# Edexcel International AS Maths: Statistics 1

Your notes

## Further Probability

### Contents

# Set notation

## What is set notation?

- **Set notation** is a formal way of writing groups of numbers (or other mathematical entities such as shapes) that share a common feature – each number in a set is called an **element** of the set
  - You should have come across common sets of numbers such as the **natural numbers**, denoted by $\mathbb{N}$ , or the set of **real numbers**, denoted by $\mathbb{R}$
- In **probability**, set notation allows us to talk about the **sample space** and **events** within in it
  - S, U, $\xi$, $\mathscr{E}$ are common symbols used for the Universal set In probability this is the entire sample space, or the rectangle in a **Venn diagram**
  - **Events** are denoted by capital letters, *A*, *B*, *C* etc
  - The **events** "*not* $A$ ", "*not* $B$", "*not* $C$" are denoted by $A'$, $B'$, $C'$ etc (Strictly pronounced "*A* prime" but often called "*A* dash") $A'$ is called the **complement** of *A*
- In probability we are often looking at combined events
  - The event *A* and *B* is called the **intersection** of events *A* and *B* , and the symbol ∩ is used i.e. *A* and *B* is written as $A \cap B$
    - On a **Venn diagram** this would be the **overlap** between the bubble for event *A* and the bubble for event $B$
    - From **Basic Probability**, for **independent** events

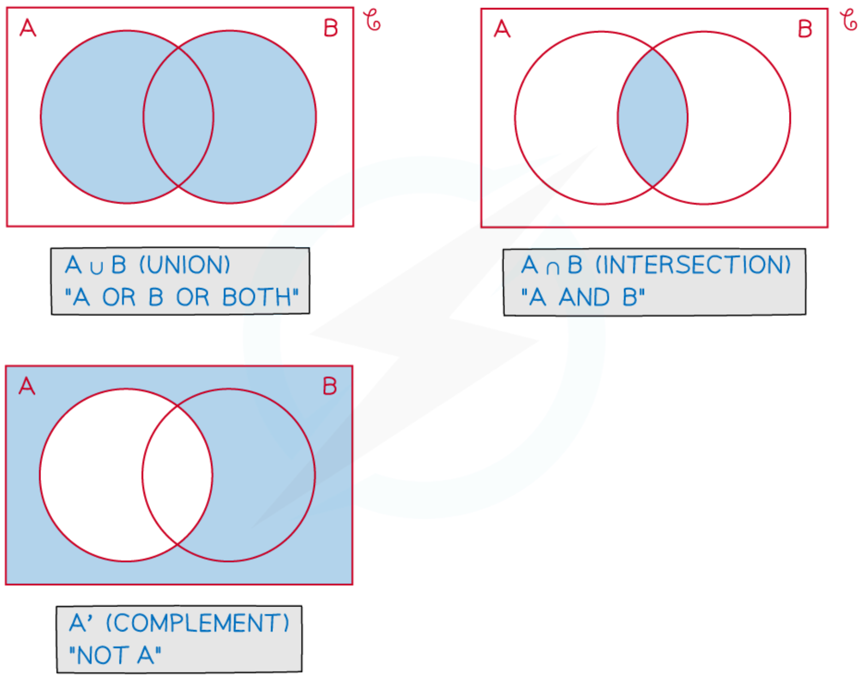
$$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$$

- The event *A* or *B* is called the **union** of events *A* and *B* , and the symbol ∪ is used i.e. *A* or *B* is written as $A \cup B$
  - On a **Venn diagram** this would be **both** the bubbles for event A and event B including their **overlap** (**intersection**)
  - From **Basic Probability**, for **mutually exclusive** events

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$$

- The other set you may come across in probability is the **empty set** The **empty set** has no **elements** and is denoted by $\varnothing$
- The **intersection** of **mutually exclusive** events is the empty set, $\varnothing$
- And finally, $\mathrm{P}(A') = 1 - \mathrm{P}(A)$






## How do I find probabilities from sets?

- Recognise the notation and symbols used and then interpret them in terms of **AND** ( $\cap$ ), **OR** ( $\cup$ ) and/or **NOT** (') statements
- Venn diagrams lend themselves particularly well to deducing which sets or parts of sets are involved- draw mini-Venn diagrams and shade them
- Practice shading various parts of Venn diagrams and then writing what you have shaded in set notation
- With combinations of **union**, **intersection** and **complement** there may be more than one way to write the set required
  - e.g. $(A \cup B)' = A' \cap B'$   $(A \cap B)' = A' \cup B'$   Not convinced? Sketch a Venn diagram and shade it in!
  - In such questions it can be the **unshaded** part that represents the solution

### Worked Example

The members of a local tennis club can decide whether to play in a singles competition, a doubles competition, both or neither.

Once all members have made their choice the chairman of the club selects, at random, one member to interview about their decision.

$S$ is the event a member selected the singles competition.

$D$ is the event a member selected the doubles competition.

