Edexcel International A Level (IAL) Maths: Pure 4

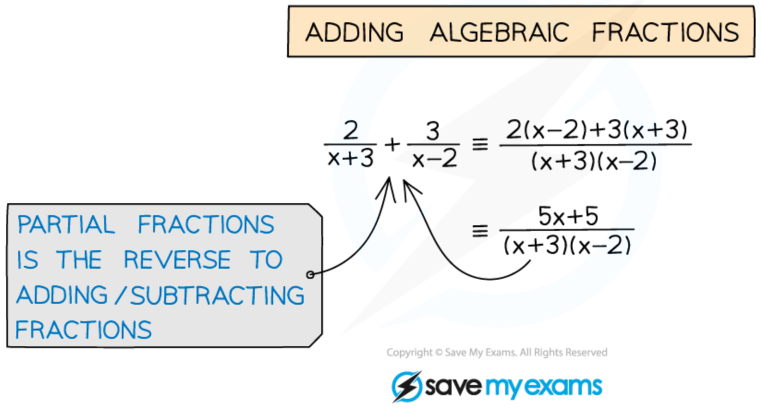
Your notes

Partial Fractions

Contents

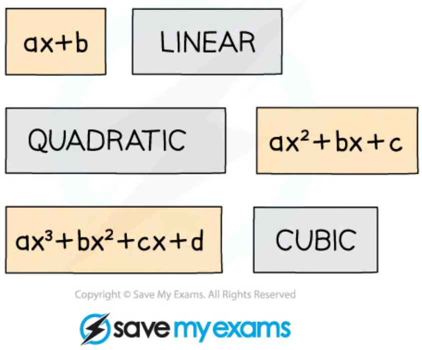
Linear Denominators

What are partial fractions?



- This is the reverse process to **adding** (or subtracting) fractions
- When adding fractions a common denominator is required
- In partial fractions the common denominator is split into parts (factors)
- Partial fractions are used in **binomial expansions** (see Multiple GBEs) and **integration** (see Integration by Parts)

What are linear denominators?



- A **linear factor** is of the form (**ax + b**)
- A **non-linear denominator** may be written as the **product** of linear factors

- If the denominator can be **factorised**

$$\frac{3x}{2x^3+5x^2+2x} \equiv \frac{3x}{x(2x+1)(x+2)}$$

FACTORISE INTO LINEAR TERMS

How do I find partial fractions?

STEP 1 **Factorise** the **polynomial** in the **denominator**

(Sometimes the numerator can be factorised too)

STEP 2 **Split** the fraction into a **sum** with **single** linear denominators

STEP 3 Multiply by the denominator to get rid of fractions

STEP 4 **Substitute** values of **x** to find **A**, **B**, etc

(An alternative method is **comparing coefficients**)

STEP 5 Write the original as partial fractions

Your notes

$$\frac{3x-2}{5x^2+3x-2} \equiv \frac{3x-2}{(5x-2)(x+1)}$$

STEP 1: FACTORISE THE DENOMINATOR

$$\equiv \frac{A}{5x-2} + \frac{B}{x+1}$$

STEP 2: SPLIT THE FRACTION

$$3x-2 = A(x+1) + B(5x-2)$$

STEP 3: MULTIPLY THROUGH BY THE DENOMINATOR

LET $x=-1$ $\quad -5=-7B$

$B=\frac{5}{7}$

LET $x=\frac{2}{5}$ $\quad \frac{-4}{5}=\frac{7}{5}A$

$A=\frac{-4}{7}$

STEP 4: CHOOSE VALUES OF x TO MAKE FINDING A AND B EASY

$$\frac{3x-2}{5x^2+3x-2} \equiv \frac{-4}{7(5x-2)} + \frac{5}{7(x+1)}$$

STEP 5: WRITE AS PARTIAL FRACTIONS

Comparing coefficients

- The quantity of each term must be equal on both sides
- "The number of $\mathbf{x^2}$ on the LHS" = "The number of $\mathbf{x^2}$ on the RHS"
- "The number of ..." is called the **coefficient** of x^2

$$3x-2 = A(x+1) + B(5x-2)$$

COMPARE COEFFICIENTS OF x: $\quad 3=A+5B \quad (*)$

COMPARE CONSTANT TERMS: $\quad -2=A-2B \quad (**)$

STEP 4 ALTERNATIVE: COMPARING COEFFICIENTS

$(*) - (**)$: $\quad 5=7B \qquad \therefore B=\frac{5}{7}$

$3=A+\frac{25}{7} \qquad \therefore A=\frac{-4}{7}$

$(*)$ & $(**)$ CAN BE SOLVED SIMULTANEOUSLY

Worked Example

Your notes

Express $\dfrac{2x+5}{3x^3-12x}$ in partial fractions.

