Edexcel International A Level (IAL) Maths: Pure 4

Your notes

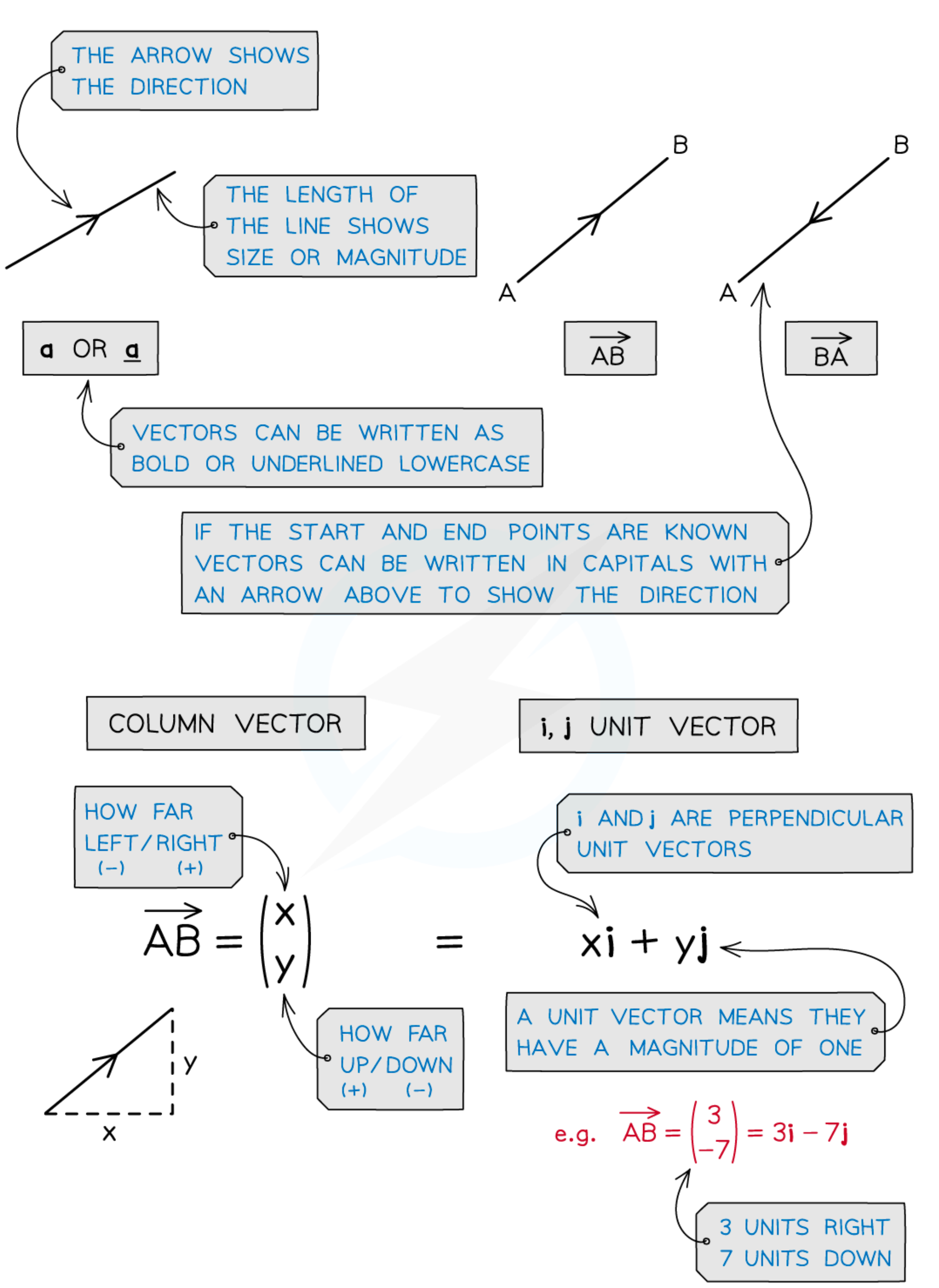
Vectors in 2D

Contents

Basic vectors

What is a vector?

- Vectors represent a movement of a certain **magnitude** (size) in a given **direction**
- You should have already come across **vectors** when translating functions of graphs
- They appear in many contexts of maths including **mechanics** for modelling forces
- Vectors can be represented in different ways such as a **column vector** or as an **i and j unit vector**



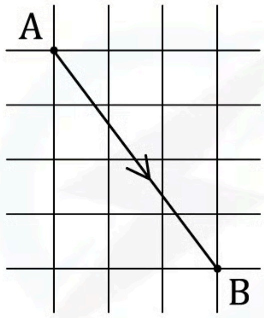
Your notes

Examiner Tips and Tricks

- Think of vectors like a journey from one place to another.
- Diagrams can help, if there isn't one, draw one.
- In your exam you can't write in **bold** so should underline your vector notation.