Given that $\mathrm{P}(S) = 2\mathrm{P}(D)$, $\mathrm{P}(S \cup D) = 0.9$ and $\mathrm{P}(S \cap D) = 0.3$, find

(i) $\mathrm{P}(S')$

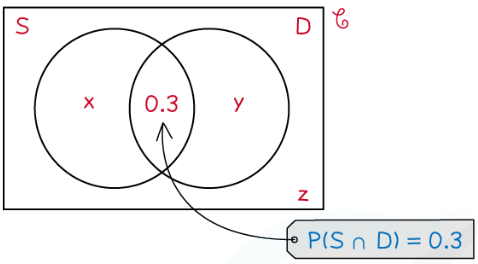
(ii) $\mathrm{P}(S' \cap D)$

(iii) $\mathrm{P}(S \cup D')$

(iv) $\mathrm{P}((S \cup D)')$

**Answer:**

Sketch a Venn diagram and use it to help work out the missing probabilities




$\mathrm{P}(S) = 2\mathrm{P}(D)$ $\quad \therefore x + 0.3 = 2(0.3 + y)$

$x - 2y = 0.3$

$\mathrm{P}(S \cup D) = 0.9$ $\quad \therefore x + 0.3 + y = 0.9$

$x + y = 0.6$

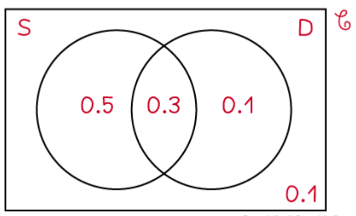
Solve simultaneously, $3y = 0.3$, $\quad y = 0.1$

$x = 0.5$

As all probabilities total 1, $0.5 + 0.3 + 0.1 + z = 1$

$z = 0.1$

Final Venn diagram



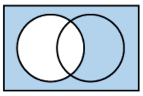





Your notes

Now use this and, if they help, shaded mini–Venn diagrams to answer the questions

(i)

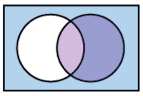


"not S"
0.1 + 0.1 = 0.2

P(S') = 1 − P(S) = 1 − (0.5 + 0.3)

P(S') = 0.2

(ii) P(S' ∩ D)

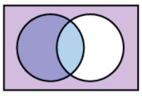


Blue shading is "not S
Purple shading is D
∩ is AND, so part shaded twice is the answer

P(S' ∩ D) = 0.1



(iii) P(S ∪ D')

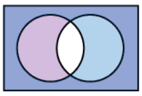


Blue shading is S
Purple shading is "not D"
∪ is OR, so all shaded parts are needed

P(S ∪ D') = 0.5 + 0.3 + 0.1 = 0.9
(You may also spot from the mini–Venn diagram this is 1 − 0.1)

(iv) P((S ∪ D)')

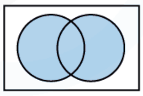
Method 1
P((S ∪ D)') = P(S' ∩ D')



Answer is the region shaded twice

P((S ∪ D)') = 0.1

Method 2
Start by shading S ∪ D



(S ∪ D)' is "not S ∪ D" so the answer is the unshaded region



### Examiner Tips and Tricks

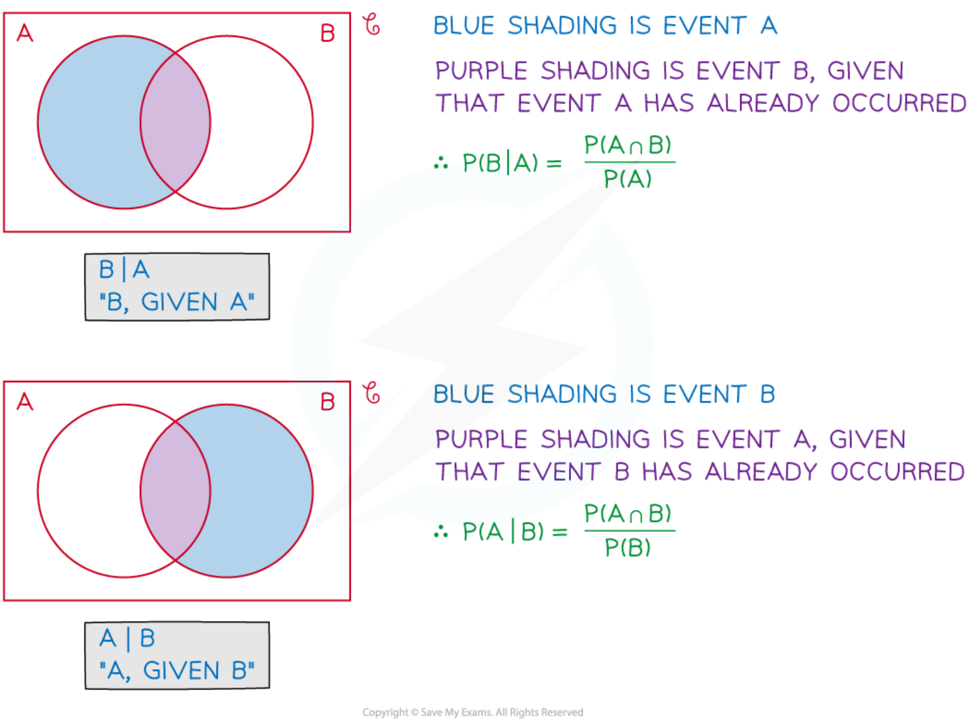
- Do not try to do everything using a single diagram – whether given one in the question or using your own; use mini-Venn diagrams and shading for each part of a question
- Do double check whether you are dealing with **union** ( ⋃ ) or **intersection** ( ⋂ ) (or both) – when these symbols are used several times near each other in a question, it is easy to get them muddled up or misread them

