

Chapter 3: Approximations

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5. A delivery company loses packages randomly at a mean rate of 10 per month.
- (e) Using a suitable approximation, find the probability that more than 27 packages are lost during a randomly selected 4-month period.

(5)

5. Cars stop at a service station randomly at a rate of 3 every 5 minutes.

Using a normal approximation, the probability that more than 40 cars will stop at the service station during a randomly selected n minute period is 0.2266 correct to 4 significant figures.

- (b) Find the value of n .

(9)

2. At a cafe, customers ordering hot drinks order either tea or coffee.
Of all customers ordering hot drinks, 80% order tea and 20% order coffee.
Of those who order tea, 35% take sugar and of those who order coffee 60% take sugar.

A randomly selected customer who orders a hot drink is chosen. The probability that the customer takes sugar is 0.4

- (d) In a random sample of 150 customers ordering hot drinks, find, using a suitable approximation, the probability that at least half of them take sugar.

(4)

6. (i) (a) State the conditions under which the Poisson distribution may be used as an approximation to the binomial distribution.

(1)

A factory produces tyres for bicycles and 0.25% of the tyres produced are defective.

A company orders 3000 tyres from the factory.

- (b) Find, using a Poisson approximation, the probability that there are more than 7 defective tyres in the company's order.

(3)

- (ii) At the company 40% of employees are known to cycle to work. A random sample of 150 employees is taken. The random variable C represents the number of employees in the sample who cycle to work.

- (a) Describe a suitable sampling frame that can be used to take this sample.

(1)

- (b) Explain what you understand by the sampling distribution of C

(1)

Louis uses a normal approximation to calculate the probability that at most α employees in the sample cycle to work. He forgets to use a continuity correction and obtains the incorrect probability 0.0668

Find, showing all stages of your working,

- (c) the value of α

(4)

- (d) the correct probability.

(2)

Question 6 continued

3.5

Answer ALL questions. Write your answers in the spaces provided.

1. A company produces NuttyChok bars that are made of chocolate and whole hazelnuts.

A 40 kg batch of the chocolate and hazelnut mixture used to make NuttyChok bars will contain 4750 hazelnuts.

The company also produces a 200 g bar of NuttyChok using the same chocolate and hazelnut mixture.

- (e) Use a suitable approximation to find the probability that a 200 g bar of NuttyChok contains at least 30 hazelnuts.

(5)

6. A machine is designed to cut bolts. The length, in millimetres, of each bolt is uniformly distributed over the interval $[39.5, 40.5]$

In a random sample of 120 bolts,

- (c) find, using a suitable approximation, the probability that more than 117 bolts can be used.

(4)

1. *Flogcar* is a car hire company that only allows cars to be hired for single days. The numbers of cars hired on different days are statistically independent. The number of cars hired on a randomly selected day from *Flogcar* is modelled by a Poisson distribution with mean 4

The probability that a car hired from *Flogcar* is returned in good condition is 0.97

In a randomly chosen period, *Flogcar* hires out 100 cars.

- (d) (i) Explain why the distribution of the number of cars returned in good condition would not be well approximated by the Poisson distribution.

(1)

- (ii) By considering the number of cars hired out that are **not** returned in good condition, use a Poisson approximation to estimate the probability that more than 95 of the cars hired out in this period are returned in good condition. Show your working clearly.

(3)

3. A manufacturer produces plates. The proportion of plates that are flawed is 45%, with flawed plates occurring independently.

George believes that the proportion of flawed plates is not 45%. To assess his belief George takes a random sample of 120 plates. The random variable F represents the number of flawed plates found in the sample.

- (b) Using a normal approximation, find the maximum number of plates, c , and the minimum number of plates, d , such that

$$P(F \leq c) \leq 0.05 \text{ and } P(F \geq d) \leq 0.05$$

where $F \sim B(120, 0.45)$

(7)

Question 3 continued

3.10

1. Jim farms oysters in a particular lake. He knows from past experience that 5% of young oysters do not survive to be harvested.

In a random sample of 30 young oysters, the random variable X represents the number that do not survive to be harvested.

A second random sample, of 200 young oysters, is taken.

The probability that at least n of these young oysters do not survive to be harvested is more than 0.8

- (d) Using a suitable approximation, find the maximum value of n .

(3)

3. The number of water fleas, in 100ml of pond water, has a Poisson distribution with mean 7

Using a normal approximation, the probability that more than 3 water fleas will be found in a random sample of n ml of the pond water is 0.9394 correct to 4 significant figures.

(c) (i) Show that $n - 1.55\sqrt{\frac{n}{0.07}} - 50 = 0$ (5)

(ii) Hence find the value of n (2)

6. The random variable $Y \sim B(225, p)$

Using a normal approximation, the probability that Y is at least 188 is 0.1056 to 4 decimal places.