Worked Example

a) From the diagram below, write the column vector for $\overrightarrow{AB}$.



b) Write the column vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$ using the unit vectors **i** and **j**.

a) $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ ← REMEMBER THE COLUMN VECTOR SIMPLY REPRESENTS THE PATH $\begin{pmatrix} \text{LEFT/RIGHT} \\ \text{UP/DOWN} \end{pmatrix}$ FROM A TO B

b) $\begin{pmatrix} -2 \\ 5 \end{pmatrix} = -2\mathbf{i} + 5\mathbf{j}$ ← THE VECTOR USING **i** AND **j**

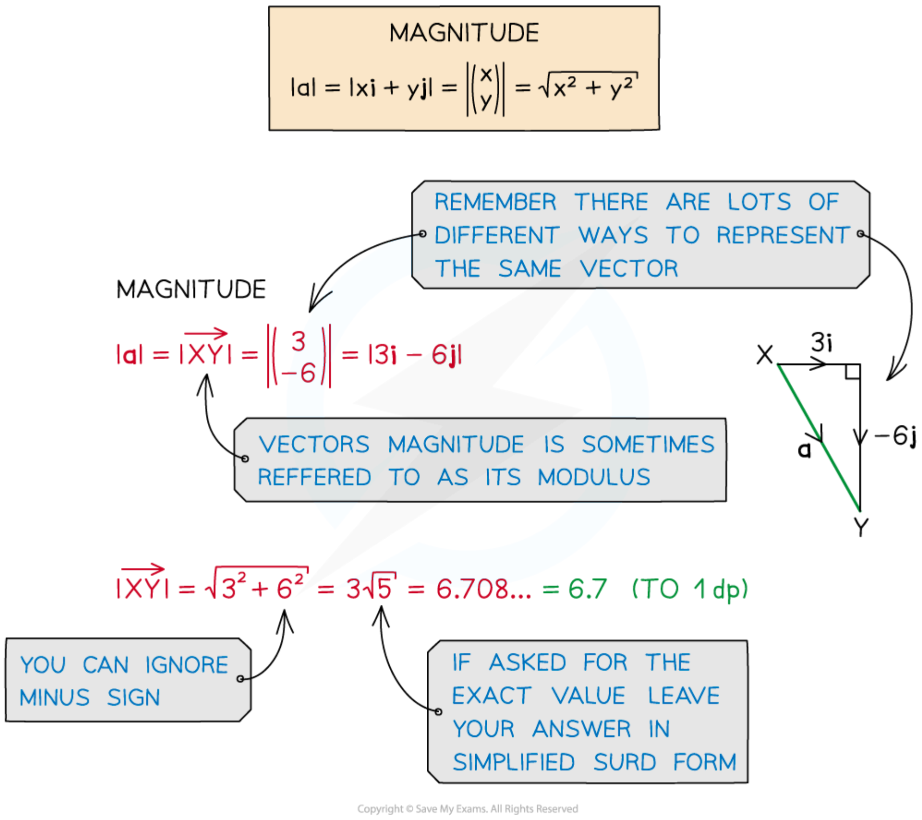
IN YOUR EXAM YOU WOULD WRITE $-2\underline{i} + 5\underline{j}$

Your notes

Magnitude & direction

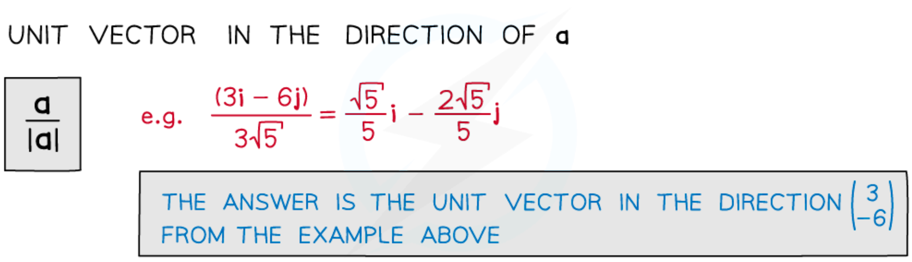
What is the magnitude of a vector?

- The magnitude of a vector is simply its size
- It also tells us the distance between two points
- You can find the magnitude of a vector using Pythagoras' theorem
- The magnitude of a vector $\mathbf{a}$ is written $|\mathbf{a}|$ when typed (or $|\underline{a}|$ when handwritten)



- To work out the **unit vector** in the direction of a given vector
- A **unit vector** has a **magnitude** of 1

  So to find the unit vector of a given vector, divide by its magnitude



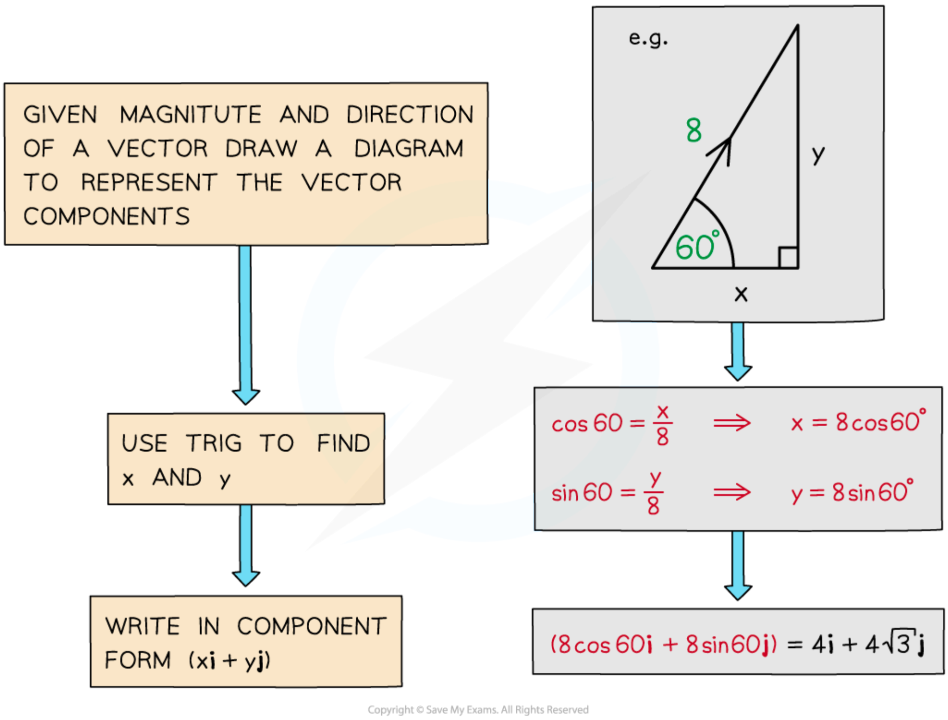
What is the direction of a vector?