# Conditional probability

Your notes

# What is conditional probability?

- **Conditional probability** is where the probability of an **event** happening can vary depending on the outcome of a prior event
- You have already been using **conditional probability** e.g. drawing more than one counter/bead/etc from a bag **without** replacement
  - Note that, mathematically, that drawing one, not replacing then drawing another is the same as drawing two at the same time.
- Consider the following example
  - e.g. Bag with 6 white and 3 red buttons. One is drawn at random and not replaced. A second button is drawn. The probability that the second button is white **given that** the first button is white is $\frac{5}{8}$.
- The key phrase here is "**given that**" – it essentially means something has already happened.
  - In set notation, "**given that**" is indicated by a vertical line ( **|** ) so the above example would be written $\mathrm{P}(2^{\text{nd}} \text{ is white} | \ 1^{\text{st}} \text{ is white}) = \frac{5}{8}$
  - There are other phrases that imply or mean the same things as "given that"
- Venn diagrams are helpful again but beware – the denominator of fractional probabilities will no longer be the total of all the frequencies or probabilities shown
  - "**given that**" questions usually reduce the sample space as an event (a subset of the outcomes of the first event) has already occurred



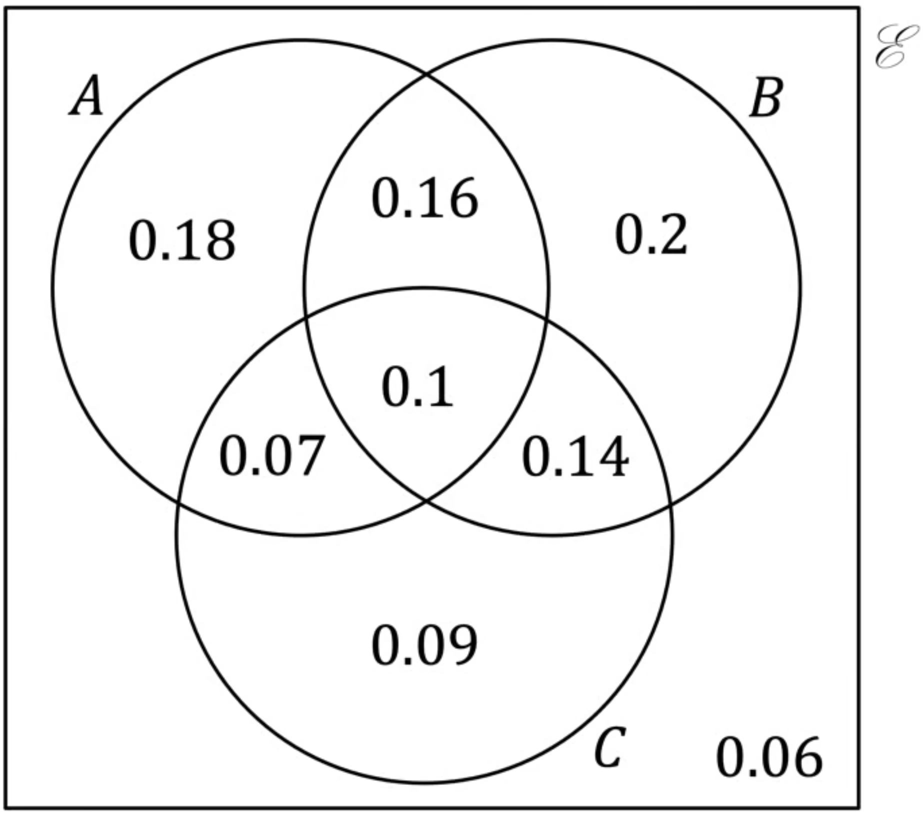

Your notes

- The diagrams above also show two more **conditional probability** results
  - $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B | A)$
  - $\mathrm{P}(A \cap B) = \mathrm{P}(B) \times \mathrm{P}(A | B)$
  - These are essentially the same as letters are interchangeable
- For **independent events** we know $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ so
  - $\mathrm{P}(B | A) = \dfrac{\cancel{\mathrm{P}(A)} \times \mathrm{P}(B)}{\cancel{\mathrm{P}(A)}} = \mathrm{P}(B)$
- and similarly $\mathrm{P}(A | B) = \mathrm{P}(A)$
- The independent result should make sense logically – if events *A* and *B* are independent then the fact that event *B* has already occurred has no effect on the probability of event *A* happening

Your notes

### Worked Example

The Venn diagram below illustrates the probabilities of three events, $A$, $B$ and $C$.




