}'(x) = \frac{6}{7}x - \frac{3}{7}x^2$	dM1
	$\frac{6}{7}x - \frac{3}{7}x^2 = 0$	dM1
	Mode = 2	A1
		(5)

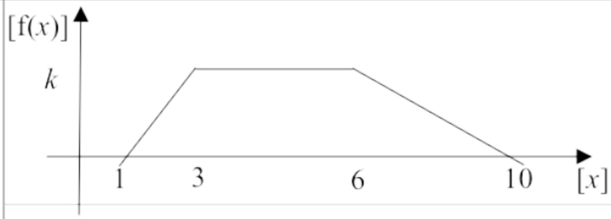
October 2021

Qu'n Number	Scheme	Marks
6(a)		B1 B1
		(2)
(b)	$\mathrm{E}(Y)=2$	B1
	$\mathrm{Var}(2Y-3)=4\mathrm{Var}(Y)$	M1
	$\mathrm{Var}(Y)=\left(\frac{131}{21}-2^2\right)$	M1
	$\mathrm{Var}(2Y-3)=\frac{188}{21}$ awrt 8.95	A1
		(4)
(c)	$\int_{-1}^{t}\frac{1}{14}(y+2)\mathrm{d}y=\frac{1}{14}\left[\frac{y^2}{2}+2y\right]_{-1}^{t}$ **or** $\int\frac{1}{14}(y+2)\mathrm{d}y=\frac{1}{14}\left[\frac{y^2}{2}+2y\right]+C$ **or** $\int\frac{1}{14}(y+2)\mathrm{d}y=\frac{1}{28}(y+2)^2+C$	M1
	$\frac{1}{14}\left[\left(\frac{t^2}{2}+2t\right)-\left(\frac{1}{2}-2\right)\right]$ **or** $\frac{1}{14}\left[\frac{(-1)^2}{2}-2\right]+C=0$ & $C=\frac{3}{28}$ **or** $\frac{1}{28}(-1+2)^2+C=0$ & $C=-\frac{1}{28}$ leading to $\frac{1}{14}\left(\frac{y^2}{2}+2y+\frac{3}{2}\right)$ *	A1*cso
		(2)
(d)	$\int_1^t\frac{3}{14}\mathrm{d}y+\mathrm{F}(1)=\left[\frac{3}{14}y\right]_1^t+\mathrm{F}(1)=\left[\left(\frac{3t}{14}\right)-\left(\frac{3}{14}\right)\right]+\mathrm{F}(1)$ or $\int\frac{3}{14}\mathrm{d}y=\left[\frac{3}{14}y\right]+C$ and use of F(1) = "their F(1)" or F(3) = " their F(3)"	M1
	$\int_3^t\frac{1}{14}(6-y)\mathrm{d}y+\mathrm{F}(3)=\frac{1}{14}\left[6y-\frac{y^2}{2}\right]_3^t+\mathrm{F}(3)=\frac{1}{14}\left[\left(6t-\frac{t^2}{2}\right)-\left(18-\frac{9}{2}\right)\right]+\mathrm{F}(3)$ or $\int\frac{1}{14}(6-y)\mathrm{d}x=\frac{1}{14}\left[6y-\frac{y^2}{2}\right]+C$ <u>or</u> $C-\frac{(6-y)^2}{28}$ and use F(5) =1	M1
	$\mathrm{F}(y)=\begin{cases}0 & y\leqslant -1\\ \frac{1}{14}\left(\frac{y^2}{2}+2y+\frac{3}{2}\right) & -1<y\leqslant 1\\ \frac{3}{14}y+\frac{1}{14} & 1<y\leqslant 3\\ \frac{3}{7}y-\frac{1}{28}y^2-\frac{1}{4} & 3<y\leqslant 5\\ 1 & y>5\end{cases}$	A1 A1 B1
		(5)