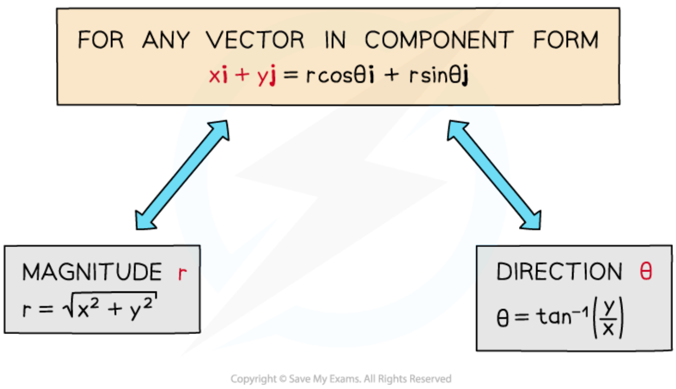
- Vectors have opposite direction if they are the same size but opposite signs
  - e.g. if **a** or $\overrightarrow{BC} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$ then $-\mathbf{a}$ or $\overrightarrow{CB} = \begin{pmatrix} 3 \\ -8 \end{pmatrix}$
- The direction of a vector is what makes it more than just a scalar
  - Eg. two objects with velocities of 7 m/s and -7 m/s are travelling at the **same speed** but in **opposite directions**
- Two vectors are **parallel** if and only if one is a **scalar multiple** of the other
- For real-life contexts such as mechanics, direction can be calculated from a given vector using **trigonometry** (see Right-Angled Triangles)
- It is usually calculated **anticlockwise** from the **positive x-axis** (unless otherwise stated eg. a bearing)

How do I write a vector in component form?

- We have already seen that vectors can be written in different forms
- Component form means writing a vector in terms of **i** and **j components**
- Given the **magnitude** and **direction** of a vector you can work out its components and vice versa

Your notes





Your notes

Examiner Tips and Tricks

- Diagrams can help, especially when working out direction – if there isn't a diagram, draw one.
- Remember, resolving a vector just means writing it in component form.

Worked Example

Your notes

A vector $\overrightarrow{XY} = \begin{pmatrix} -9 \\ 6 \end{pmatrix}$.

a) Find $\left|\overrightarrow{XY}\right|$, giving your answer to 3 significant figures.

b) Write the exact value of the unit vector in the direction of XY, giving your answer in simplified surd form.

a) $|\overrightarrow{XY}| = \sqrt{9^2 + 6^2}$

$= 3\sqrt{13}$

$= 10.8$ (3 sf)

$|\overrightarrow{XY}|$ MEANS FIND THE MAGNITUDE USING PYTHAGORAS

b) USING $\overrightarrow{XY} = \begin{pmatrix} -9 \\ 6 \end{pmatrix}$

$= -9\mathbf{i} + 6\mathbf{j}$

WRITE VECTOR USING i, j NOTATION

AND $|\overrightarrow{XY}| = 3\sqrt{13}$

$\therefore \dfrac{(-9\mathbf{i} + 6\mathbf{j})}{3\sqrt{13}} = \dfrac{-9}{3\sqrt{13}}\mathbf{i} + \dfrac{6}{3\sqrt{13}}\mathbf{j}$