(a) Find

(i) $P(A|B)$

(ii) $P(B|A')$

(iii) $P(C'|A')$

(b) Show, in two different ways, that the events $B$ and $C$ are independent.

**Answer:**

Before starting we can work out some basic probabilities that are likely to be useful

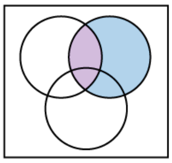
$P(A) = 0.18 + 0.16 + 0.1 + 0.07 = 0.51$

$P(B) = 0.16 + 0.2 + 0.1 + 0.14 = 0.6$

$P(C) = 0.07 + 0.1 + 0.14 + 0.09 = 0.4$



a) i) Mini 'Venn'

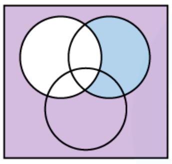


Formula: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$\therefore P(A|B) = \dfrac{0.16 + 0.1}{0.6}$ $\quad P(A|B) = \dfrac{13}{30}$

Use fractions rather than rounding

ii) Mini 'Venn'

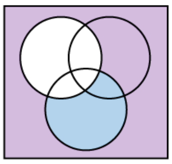


Formula: $P(B|A') = \dfrac{P(A' \cap B)}{P(A')}$

$\therefore P(B|A') = \dfrac{0.2 + 0.14}{1 - 0.51}$ $\quad P(B|A') = \dfrac{34}{49}$

$P(A') = 1 - P(A)$

iii) Mini 'Venn'



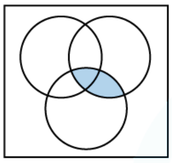
Formula: $P(C'|A') = \dfrac{P(A' \cap C')}{P(A')}$

$\therefore P(C'|A') = \dfrac{0.2 + 0.06}{1 - 0.51}$ $\quad P(C'|A') = \dfrac{26}{49}$

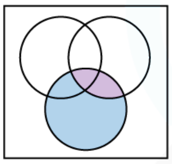


Your notes

b) For independent events, "$P(A \cap B) = P(A) \times P(B)$"
and, "$P(A|B) = P(A)$"



$P(B \cap C) = 0.1 + 0.14 = 0.24$
$P(B) \times P(C) = 0.6 \times 0.4 = 0.24$
∴ Events B and C are independent



$P(B|C) = \frac{0.1 + 0.14}{0.4} = 0.6$
$P(B) = 0.6$
∴ Events B and C are independent

(For the second method, you could show $P(C|B) = P(C)$ instead:
$P(C|B) = \frac{0.24}{0.6} = 0.4$, $P(C) = 0.4$)



Your notes

### Examiner Tips and Tricks

- There are now several symbols used from set notation in probability – make sure you are familiar with them
  - **union** ( $\cup$ )
  - **intersection** ( $\cap$ )
  - **not** (‘)
  - **given that** ( | )
- If given a Venn diagram with all the separate probabilities you may find it easier to work out P(A), P(B) etc first

# Two-way tables

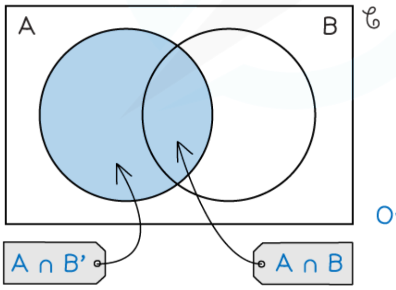
## What are two-way tables?

- In **probability**, **two-way tables** list the frequencies for the outcomes of two events – one event along the top (columns), one event down the side (rows)
- The frequencies, along with a “Total” row and “Total” column instantly show the values involved in finding probabilities

For two events, A and B,

| | A | A’ | Total |
|---|---|---|---|
| B | A ∩ B | A’ ∩ B | B |
| B’ | A ∩ B’ | A’ ∩ B’ | B’ |
| Total | A | A’ | ℰ |

Try sketching a Venn diagram
for each ‘total’ cell.
Here’s the one for A:




Overall: A



## How do I find probabilities from two-way tables?

- Questions will usually be wordy – and may not even mention two-way tables
  - Questions will need to be interpreted in terms of **AND** ( ⋂ , **intersection**), **OR** ( ⋃ , **union**), **NOT** (‘) and **GIVEN THAT** ( | )
- Complete as much of the table as possible from the information given in the question
  - If any empty cells remain, see if they can be calculated by looking for a row or column with just one missing value
- Each cell in the table is **similar** to a region in a Venn diagram
  - With event *A* outcomes on columns and event *B* outcomes on rows
    - $P \cap Q$ (**intersection, AND**) will be the cell where outcome $P$ meets outcome $Q$
    - $P \cup Q$ (**union**, **OR**) will be all the cells for outcomes $P$ and $Q$ including the cell for both
  - Beware! As **union** includes the cell for **both** outcomes, avoid counting this cell **twice** when calculating **frequencies** or **probabilities**
  - (see Worked Example Q(b)(ii))
- You may need to use the results
  - $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B \mid A)$

- $\mathrm{P}(A|B) = \mathrm{P}(A)$ (for independent events)

Your notes

## Worked Example

The incomplete two-way table below shows the type of main meal provided by 80 owners to their cat(s) or dog(s).

| | Dry Food | Wet Food | Raw Food | **Total** |
|---|---|---|---|---|
| Dog | 11 | | 8 | |
| Cat | | 19 | | 33 |
| **Total** | 21 | | | |

(a) Complete the two-way table

(b) One of the 80 owners is selected at random. Find the probability

(i) the selected owner has a cat and feeds it raw food for its main meal.