(i) Show that p satisfies $145p^2 - 241p + 100 = 0$ when the normal probability tables are used.

(ii) Hence find the value of p , justifying your answer.

(10)

Question 6 continued

3.14

- 1 A local pottery makes cups. The number of faulty cups made by the pottery in a week follows a Poisson distribution with a mean of 6

- (b) Use a normal approximation to find the probability that in 6 randomly chosen weeks the total number of faulty cups made is fewer than 32

(4)

2. The time, in minutes, spent waiting for a call to a call centre to be answered is modelled by the random variable T with probability density function

$$f(t) = \begin{cases} \frac{1}{192}(t^3 - 48t + 128) & 0 \leq t \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$P(1 < T < 3) = \frac{7}{16}$$

A supervisor randomly selects 256 calls to the call centre.

- (c) Use a suitable approximation to find the probability that more than 125 of these calls take between 1 and 3 minutes to be answered.

(5)

4. The probability that a person completes a particular task in less than 15 minutes is 0.4

Jeffrey selects 20 people at random and asks them to complete the task. The random variable, X , represents the number of people who complete the task in less than 15 minutes.

Mia takes a random sample of 140 people.

Using a normal approximation, the probability that fewer than n of these 140 people complete the task in less than 15 minutes is 0.0239 to 4 decimal places.

- (b) Find the value of n

Show your working clearly.

(6)

Defects in the steel cable produced by this company occur at random, at a constant rate of 1 defect per 16 metres.

Using a normal approximation, the probability that this piece of steel cable has fewer than 26 defects is 0.5398

(8)

- (1)

- (1)

A random sample of 125 candles produced by the manufacturer is taken.

- (4)

7. The discrete random variable X is given by

$$X \sim B(n, p)$$

The value of n and the value of p are such that X can be approximated by a normal random variable Y where

$$Y \sim N(\mu, \sigma^2)$$

Given that when using a normal approximation

$$P(X < 86) = 0.2266 \quad \text{and} \quad P(X > 97) = 0.1056$$

- (a) show that $\sigma = 6$

(7)

- (b) Hence find the value of n and the value of p

(3)

Question 7 continued

3.22

1. The manager of a supermarket is investigating the number of complaints per day received from customers.

A random sample of 180 days is taken and the results are shown in the table below.

Number of complaints per day	0	1	2	3	4	5	6	≥ 7
Frequency	12	28	37	38	29	17	19	0

The manager uses a Poisson distribution with mean 3 to model the number of complaints per day.

A week consists of 7 consecutive days.

- (d) Using the manager's model and a suitable approximation, show that the probability that there are less than 19 complaints in a randomly selected week is 0.29 to 2 decimal places.

Show your working clearly.

(Solutions relying on calculator technology are not acceptable.)

(5)

A period of 13 weeks is selected at random.

- (e) Find the probability that in this period there are exactly 5 weeks that have less than 19 complaints.

Show your working clearly.

(3)

Question 1 continued

3.24

2. The length of pregnancy for a randomly selected pregnant sheep is D days where

$$D \sim N(112.4, \sigma^2)$$

Given that 5% of pregnant sheep have a length of pregnancy of less than 108 days,

Qiang selects 25 pregnant sheep at random from a large flock.

- (b) Find the probability that more than 3 of these pregnant sheep have a length of pregnancy of less than 108 days.

(2)

Charlie takes 200 random samples of 25 pregnant sheep.

- (c) Use a Poisson approximation to estimate the probability that at least 2 of the samples have more than 3 pregnant sheep with a length of pregnancy of less than 108 days.

(3)

Question 2 continued

3.26

- 5 A receptionist receives incoming telephone calls and should connect them to the appropriate department. The probability of them being connected to the wrong department on the first attempt is 0.05

A random sample of 8 calls is taken.

- (a) Find the probability that at least 2 of these calls are connected to the wrong department on the first attempt.

(3)

The receptionist receives 1000 calls each day.

- (b) Use a Poisson approximation to find the probability that exactly 45 callers are connected to the wrong department on the first attempt in a day.

(3)

The total time, T seconds, taken for a call to be answered by a department has a continuous uniform distribution over the interval $[10, 50]$

- (c) Find $P(T > 16)$

(2)

The number of calls the receptionist receives in a one-minute interval is modelled by a Poisson distribution with mean 6

The receptionist receives a call from Jia and tries to connect it to the right department.

- (d) Find the probability that in the next 40 seconds Jia's call is answered by the right department on the first attempt and the receptionist has received no other calls.

(4)

Question 5 continued