January 2022

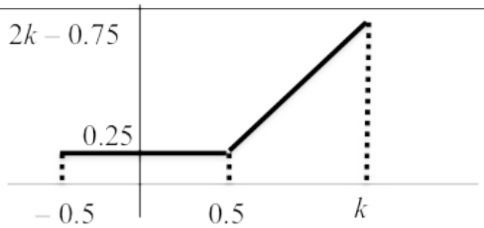
Question Number	Scheme	Marks
2 (a)	$\mathrm{f}(x)=\begin{cases}\dfrac{1}{4k} & -k \leqslant x \leqslant 3k \\ 0 & \text{otherwise}\end{cases}$	M1 A1
		(2)
(b)	$[\mathrm{E}(X)]=k$	B1
		(1)
(c)	$[\mathrm{Var}(X)]=\dfrac{(3k--k)^2}{12}=\dfrac{16k^2}{12}$ or $\left[\dfrac{x^3}{3}\ \text{"f}(x)\text{"}\right]_{-k}^{3k}-(\text{"}k\text{"})^2$	M1
	$=\dfrac{4k^2}{3}$ *	A1* cso
		(2)
(d)	$\mathrm{E}(X^2)=\mathrm{Var}(X)+\mathrm{E}(X)^2=\dfrac{4k^2}{3}+(\text{"}k\text{"})^2$	M1
	$=\dfrac{7k^2}{3}$	A1
	$\mathrm{E}(3X^2)=3\mathrm{E}(X^2)=3\times\dfrac{7k^2}{3}\ =7k^2$	A1
		(3)

January 2022

Question Number	Scheme	Marks
4 (a)		B1 B1
		(2)
(b)	$\frac{1}{2}(3+9)\times k=1$ or $\frac{1}{2}(3-1)k+(6-3)k+\frac{1}{2}(10-6)k=1$ or $\frac{1}{2}k\left[\frac{x^2}{2}-x\right]_1^3+k\left[x\right]_3^6+\frac{1}{4}k\left[10x-\frac{x^2}{2}\right]_6^{10}=1$	M1
	$k=\frac{1}{6}$ *	A1* cso
		(2)
(c)	$\int_1^x \frac{1}{12}(x-1)\,\mathrm{d}x$ or $\int \frac{1}{12}(x-1)\,\mathrm{d}x$ and using $\mathrm{F}(1)=0$	M1
	$\int_3^x \frac{1}{6}\mathrm{d}x+\text{"F(3)"}$ or $\int \frac{1}{6}\mathrm{d}x$ and using "$\mathrm{F}(3)=\frac{1}{6}$"	M1
	$\int_6^x \left(\frac{5}{12}-\frac{1}{24}x\right)\mathrm{d}x+\text{"F(6)"}$ or $\int\left(\frac{5}{12}-\frac{1}{24}x\right)\mathrm{d}x$ and using either "$\mathrm{F}(6)=\frac{2}{3}$" or $\mathrm{F}(10)=1$	M1
	$\mathrm{F}(x)=\begin{cases}0 & x<1\\ \frac{1}{24}(x^2-2x+1) & 1\leqslant x\leqslant 3\\ \frac{1}{6}(x-2) & 3<x\leqslant 6\\ \frac{1}{48}(20x-x^2-52) \text{ or } 1-\frac{(10-x)^2}{48} & 6<x\leqslant 10\\ 1 & x>10\end{cases}$	A1oe A1oe A1 oe B1
		(7)
(d)	$\mathrm{P}(X>\mathrm{E}(X))=1-\mathrm{F}\left(\frac{61}{12}\right)=1-0.51388...=0.4861...$ awrt 0.486	M1 A1 (2)
(e)	Since (d) < 0.5 [the mean is greater than the median] therefore positive (skew) or follow through their sketch in part (a)	M1 A1ft
		(2)

Question	Scheme	Marks
2. (a)	$\mathrm{E}(T)=\int_0^4 \frac{1}{192}t(t^3-48t+128)\,\mathrm{d}t$	M1
	$=\frac{1}{192}\left[\frac{t^5}{5}-16t^3+64t^2\right]_0^4$ **or** $\left[\frac{t^5}{960}-\frac{1}{12}t^3+\frac{1}{3}t^2\right]_0^4$ oe	dM1
	$=\frac{1}{192}\left(\frac{4^5}{5}-16(4^3)+64(4^2)-0\right)=\frac{16}{15}\text{min}\rightarrow 1$ minute 4 seconds	A1
		(3)
(b)	P(call takes between 1 and 3 minutes) $=\int_1^3 \frac{1}{192}(t^3-48t+128)\,\mathrm{d}t$	
	mor $\left[\frac{t^4}{768}-\frac{1}{8}t^2+\frac{2}{3}t\right]_1^3$ oe	M1
	$=\frac{1}{192}\left(\left(\frac{3^4}{4}-24(3^2)+128(3)\right)-\left(\frac{1^4}{4}-24(1^2)+128(1)\right)\right)=\frac{7}{16}$ *	dM1 A1*cso
		(3)
(c)	$C\sim\mathrm{B}(256,\frac{7}{16})\approx\mathrm{N}(112,63)$	M1 A1
	$\mathrm{P}(C>125)\approx\mathrm{P}\left(Z>\frac{125.5-112}{\sqrt{63}}\right)$	M1M1
	$\mathrm{P}(Z>1.70)=1-0.9554=0.0446$	A1
		(5)