(ii) the selected owner has a dog or feeds it wet food for its main meal.

(iii) the owner feeds raw food to its pet, given it is a dog.

(iv) the owner has a cat, given that they feed it dry food.

**Answer:**

a) The key valve here is the grand total of 80 – given in the wording of the question

| | Dry food | Wet food | Raw food | Total |
|---|---|---|---|---|
| Dog | 11 | 28 | 8 | 47 |
| Cat | 10 | 19 | 4 | 33 |
| Total | 21 | 47 | 12 | 80 |

b) (i) $P(\text{cat} \cap \text{raw}) = \frac{4}{80}$

(ii) $P(\text{dog} \cup \text{wet}) = \underbrace{\frac{47}{80}}_{P(\text{dog})} + \underbrace{\frac{47}{80}}_{P(\text{wet})} - \frac{28}{80} = \frac{33}{40}$

Beware! $P(\text{dog} \cap \text{wet})$ would've been counted twice

(iii) $P(\text{raw} \mid \text{dog}) = \frac{8}{47}$

'Dog' 'raw food'

It's given "it's a dog"

(iv) $P(\text{cat} \mid \text{dry}) = \frac{10}{21}$

'Cat' 'dry food'

It's given "dry food"



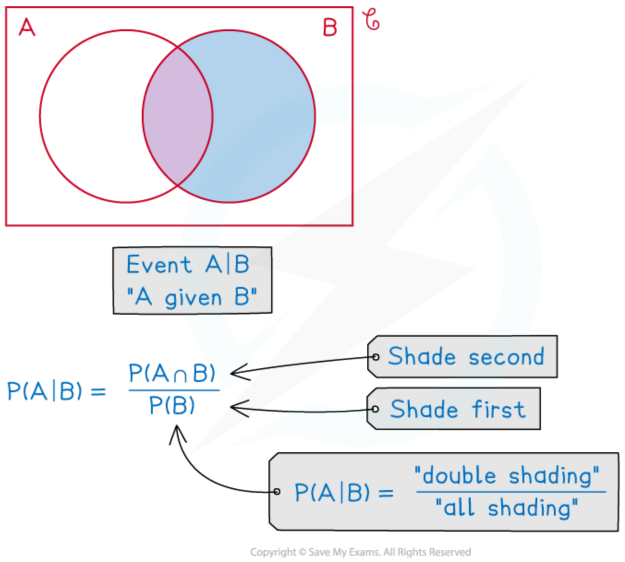
## Examiner Tips and Tricks

- Ensure any table – given or drawn - has a "Total" row and a "Total" column
- Do not confuse a two-way table with a **sample space** diagram – a two-way table does not necessarily display **all outcomes** from an **experiment**, just those (**events**) we are interested in

# Venn diagrams with conditional probability

## How do I find conditional probabilities from a Venn diagram?

- Interpreting questions in terms of **OR** ( $\cup$ ), **AND** ( $\cap$ ), **complement** ( ' ) and **"given that"** ( | )
- **Conditional probability** may now be involved too
- Use mini-Venn diagrams to sketch and shade the regions you are dealing with – use different colours if available or different styles of shading if not
  - Shading can help you see the answer
- since $\mathrm{P}(A|B)=\dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$ shade $B$ first, then shade $A \cap B$
  - the answer will then be $\dfrac{\text{" double shading "}}{\text{" all shading "}}$




### Worked Example

Three events, $A$, $B$ and $C$ are such that events $A$ and $C$ are mutually exclusive

$$P(A \cap B) = 0.2$$
$$P(B \cap C) = 0.3$$
$$P((A \cup B \cup C)') = 0.1$$
$$P(B) = 0.7$$
$$P(A') = 0.75.$$

(a) Draw a complete Venn diagram to show the probabilities connecting the three events.

(b) Find

(i) $P(A \mid B)$

(ii) $P(A' \mid C')$

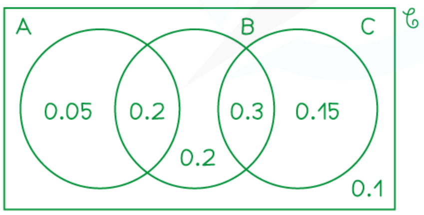
(iii) $P(C \mid (A \cup B)')$

**Answer:**

a) Events A and C are mutually exclusive so their bubbles will not overlap.