$$\frac{2x+5}{3x^3-12x} \equiv \frac{2x+5}{3x(x^2-4)}$$

$$\equiv \frac{2x+5}{3x(x+2)(x-2)}$$

STEP 1: FACTORISE THE DENOMINATOR

STEP 2: SPLIT THE FRACTION

$$\frac{2x+5}{3x(x+2)(x-2)} \equiv \frac{A}{3x} + \frac{B}{x+2} + \frac{C}{x-2}$$

STEP 3: GET RID OF FRACTIONS

$\therefore\ 2x+5 \equiv A(x+2)(x-2) + B(3x)(x-2) + C(3x)(x+2)$

LET $x=0$: $5=-4A$ $\quad\therefore A=\frac{-5}{4}$

LET $x=2$: $9=24C$ $\quad\therefore C=\frac{9}{24}=\frac{3}{8}$

LET $x=-2$: $1=24B$ $\quad\therefore B=\frac{1}{24}$

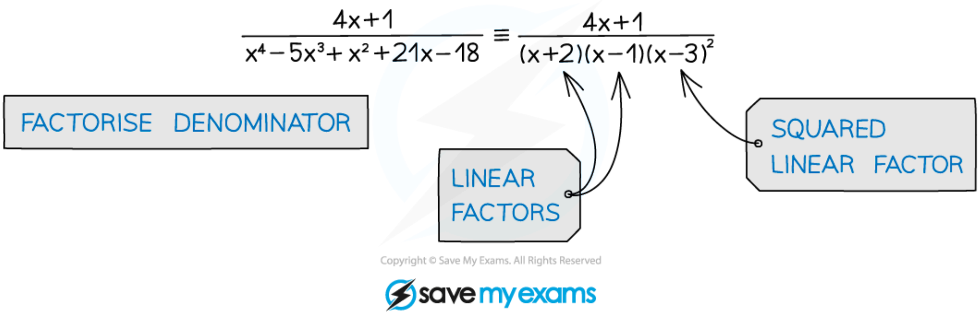
STEP 4: CHOOSE SUITABLE VALUES OF x

STEP 5: WRITE ORIGINAL FRACTION AS PARTIAL FRACTIONS

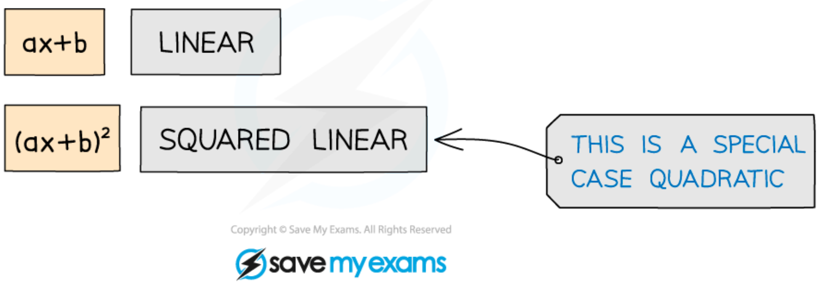
$$\therefore \frac{2x+5}{3x^3-12x} \equiv \frac{-5}{12x} + \frac{1}{24(x+2)} + \frac{3}{8(x-2)}$$

Partial fractions with squared linear denominators

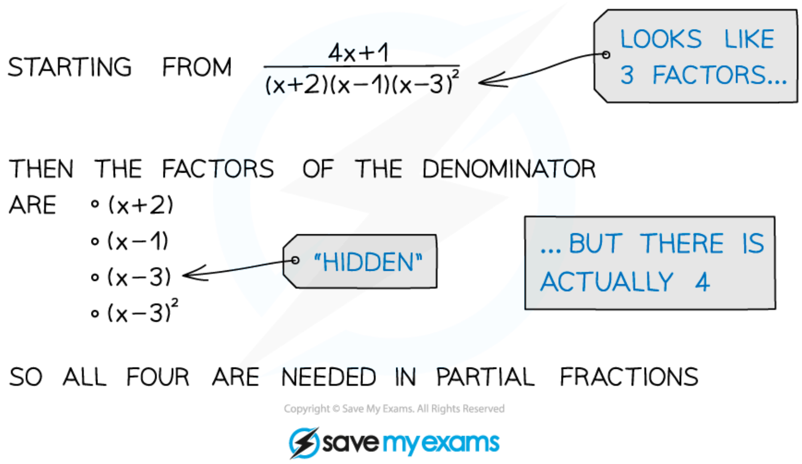
What are partial fractions with squared linear denominators?