Question	Scheme	Marks
6. (a)	$\int_0^2 0.1x\,\mathrm{d}x + \int_2^4 kx(8-x)\,\mathrm{d}x = \frac{31}{45}$	M1
	$\left[\frac{0.1x^2}{2}\right]_0^2 + k\left[4x^2 - \frac{x^3}{3}\right]_2^4 = \frac{31}{45}$	M1
	$0.2 + k\left(64 - \frac{64}{3} - (16 - \frac{8}{3})\right) = \frac{31}{45} \rightarrow k = \frac{1}{60}$	dM1 A1
		(4)
(b)(i)	$a = \left[\left(1 - \frac{31}{45}\right) \div 2 = \right]\ \frac{7}{45}$	B1
(ii)	$\mathrm{P}(0 \leqslant X \leqslant 5.5) = \frac{31}{45} + "a" \times 1.5 = \frac{83}{90}$	M1 A1
		(3)
(c)	$\int_0^x 0.1t\,\mathrm{d}t = \frac{0.1x^2}{2}$	B1
	$\int_0^2 0.1t\,\mathrm{dt} + \int_2^x "\frac{1}{60}" t(8-t)\,\mathrm{d}t$, $\qquad \frac{31}{45} + \int_4^x "\frac{7}{45}"\,\mathrm{d}t$	M1, M1
	$[\mathrm{F}(x) =] \begin{cases} 0 & x < 0 \\ 0.05x^2 & 0 \leqslant x < 2 \\ \frac{1}{60}(4x^2 - \frac{x^3}{3} - \frac{4}{3}) & 2 \leqslant x < 4 \\ \frac{7}{45}x + \frac{1}{15} & 4 \leqslant x < 6 \\ 1 & x \geqslant 6 \end{cases}$	B1 A1 A1 (6)

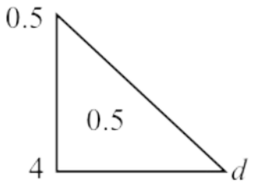
Question Number	Scheme	Marks
2 (a)		M1 A1
		(2)
(b)	Area $= \frac{1}{4}+\frac{1}{2}\left(\frac{1}{4}+2k-\frac{3}{4}\right)\left(k-\frac{1}{2}\right)=1$ **or** $\frac{1}{4}\left(k+\frac{1}{2}\right)+\frac{1}{2}(2k-1)\left(k-\frac{1}{2}\right)=1$ **or** $\frac{1}{4}+\left[\left(k^2-\frac{3}{4}k\right)-\left(\frac{1}{2}^2-\frac{3}{8}\right)\right]=1$	M1
	$8k^2-6k-5=0$ or $k^2-\frac{3}{4}k-\frac{5}{8}=0$ oe	A1
	$(4k-5)(2k+1)=0$ or $k=\dfrac{\frac{3}{4}\pm\sqrt{\left(-\frac{3}{4}\right)^2+4\times\left(\frac{5}{8}\right)}}{2}$ oe	M1
	$k=1.25$*	A1 *
		(4)
(c)	$\left[\int_{-0.5}^{0.5}\frac{1}{4}x\,\mathrm{d}x\right]+\int_{0.5}^{1.25}2x^2-\frac{3}{4}x\,\mathrm{d}x=\ [0]+\left[\frac{2x^3}{3}-\frac{3}{8}x^2\right]_{0.5}^{1.25}$	M1A1
	$=\left(\frac{2\times1.25^3}{3}-\frac{3}{8}\times1.25^2\right)-\left(\frac{2\times0.5^3}{3}-\frac{3}{8}\times0.5^2\right)$	dM1
	$=\frac{93}{128}$ awrt 0.727	A1
		(4)
(d)	$[Q_1=]\ 0.5$	B1
	${Q_3}^2-\frac{3}{4}Q_3+0.125=0.5$ or ${Q_3}^2-\frac{3}{4}Q_3-0.375=0$ oe	M1
	$Q_3=\dfrac{\frac{3}{4}\pm\sqrt{\left(-\frac{3}{4}\right)^2+4\times\left(\frac{1}{8}\right)}}{2}$	M1
	IQR ="1.093…" – 0.5	M1
	= 0.593… awrt 0.59	A1
		(5)