$P(A \cap B) = 0.2$
$P(B \cap C) = 0.3$
$P(A \cup B \cup C)' = 0.1$ $\therefore$ P("not (A or B or C)") = 0.1
$P(B) = 0.7$, $\therefore$ P("only B") = 0.2
$P(A') = 0.75$, $\therefore$ P("only A") = 1 − 0.75 − 0.2 = 0.05
and so P("only C")
= 1 − (0.05 + 0.2 + 0.2 + 0.3 + 0.1) = 0.15

(Check by seeing if all probabilities total 1 and so no single probability is greater than 1)






Your notes

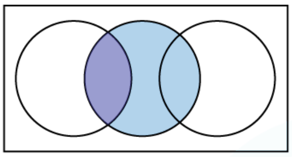
Your notes

b) Use mini–Venn diagrams and/or formulae
Working out P(A), etc. first can help cut down later work.

$P(A) = 1 - 0.75 = 0.25$ (From information given)
$P(B) = 0.7$ (Given in question)
$P(C) = 0.3 + 0.15 + 0.45$ (From Venn digaram)

i) $P(A|B)$

Mini–Venn:



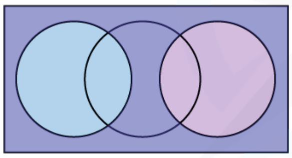
Formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.7}$$

$$P(A|B) = \frac{2}{7}$$

ii) $P(A'|C')$

Mini–Venn:



Formula:

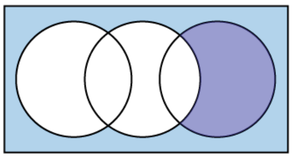
$$P(A'|C') = \frac{P(A' \cap C')}{P(C')}$$

$$= \frac{0.2 + 0.1}{1 - 0.45}$$

$$P(A'|C') = \frac{6}{11}$$

iii) $P(C|(A \cup B)')$

Mini–Venn:



Formula:

$$P(C|(A \cup B)') = \frac{P(C \cap (A \cup B)')}{P((A \cup B)')}$$

$$= \frac{0.15}{0.15 + 0.1}$$

$$P(C|(A \cup B)') = \frac{3}{5}$$



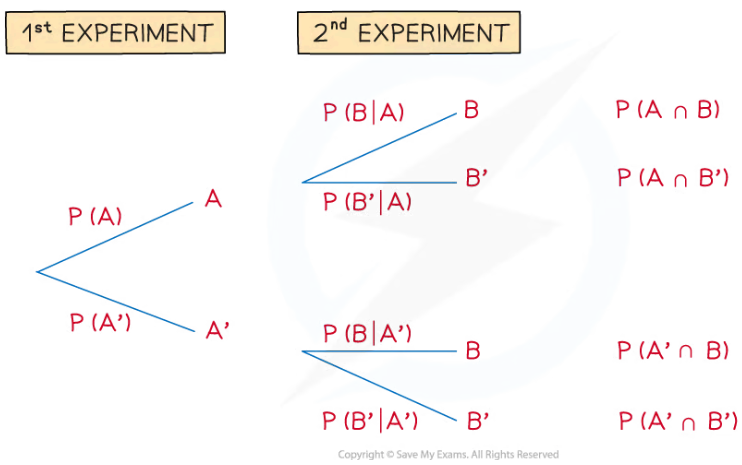
## Examiner Tips and Tricks

- Always draw the box in a Venn diagram; it represents all possible outcomes of the experiment so is a crucial part of the diagram, the bubbles merely represent the events we are particularly interested in
- You may be able to answer some questions by applying formulae or you may prefer to use shaded mini-Venn diagrams; complicated questions tend to be easier with mini-Venn diagrams

# Tree diagrams with conditional probability

## How do I find conditional probabilities problems from tree diagrams?

- Interpreting questions in terms of **AND** ( $\cap$ ), **OR** ( $\cup$ ) and **complement** ( ' )
- **Conditional probability** may now be involved too – **"given that"** ( | )
- This makes it harder to know where to start and how to complete the probabilities on a tree diagram
    - e.g. If given, possibly in words, $P(B|A)$ then **event *A*** has already occurred so start by looking for the branch **event *A*** in the **1st experiment,** and then there would be the branch for **event *B*** in the **2nd experiment**
- Similarly, $P(B|A')$ would require starting with **event "*not A*"** in the **1st experiment** and **event *B*** in the **2nd experiment**




- The diagram above gives rise to some probability formulae you will see in the next revision note
- $\mathbf{P(B|A)}$ ("**given that**") is the probability on the branch of the 2nd experiment
- However, the "**given that**" statement $\mathbf{P(A|B)}$ is more complicated and a matter of working backwards
    - from Conditional Probability, $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

- from the diagram above, $P(B) = P(A \cap B) + P(A' \cap B)$
- leading to $\mathrm{P}(A|B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B)}$
- This is quite a complicated looking formula to try to remember so use the logical steps instead – and a clearly labelled tree diagram!

Your notes

## Worked Example

The event $F$ has a 75% probability of occurring.

The event $W$ follows event $F$, and if event $F$ has occurred, event $W$ has an 80% chance of occurring.

It is also known that $\mathrm{P}(F' \cap W) = 0.15$.