UNIT VECTOR IN GIVEN DIRECTION $= \dfrac{a}{|a|}$

$= \dfrac{-3}{\sqrt{13}}\mathbf{i} + \dfrac{2}{\sqrt{13}}\mathbf{j}$

SIMPLIFYING SURD FORM AS MUCH AS POSSIBLE

$= \dfrac{-3\sqrt{13}}{13}\mathbf{i} + \dfrac{2\sqrt{13}}{13}\mathbf{j}$

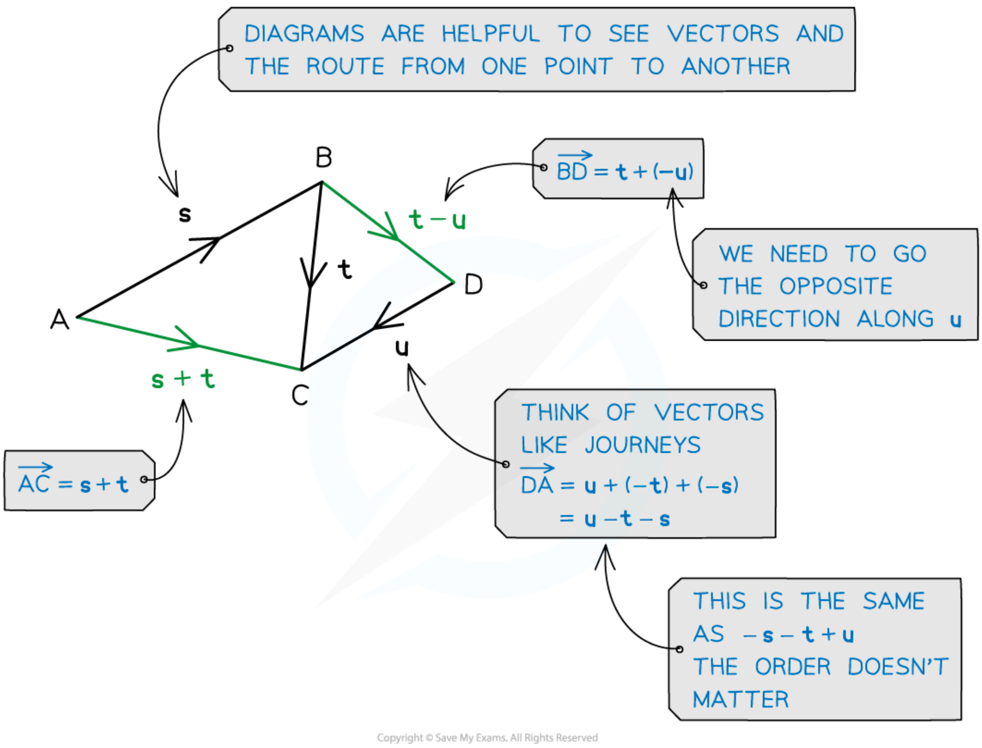
WITH RATIONALISED DENOMINATORS

Your notes

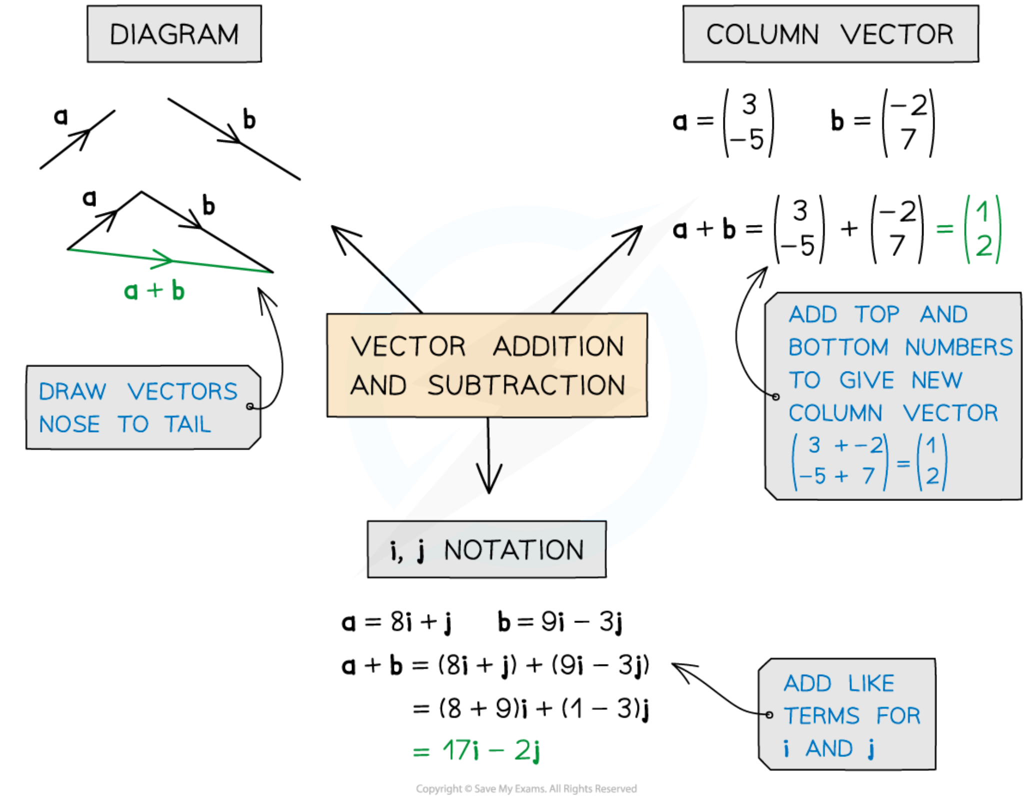
Vector addition

What is vector addition?

- **Adding** vectors together lets us describe the **movement between two points**
- The **single** vector this creates is called the **resultant** vector
- **Subtracting** a vector is the same as **adding the negative** vector
- Adding the vectors **PQ** and **QP** gives the zero vector, denoted by a bold zero **0**



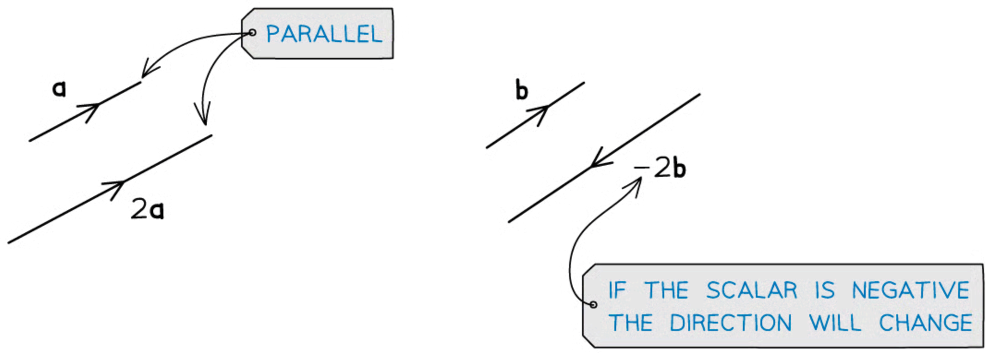
- You can add and subtract vectors in any of the forms previously described:

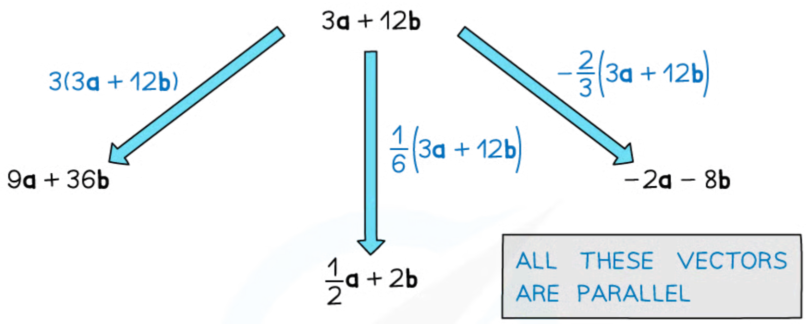


Scalars and parallel vectors

- Multiplying by a positive **scalar** only changes the **size** of a vector, not its **direction**
- Two vectors are **parallel** if and only if one is a **scalar multiple** of the other