Question Number	Scheme		Marks
5(a)	$d = 7$		B1
	$\frac{c}{3} - \frac{7}{6} = 1 - \frac{1}{6}("7" - c)^2$		M1
	$\frac{c}{3} - \frac{7}{6} = 1 - \frac{1}{6}("49" - 2 \times "7"c + c^2)$ oe or $c^2 - 12c + 36 = 0$ oe		dM1
	$(c-6)^2 = 0 \quad \therefore c = 6$ *		A1*
			(4)
(b)	$\mathrm{P}(X > 3.5) = 1 - \frac{1}{6}(3.5 - 3)^2$		M1
	$= \frac{23}{24}$ oe	awrt 0.958	A1
			(2)
(c)	$\mathrm{P}(3.5 < X < 5.5) = \left(\frac{5.5}{3} - \frac{7}{6}\right) - \left(\frac{1}{6}(3.5-3)^2\right)\left[= \frac{5}{8}\right]$ oe		M1
	$\mathrm{P}(X > 4.5 \mid 3.5 < X < 5.5) = \dfrac{\left(\frac{5.5}{3} - \frac{7}{6}\right) - \left(\frac{4.5}{3} - \frac{7}{6}\right)}{"5/8"}$		M1
	$= \frac{8}{15}$ oe	awrt 0.533	A1
			(3)

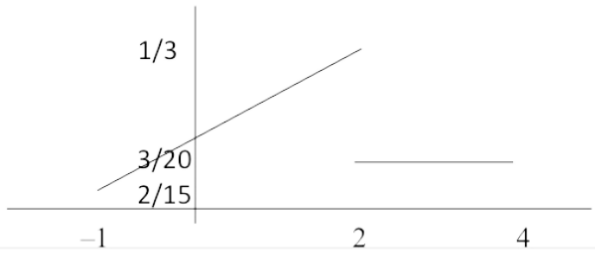
Number		
6 (a)	$[\mathrm{F}(k)=1\Rightarrow]ak+bk^2=1\Rightarrow ak=1-bk^2$ *	B1*
		(1)
(b)	$\mathrm{f}(x)=a+2bx$	B1
	$\mathrm{E}(X)=\int_0^k\left(ax+2bx^2\right)\,\mathrm{d}x\left[=\frac{6}{5}\right]\Rightarrow\left[\frac{ax^2}{2}+\frac{2bx^3}{3}\right]_0^k\left[=\frac{6}{5}\right]$	M1
	$\frac{ak^2}{2}+\frac{2bk^3}{3}=\frac{6}{5}$	dM1, A1
	$15ak^2+20bk^3=36$	
	$15k\left(1-bk^2\right)+20bk^3=36$	M1
	$5bk^3=36-15k$ *	A1*
		(6)
(c)	$\mathrm{E}(X^2)=\int_0^k\left(ax^2+2bx^3\right)\,\mathrm{d}x\Rightarrow\left[\frac{ax^3}{3}+\frac{bx^4}{2}\right]_0^k$	M1
	$\mathrm{Var}(X)=\frac{ak^3}{3}+\frac{bk^4}{2}-\frac{36}{25}=\frac{22}{75}$	dM1 A1
	$10ak^3+15bk^4=52$	
	$10k^2\left(1-bk^2\right)+15bk^4=52$	M1
	$5bk^4=52-10k^2$ *	A1*
		(5)
(d)	$\frac{1}{k}=\frac{36-15k}{52-10k^2}$	M1
	$5k^2-36k+52=0$	A1
	$(k-2)(5k-26)=0$	M1
	$k=2$	A1
		(4)
(e)	$'40'b=36-'30'\Rightarrow b=\frac{3}{20}$ or $'80'b=52-'40'\Rightarrow b=\frac{3}{20}$	B1ft
	$2a+\frac{3}{5}=1\Rightarrow a=\frac{1}{5}$	B1ft
		(2)