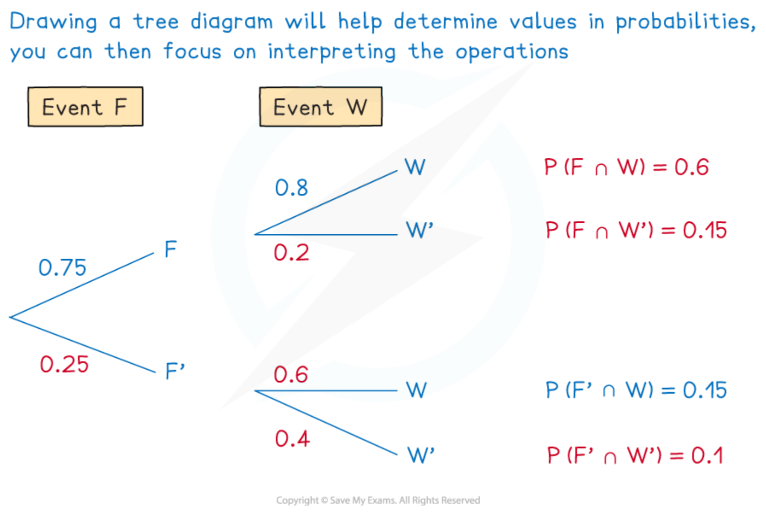
Find

(i) $\mathrm{P}(W|F')$

(ii) $\mathrm{P}(F|W')$

(iii) the probability that event $F$ didn't occur, given that event $W$ didn't occur.

**Answer:**



i) $P(W|F')$ F' on "$1^{st}$ experiment', W on "$2^{nd}$"

$\therefore P(W|F') = 0.6$

ii) $P(F|W')$ Use "$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$" and logic...

$P(F \cap W') = 0.15$

$P(W') = P(F \cap W') + P(F' \cap W')$

$= 0.15 + 0.1 = 0.25$

$\therefore P(F|W') = \dfrac{0.15}{0.25}$

$\therefore P(F|W') = 0.6$

iii) This is the same as $P(F'|W')$ and so a similar formula and logic would be recommended

But to show how the formula in the revision note would apply, we've used that below

$$P(F'|W') = \frac{P(F' \cap W')}{P(F \cap W') + P(F' \cap W')}$$

$$= \frac{0.1}{0.15 + 0.1}$$

$$= \frac{0.1}{0.25}$$

$\therefore P(F'|W') = 0.4$



Your notes

## Examiner Tips and Tricks

- It can be tricky to get a tree diagram looking neat and clear first attempt – it can be worth drawing a rough one first, especially if there are more than two outcomes or more than two events; do keep an eye on the exam clock though!
- Always worth another mention – tree diagrams make particularly frequent use of the result $\mathrm{P}(\text{not } A) = 1 - \mathrm{P}(A)$
- Tree diagrams have built-in checks
  - the probabilities for each pair of branches should add up to 1
  - the probabilities for each outcome of combined events should add up to 1

# Probability formulae

## What are probability formulae?

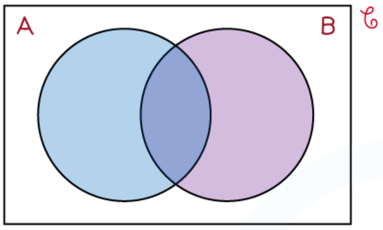
- If you have seen any of the other Revision Notes on probability, almost every one contains a formula
  - These however are often disguised in Venn diagrams, tree diagrams etc
  - Sometimes a diagram can be easier to use than a formula
- This Revision Note rounds up all these formulae and introduces one or two new ones

## What probability formulae do I need to know?

- Some formulae only apply under certain conditions
- The formulae for probability you should've encountered so far
  - for **independent** events, $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ **("AND", intersection)**
  - for **mutually exclusive** events, $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$ **("OR", union)**
  - $\mathrm{P}(A') = 1 - \mathrm{P}(A)$ **("NOT", complement/prime/dash)**
  - $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ or $\mathrm{P}(A \cap B) = \mathrm{P}(B) \times \mathrm{P}(A|B)$ **("GIVEN THAT")**

    This second version in particular is referred to as the **MULTIPLICATION FORMULA** (see diagram below)
  - For **independent** events, $\mathrm{P}(A|B) = \mathrm{P}(A)$

Consider the shaded parts of the Venn diagram




P(A) is shaded blue
P(B) is shaded purple
P(A ∩ B) is shaded twice
P(A ∪ B) is shaded at least once

From this we can deduce
P(A ∪ B) = P(A) + P(B) − P(A ∩ B)

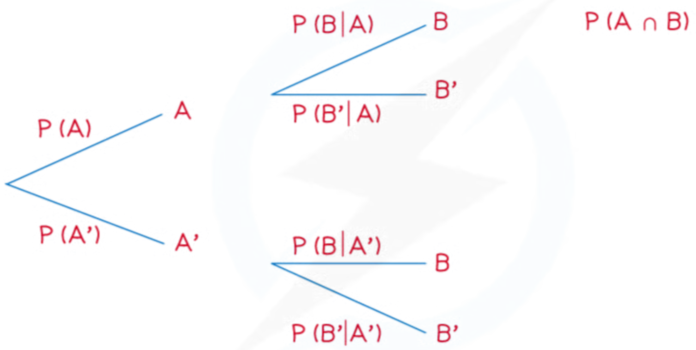
Subtract the region that gets counted twice