Your notes





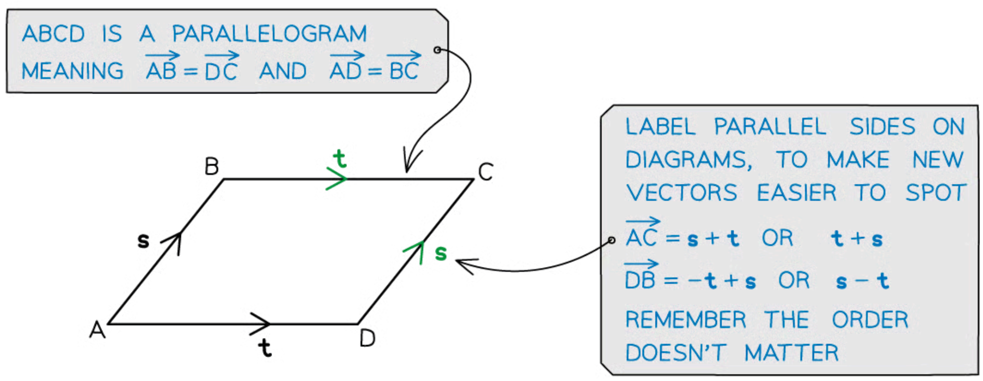
$\mathbf{a} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ $\quad 2\mathbf{a} = 2 \times \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} 6 \\ -10 \end{pmatrix}$

FOR COLUMN VECTORS JUST MULTIPLY TOP AND BOTTOM NUMBERS BY THE SCALAR

$\mathbf{a} = 3\mathbf{i} + 7\mathbf{j}$

$3\mathbf{a} = 3(3\mathbf{i} + 7\mathbf{j}) = 9\mathbf{i} + 21\mathbf{j}$

FOR **i j** VECTORS JUST MULTIPLY BOTH NUMBERS BY THE SCALAR



Examiner Tips and Tricks

- Think of vectors like a journey from one place to another – you may have to take a detour eg. A to B might be A to O then O to B.
- Diagrams can help, so if there isn't one, draw one. If there are any, labelling parallel vectors will help.

Your notes

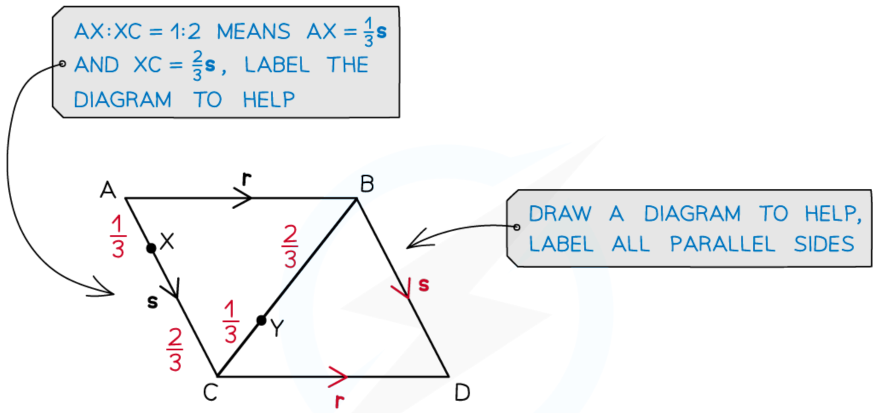
Worked Example

ABCD is a parallelogram with $\overrightarrow{AB} = \mathbf{r}$ and $\overrightarrow{AC} = \mathbf{s}$.

Point X lies on $\overrightarrow{AC}$ such that AX : XC = 1 : 2 and

point Y lies on $\overrightarrow{CB}$ such that CY : YB = 1 : 2

Show that $\overrightarrow{XY}$ is parallel to $\overrightarrow{AD}$.



$\overrightarrow{XY} = \frac{2}{3}(\overrightarrow{AC}) + \frac{1}{3}(\overrightarrow{CB})$

THINK ABOUT THE SEPARATE PARTS OF THE "JOURNEY"

$\overrightarrow{AD} = \mathbf{s} + \mathbf{r}$

$\overrightarrow{CB} = \mathbf{r} - \mathbf{s}$

$\overrightarrow{XY} = \frac{2}{3}(\mathbf{s}) + \frac{1}{3}(\mathbf{r} - \mathbf{s})$

$\overrightarrow{XY} = \frac{2}{3}\mathbf{s} + \frac{1}{3}\mathbf{r} - \frac{1}{3}\mathbf{s} = \frac{1}{3}\mathbf{r} + \frac{1}{3}\mathbf{s} = \frac{1}{3}(\mathbf{r} + \mathbf{s})$