Question Number	Scheme	Marks
3. (a)	$\int_2^5 \frac{1}{48}\left(x^2-8x+c\right)\mathrm{d}x=1$	M1
	$1=\frac{1}{48}\left[\frac{x^3}{3}-4x^2+cx\right]_2^5$	M1
	$1=\frac{1}{48}\left(\left(\frac{5^3}{3}-4(5^2)+5c\right)-\left(\frac{2^3}{3}-4(2^2)+2c\right)\right)$ <u>or</u> $48=39-84+3c$	
	$(\Rightarrow 3c=93\Rightarrow)c=31*$	A1cso* (3)
(b)	$\mathrm{P}(2<X<3)=\frac{1}{48}\left[\frac{x^3}{3}-4x^2+31x\right]_2^3$	M1
	$\frac{1}{48}\left(\left(\frac{3^3}{3}-4(3^2)+31(3)\right)-\left(\frac{2^3}{3}-4(2^2)+31(2)\right)\right)=\frac{13}{36}$ (=awrt 0.361)	A1
		(2)
(c)	Less than 3 since $"\frac{13}{36}">0.25$	B1 (1)
(d)	$x=4$ leads to the minimum/lowest value of f(x) / f(x) is a positive quadratic	B1 (1)
(e)	Considers $x=2$ and $x=5$ by e.g. • $\mathrm{f}(2)=0.39(58\dot{3})\,[=\frac{19}{48}]$ and $\mathrm{f}(5)=0.\dot{3}\,[=\frac{16}{48}]$ (so f(2) > f(5)) • Sketch of f(x) from $x=2$ to $x=5$ • $x=2$ is further than $x=4$ (then $x=5$)	M1
	Mode is $x=2$	A1 (2)
		[9 marks]

Question Number	Scheme	Marks
5. (a)	$\mathrm{F}(3) = 0 \rightarrow \frac{1}{16}\left(3^2 - 6(3) + a\right) = 0$	M1
	$a = 9$	A1
	$\mathrm{F}(10) = 1 \rightarrow \frac{1}{12}\left(100(10) - (5)10^2 + c\right) = 1$	M1
	$c = -488$	A1 (4)
(b)	$\frac{1}{16}\left(5^2 - 6(5) + "9"\right) = \frac{1}{12}\left(5 + b\right)$ | $\frac{1}{12}(9+b) = \frac{1}{12}\left(100(9) - 5(9^2) + "-488"\right)$	M1
	$b = -2$	A1
		(2)
(c)	$\mathrm{P}(6 < Y \leqslant 9) = \mathrm{F}(9) - \mathrm{F}(6)$	M1
	$= \frac{1}{12}(9 + "-2") - \frac{1}{12}(6 + "-2")$	M1
	$= \frac{1}{4}$	A1 (3)
(d)	$\mathrm{f}(y) = \frac{1}{12}$	B1 (1)
(e)	$\mathrm{E}(6Y - 5) = [26.5+]\int_5^9 (6y-5)"\tfrac{1}{12}"\,\mathrm{d}y$	M1
	$= [26.5+]\tfrac{1}{12}[(3y^2 - 5y)]_5^9$	dM1
	$= 26.5 + \tfrac{1}{12}[(3(9^2) - 5(9)) - (3(5^2) - 5(5))]$	dM1
	$= \frac{233}{6}$	A1
		(4)
		[Total 14]

Question Number	Scheme	Marks
2 (a)	[Mode =] 4	B1
		(1)
(b)	$\left[a\int_0^4 x^3\,\mathrm{d}x = \frac{1}{2} \Rightarrow\right] a\left[\frac{x^4}{4}\right]_0^4 = \frac{1}{2}$	M1
	$64a = \frac{1}{2} \Rightarrow a = \frac{1}{128}$ *	A1*
		(2)
(c)	 $\frac{1}{2}\times\frac{1}{2}\times(d-4) = \frac{1}{2}$ or $\frac{1}{2}\times\frac{1}{2}\times(d-4) + \int_0^4 ax^3\,\mathrm{d}x = 1$	M1
	$d = 6$	A1
		(2)
(d)	$b = \dfrac{-\frac{1}{2}}{'6'-4}\left[= -\frac{1}{4}\right]$ $4b + c = 0.5$ oe	M1
	$0 = '-\frac{1}{4}' \times '6' + c$ or $\frac{1}{2} = '-\frac{1}{4}' \times 4 + c$ $10b + 2c = 0.5$ oe or $'6'b + c = 0$ oe	M1
	$b = -\frac{1}{4}$ and $c = \frac{3}{2}$	A1
		(3)