This is called the ADDITION FORMULA
It can easily be rearranged to

P(A ∩ B) = P(A) + P(B) − P(A ∪ B)



There is also a MULTIPLICATION FORMULA
Consider the top branches (or any pair really) of this tree diagram




So we can deduce

P(A ∩ B) = P(A) × P(B|A)



- All of the formulae above are given in the formulae booklet but some are "hidden"
  - e.g. The formulae booklet lists the **addition formula** $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
  - but it does not list the rearranged version of it
  - nor does it point out that if $A$ and $B$ are mutually exclusive then $P(A \cap B) = 0$

Your notes

- and so $P(A \cup B) = P(A) + P(B)$

## How do I use probability formulae?

- This is a combination of
  - Recognising the **set notation** used
  - Taking note of any **independent** or **mutually exclusive** events
  - Using an appropriate diagram (Venn, mini-Venn, tree, two-way table)
  - Converting worded questions into **AND**, **OR**, **GIVEN THAT** etc statements
  - Knowing when a formula could be used and using the **formulae booklet** to double-check and confirm
- In the majority of cases a diagram can be used – so if you do not like using the formulae or find them confusing
  - For events that happen in succession, use a tree diagram; Venn diagrams suit most other purposes but two-way tables can be easier to follow
  - Of course, a question may give or demand the use of a particular diagram

Your notes

### Worked Example

In the fictional World Stare Out Championships players compete by staring at each other with the player blinking first losing. A match cannot be drawn.

During each day of the championships three matches are scheduled to take place but if the first two matches both take more than an hour each, then the third match cannot take place that day.

A statistician notices from past fictional championship records that the probability of the first match taking longer than an hour is 0.15 and that the probability of only two matches taking place on any day is 0.06. They also notice that the probability that at least one of the first two matches takes longer than an hour is 0.32.

Find the probability that

(i) the second match of a day takes longer than an hour to complete

(ii) the first match takes longer than an hour given that the second match takes longer than an hour.

**Answer:**

Let $M_i$ be the event match i takes more than an hour to complete
The question tells us

$P(M_1) = 0.15$ (first match more than an hour)

$P(M_1 \cap M_2) = 0.06$ (only 2 matches – i.e both 1st two matches take more than an hour)

$P(M_1 \cup M_2) = 0.32$ (at least 1 match takes longer than an hour)

(i) $P(M_1 \cap M_2)$ is required. Using "$P(A \cap B) = P(A) + P(B) - P(A \cup B)$"

$P(M_1 \cap M_2) = P(M_1) + P(M_2) - P(M_1 \cup M_2)$

$0.06 = 0.15 + P(M_2) - 0.32$

$\therefore P(M_2) = 0.23$

(ii) $P(M_1 \mid M_2)$ is required. Using "$P(A \cap B) = P(B) \times P(A \mid B)$"

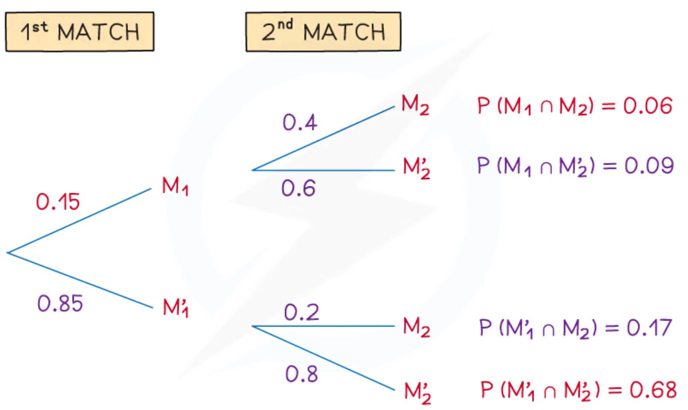
$P(M_1 \cap M_2) = P(M_2) \times P(M_1 \mid M_2)$

$0.06 = 0.23 \times P(M_1 \mid M_2)$

$\therefore P(M_1 \mid M_2) = \dfrac{6}{23}$

Note:
The context of this question has one experiment (match 2) following another (match 1) so lends itself to a tree diagram.
If you're confident using tree diagrams, drew one and use it. Do be aware that some questions like part (ii) above are not obvious from a diagram and so the formulae are still important. For completeness the final tree diagram would be.








## Examiner Tips and Tricks

- If in any doubt always start with a diagram
  - a **Venn diagram** can be used for most problems
  - a **two-way table** can be easier to read if it's possible to construct one
  - a **tree diagram** is useful if you are looking at an event that follows another

Your notes

- Remember that all probability formulae are given in the formulae booklet but not necessarily in the most user-friendly way; a quick look could just be enough to jog your memory though!

Your notes