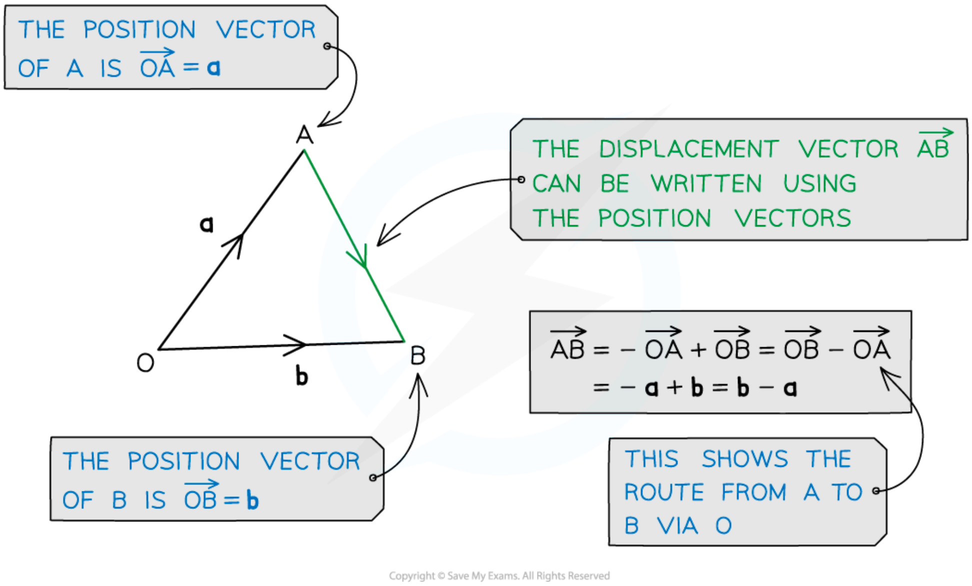
$\overrightarrow{XY} = \frac{1}{3}(\overrightarrow{AD})$

THEY ARE SCALAR MULTIPLES SO MUST BE PARALLEL

Position vectors

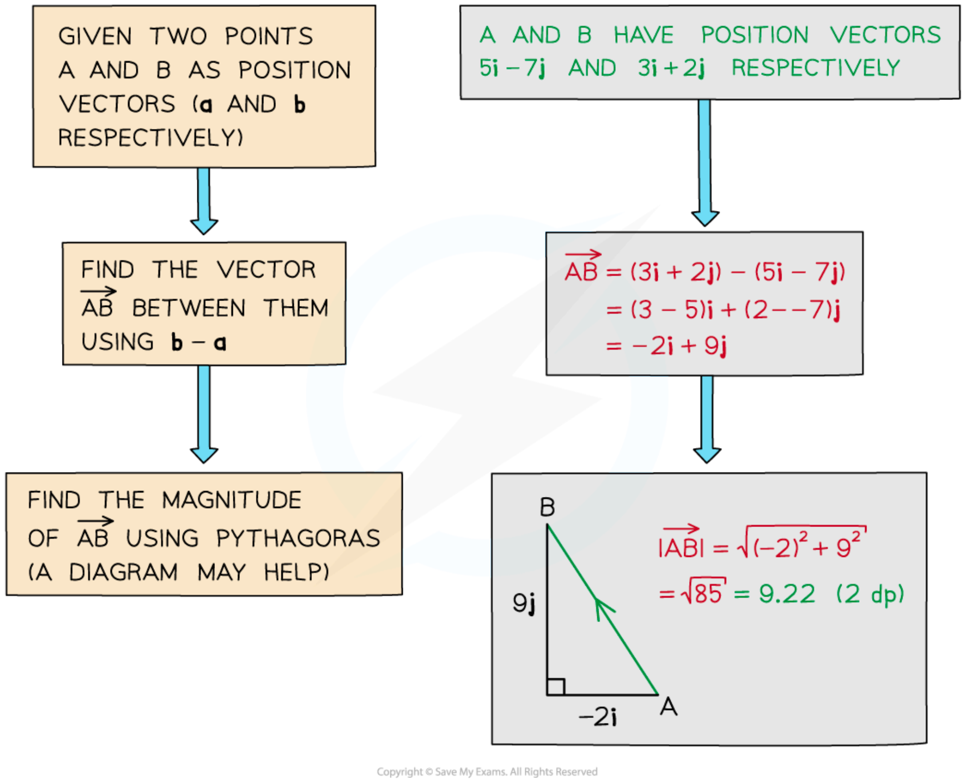
What is a position vector?

- Position vectors describe the **position of a point** in relation to the origin
- They are different to **displacement vectors** which describe the **direction and distance** between any two points



Distance between two points

- The distance between two points is the **magnitude** of the vector between them (see Magnitude Direction)



How do I find the magnitude of a displacement vector?

- You can use **coordinate geometry** to find **magnitudes** of **displacement vectors** from *A* to *B*
  - From the **position vectors** of *A* and *B* you know their coordinates
    - If $\mathbf{a} = \overrightarrow{OA} = x_1\mathbf{i} + y_1\mathbf{j} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, then point A has coordinates (x_1, y_1)
    - If $\mathbf{b} = \overrightarrow{OB} = x_2\mathbf{i} + y_2\mathbf{j} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$, then point B has coordinates (x_2, y_2)
  - The **distance** between two points is given by
    $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
    - So $\left|\overrightarrow{AB}\right| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
  - For example, if points A and B have position vectors $5\mathbf{i} + 3\mathbf{j}$ and $3\mathbf{i} - 6\mathbf{j}$ respectively

Your notes

- then $\left|\overrightarrow{AB}\right| = \sqrt{(5-3)^2 + (3-(-6))^2} = \sqrt{85} = 9.22\ (3\text{ s.f.})$

- Alternatively, you could find $\left|\overrightarrow{AB}\right|$ by
  - first using $\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$ to find $\overrightarrow{AB}$ in **vector form**
    - and then calculating its **magnitude** directly
  - See the Worked Example below

Examiner Tips and Tricks

- Remember if asked for a position vector, you must find the vector all the way from the origin.
- Diagrams can help, if there isn't one, draw one.