Question Number	Scheme		Marks
6(a)	$\left[\mathrm{P}\left(Y<\frac{1}{4}k \mid Y<k\right)=\right]\frac{\mathrm{F}\left(\frac{1}{4}k\right)}{\mathrm{F}(k)}=\frac{\frac{1}{21}\left(\frac{k}{4}\right)^2}{\frac{1}{21}k^2}=\frac{1}{16}$ oe		M1 A1
			(2)
(b)	$\frac{1}{21}k^2=-\frac{1}{15}k^2+\frac{4}{5}k-\frac{7}{5}$	$\frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{1}{21}y^2\right)=\frac{2}{21}y$ or $\frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{2}{15}\left(6y-\frac{y^2}{2}\right)-\frac{7}{5}\right)=\frac{2}{15}(6-y)$	M1
	$\Rightarrow 4k^2-28k+49=0$ oe	$\frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{1}{21}y^2\right)=\frac{2}{21}y$ & $\frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{2}{15}\left(6y-\frac{y^2}{2}\right)-\frac{7}{5}\right)=\frac{2}{15}(6-y)$	A1
	$\Rightarrow (2k-7)^2=0$	$\frac{2}{21}k=\frac{2}{15}(6-k)$	M1
	$k=\frac{7}{2}$ oe		A1
			(4)
(c)	$\mathrm{f}(y)=\begin{cases}\frac{2}{21}y & 0\leqslant y\leqslant \text{'3.5'}\\ \frac{2}{15}(6-y) & \text{'3.5'}<y\leqslant 6\\ [0] & [\text{otherwise}]\end{cases}$		M1 M1
	$\mathrm{E}(Y)=\frac{2}{21}\int_0^{\text{'3.5'}}y^2\,\mathrm{d}y+\frac{2}{15}\int_{\text{'3.5'}}^6\left(6y-y^2\right)\mathrm{d}y \quad \Rightarrow \quad \frac{2}{21}\left[\frac{y^3}{3}\right]_0^{\text{'3.5'}}+\frac{2}{15}\left[3y^2-\frac{y^3}{3}\right]_{\text{'3.5'}}^6$		M1 M1
	$\frac{2}{21}\left(\frac{343}{24}\right)+\frac{2}{15}\left(\frac{325}{24}\right)=\frac{19}{6}=3.166...$ awrt 3.17		dM1 dA1
			(6)

Number		
4 (a)		M1 A1
		(2)
(b)	$\left[\mathrm{P}(G \leqslant 2) =\right] 1 - 2\times\frac{3}{20}[=0.7]$ or $\frac{1}{2}\times 3\times\left(\frac{2}{15}+\frac{1}{3}\right)$ or $\frac{1}{15}\int_{-1}^{2}(g+3)\,\mathrm{d}g[=0.7]$ or $\frac{1}{30}\times 2^2+\frac{1}{5}\times 2+\frac{1}{6}[=0.7]$ or $\left[\mathrm{P}\left(G \leqslant \frac{1}{2}\right)\right]=\frac{1}{2}\times 1.5\times\left(\frac{2}{15}+\frac{3.5}{15}\right)[=0.275]$ or $\frac{1}{15}\int_{-1}^{0.5}(g+3)\,\mathrm{d}g[=0.275]$ or $\frac{1}{30}\times 0.5^2+\frac{1}{5}\times 0.5+\frac{1}{6}[=0.275]$ or $\left[\mathrm{P}\left(\frac{1}{2} \leqslant G \leqslant 2\right)=\right]\ \frac{1}{2}\times 1.5\times\left(\frac{7}{30}+\frac{1}{3}\right)[=0.425]$ or $\frac{1}{15}\int_{0.5}^{2}(g+3)\,\mathrm{d}g[=0.425]$ or $\frac{1}{30}\times\left(2^2-0.5^2\right)+\frac{1}{5}\times(2-0.5)[=0.425]$	M1
	$\left[\mathrm{P}(1 \leqslant 2G \leqslant 6 \mid G \leqslant 2)=\right]\frac{\mathrm{P}\left(\frac{1}{2} \leqslant G \leqslant 2\right)}{\mathrm{P}(G \leqslant 2)}=\frac{0.425}{0.7}$ or $1-\frac{0.275}{0.7}$ oe	M1M1
	$=\frac{17}{28}$ or $0.607\ldots$ awrt 0.607	A1
		(4)
(c)	$\left[\mathrm{E}\left(H^2\right)=\right]2.4+12^2[=146.4]$	M1
	$\left[\mathrm{E}(G)=\right]\int_{-1}^{2}\frac{1}{15}\left(g^2+3g\right)\mathrm{d}g+\int_{2}^{4}\frac{3}{20}g\ \mathrm{d}g$	M1
	$\left[\mathrm{E}(G)=\right]\left(\frac{1}{15}\left(\frac{1}{3}g^3+\frac{3}{2}g^2\right)\right)_{-1}^{2}+\left(\frac{3}{40}g^2\right)_{2}^{4}$	M1
	$=\frac{1}{15}\left(\frac{8}{3}+\frac{12}{2}+\frac{1}{3}-\frac{3}{2}\right)+\left(\frac{48}{40}-\frac{12}{40}\right)[=1.4]$	dM1
	$\left[\mathrm{E}\left(2H^2+3G+3\right)=\right]2\times\text{"146.4"}+3\times\text{"1.4"}+3$	M1
	$=300$	A1 (6)
		Total 12