- In partial fractions the common denominator is split into parts (factors)
- This is the reverse process to **adding** (or subtracting) fractions
- In harder questions there is a repeated factor, this is a **squared linear factor**



- A **linear factor** is of the form (**ax + b**)
- It is possible **b = 0** so a linear factor could be of the form **ax** (eg **4x**)
- A **squared linear factor** is of the form (**ax + b**)^2
- With b = 0 this would be of the form (**ax**)^2 (**x**^{2} would be too!)



How do I find partial fractions with squared linear denominators?

STEP 1 **Factorise** the denominator (Sometimes the numerator can be factorised too)

STEP 2 **Split** the fraction into a **sum** with a **squared linear denominator** and any other single linear denominators

STEP 3 Multiply by the denominator to get rid of fractions

STEP 4 Substitute values of **x** to find **A**, **B**, etc (or use comparing coefficients)

STEP 5 Write the **original** as partial fractions

Your notes

Your notes

$$\frac{4x+1}{x^4-5x^3+x^2+21x-18} \equiv \frac{4x+1}{(x+2)(x-1)(x-3)^2}$$

STEP 1: FACTORISE THE DENOMINATOR

MAY INVOLVE
- FACTOR THEOREM
- ALGEBRAIC DIVISION

STEP 2: SQUARED LINEAR DENOMINATOR

$$\equiv \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{x-3} + \frac{D}{(x-3)^2}$$

STEP 2: OTHER LINEAR DENOMINATORS

$$4x+1 = A(x-1)(x-3)^2 + B(x+2)(x-3)^2 + C(x+2)(x-1)(x-3) + D(x+2)(x-1)$$

STEP 3: MULTIPLY BY DENOMINATOR

LET $x=1$: $5=12B$ $\quad B=\frac{5}{12}$

LET $x=3$: $13=10D$ $\quad D=\frac{13}{10}$

LET $x=-2$: $-7=-75A$ $\quad A=\frac{7}{75}$

STEP 4: CHOOSE VALUES OF x

LET $x=0$: $1=-9A+18B+6C-2D$

$$1=\frac{-21}{25}+\frac{15}{2}+6C-\frac{13}{5}$$

$$6C=\frac{-153}{50}$$

$$C=\frac{-51}{100}$$

NO VALUE OF x GIVES AN EQUATION IN C ONLY SO CHOOSE AN EASY NUMBER TO WORK WITH

STEP 5: WRITE AS PARTIAL FRACTIONS

$$\frac{4x+1}{x^4-5x^3+x^2+21x-18} \equiv \frac{7}{75(x+2)} + \frac{5}{12(x-1)} - \frac{51}{100(x-3)} + \frac{13}{10(x-3)^2}$$

Worked Example

Your notes

Express $\dfrac{x^2 - 9x + 38}{(x+1)(x-3)^2}$ in partial fractions.

STEP 1: NUMERATOR CAN'T BE FACTORISED, DENOMINATOR ALREADY IS

$$\frac{x^2-9x+38}{(x+1)(x-3)^2} \equiv \frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

STEP 2: THERE IS A SQUARED LINEAR DENOMINATOR

STEP 3: GET RID OF FRACTIONS!

$$x^2 - 9x + 38 \equiv A(x-3)^2 + B(x+1)(x-3) + C(x+1)$$

LET $x=3$: $20 = 4C$ $\therefore C = 5$

LET $x=-1$: $48 = 16A$ $\therefore A = 3$

STEP 4: CHOOSE SUITABLE VALUES OF x

NOTICE WE CAN NOT GET AN EQUATION IN B ONLY BY CHOOSING A VALUE OF x

LET $x=0$: $38 = 9A - 3B + C$

$3B = 27 + 5 - 38$

$\therefore B = -2$

STEP 5: WRITE ORIGINAL FRACTION AS PARTIAL FRACTIONS

$$\therefore \frac{x^2-9x+38}{(x+1)(x-3)^2} \equiv \frac{3}{x+1} - \frac{2}{x-3} + \frac{5}{(x-3)^2}$$