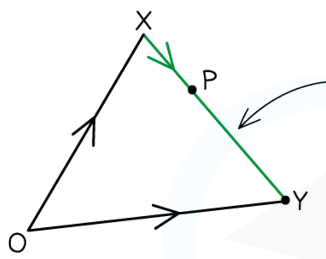
Worked Example

Your notes

The points X and Y have position vectors $3\mathbf{i} + 5\mathbf{j}$ and $6\mathbf{i} - 4\mathbf{j}$ respectively.

Point P is on the line XY such that XP : PY = 1 : 2.

Determine the position vector of P.



START BY DRAWING A DIAGRAM TO HELP

FIND VECTORS FOR ANY UNKNOWN PARTS OF THE DIAGRAM

$$\overrightarrow{XY} = \overrightarrow{OY} - \overrightarrow{OX} = (6\mathbf{i} - 4\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j})$$

$$= (6-3)\mathbf{i} + (-4-5)\mathbf{j}$$

$$\overrightarrow{XY} = 3\mathbf{i} - 9\mathbf{j}$$

XP:PY = 1:2 MEANS P IS $\frac{1}{3}$ OF THE WAY ALONG $\overrightarrow{XY}$

$$\overrightarrow{XP} = \frac{1}{3}(3\mathbf{i} - 9\mathbf{j})$$

$$= \mathbf{i} - 3\mathbf{j}$$

WE ARE ASKED FOR POSITION VECTOR OF P SO MUST START AT THE ORIGIN

$$\overrightarrow{OP} = \overrightarrow{OX} + \overrightarrow{XP} = (3\mathbf{i} + 5\mathbf{j}) + (\mathbf{i} - 3\mathbf{j})$$

$$= (3+1)\mathbf{i} + (5-3)\mathbf{j}$$

$$= (4\mathbf{i} + 2\mathbf{j})$$

Your notes

Problem solving using vectors

How are vectors used for problem solving?

- Vectors can be used to prove two lines are **parallel** (see Vector Addition)
- They can also be used to show points are **collinear** (lie on the same straight line)
- Vectors can be used to find missing vertices of a given shape
- You will need a good understanding of how to divide a line segment into a given ratio

VECTORS ARE PARALLEL IF ONE IS A SCALAR **MULTIPLE** OF THE OTHER

$\overrightarrow{AB}$ IS **PARALLEL** TO $\overrightarrow{CD}$

$\overrightarrow{AB} = \mathbf{s}$ $\quad \overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{AB} + \overrightarrow{BD}$

$= \mathbf{s} - \mathbf{t} + \mathbf{s} + \mathbf{t}$

$= 2\mathbf{s}$

$2\mathbf{s}$ IS A MULTIPLE OF $\mathbf{s}$

$\overrightarrow{CD} = 2\overrightarrow{AB}$

FIND SIMPLIFIED VECTOR FOR BOTH $\overrightarrow{AB}$ AND $\overrightarrow{CD}$ TO PROVE THEY ARE PARALLEL

THREE POINTS P, Q, R ARE **COLLINEAR** IF $\overrightarrow{PQ}$ AND $\overrightarrow{PR}$ ARE PARALLEL

A, X AND D ARE **COLLINEAR**

$\overrightarrow{AX} = \overrightarrow{AC} + \overrightarrow{CX}$

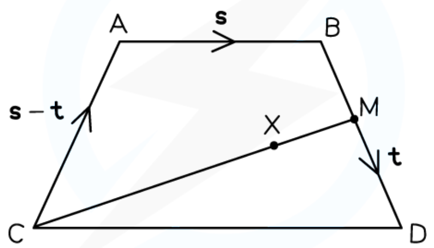
$= \mathbf{t} - \mathbf{s} + \frac{4}{5}(2\mathbf{s} - \frac{1}{2}\mathbf{t})$

$= \frac{3}{5}\mathbf{s} + \frac{3}{5}\mathbf{t}$

$\overrightarrow{AD} = \mathbf{s} + \mathbf{t}$

$\overrightarrow{AX} = \frac{3}{5}\overrightarrow{AD}$ SO $\overrightarrow{AX}$ IS PARALLEL TO $\overrightarrow{AD}$

FIND SIMPLIFIED VECTOR FOR BOTH $\overrightarrow{AX}$ AND $\overrightarrow{AD}$ TO PROVE THEY ARE PARALLEL THEREFORE COLLINEAR



ABCD IS A QUADRILATERAL

$\overrightarrow{AB} = \mathbf{s}$, $\overrightarrow{BD} = \mathbf{t}$ AND $\overrightarrow{CA} = \mathbf{s} - \mathbf{t}$

M IS THE MIDPOINT OF BD

X IS A POINT ON CM SUCH THAT CX:XM = 4:1

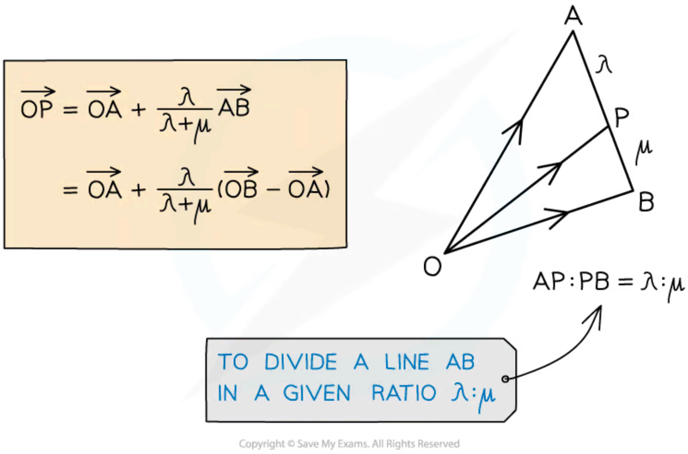
THE POINT E FORMS A PARALLELOGRAM CXDE

PARALLEL SIDES OF A PARALLELOGRAM HAVE THE **SAME VECTOR**

$\overrightarrow{CE} = \overrightarrow{XD} = \frac{2}{5}\mathbf{s} + \frac{2}{5}\mathbf{t}$