<table>
<tr><th>Question Number</th><th colspan="3">Scheme</th><th>Marks</th></tr>
<tr><td>7(a)</td><td colspan="3">$\mathrm{f}(x)=[k]\left(a+3bx^2-4x^3\right)$</td><td>M1</td></tr>
<tr><td></td><td colspan="3">$[k]\left(6bx-12x^2\right)=0$</td><td>M1</td></tr>
<tr><td></td><td colspan="3">$9b-27=0 \Rightarrow b=3$ or $6\times3\times1.5-12\times1.5^2=0 \Rightarrow \therefore b=3$ *</td><td>A1*</td></tr>
<tr><td></td><td colspan="3"></td><td>(3)</td></tr>
<tr><td>(b)</td><td colspan="3">$a+3-1-4=0$ oe $[\Rightarrow a=2]$</td><td>B1*</td></tr>
<tr><td></td><td colspan="3"></td><td>(1)</td></tr>
<tr><td>(c)</td><td colspan="3">$k\left(2\times2+3\times2^3-2^4-4\right)=1 \quad \left[\Rightarrow k=\frac{1}{8}\right]$</td><td>M1</td></tr>
<tr><td></td><td>$\mathrm{F}(x)=0.5$</td><td>$\mathrm{F}(x)=4$</td><td>$\mathrm{F}(x)=0$</td><td></td></tr>
<tr><td></td><td>$\mathrm{F}(1.4)=0.3988\ldots$
$\mathrm{F}(1.5)=0.5078\ldots$</td><td>$\mathrm{F}(1.4)=3.1904\ldots$
$\mathrm{F}(1.5)=4.0625\ldots$</td><td>$\mathrm{F}(1.4)=-0.8(096\ldots)$
$\mathrm{F}(1.5)=0.06(25\ldots)$</td><td>M1A1</td></tr>
<tr><td></td><td>$0.399<0.5<0.508$
therefore, the median lies between 1.4 and 1.5</td><td>$3.1904<4<4.0625$
therefore, the median lies between 1.4 and 1.5</td><td>$-0.8(096)<0<0.06(25)$
therefore, the median lies between 1.4 and 1.5</td><td>A1</td></tr>
<tr><td></td><td colspan="3">ALTERNATIVE M1A1A1 for $\mathrm{F}(x)=0$</td><td></td></tr>
<tr><td></td><td colspan="3">$x_1=2.91\ldots \quad x_2=1.49\ldots \quad x_3=-0.70\ldots$ So $x=1.49\ldots$ as $1\leqslant x\leqslant 2$</td><td>M1 A1</td></tr>
<tr><td></td><td colspan="3">$1.4<1.49\ldots<1.5$ [therefore, the median lies between 1.4 and 1.5]</td><td>dA1</td></tr>
<tr><td></td><td colspan="3"></td><td>(4)</td></tr>
</table>