TO FIND THE MISSING VECTOR OF A PARALLELOGRAM USE THE PARALLEL VECTOR YOU ALREADY HAVE

Your notes

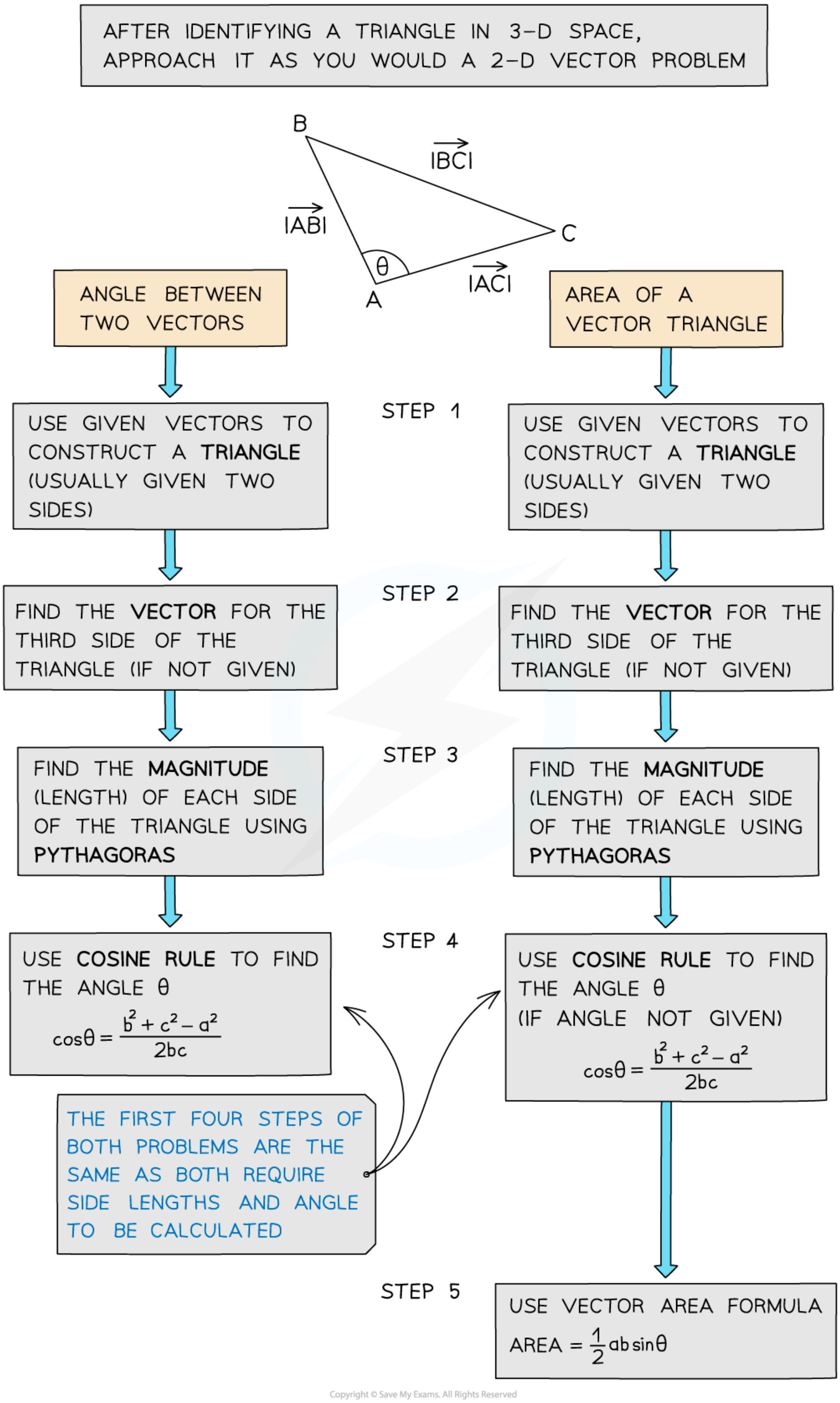


When do I use trigonometry in vector problems?

- When problem-solving with vectors, trigonometry can help us:
  - convert between **component form** and magnitude/direction form (see Magnitude Direction)
  - find the **angle between two vectors** using **Cosine Rule** (see Non-Right-Angled Triangles)
  - find the **area of a triangle** using a variation of **Area Formula** (see Non-Right-Angled Triangles)

Your notes

Your notes


AFTER IDENTIFYING A TRIANGLE IN 3-D SPACE, APPROACH IT AS YOU WOULD A 2-D VECTOR PROBLEM
B
$|\overrightarrow{BC}|$
$|\overrightarrow{AB}|$
C
θ
A
$|\overrightarrow{AC}|$
ANGLE BETWEEN TWO VECTORS
AREA OF A VECTOR TRIANGLE
STEP 1
USE GIVEN VECTORS TO CONSTRUCT A TRIANGLE (USUALLY GIVEN TWO SIDES)
USE GIVEN VECTORS TO CONSTRUCT A TRIANGLE (USUALLY GIVEN TWO SIDES)
STEP 2
FIND THE VECTOR FOR THE THIRD SIDE OF THE TRIANGLE (IF NOT GIVEN)
FIND THE VECTOR FOR THE THIRD SIDE OF THE TRIANGLE (IF NOT GIVEN)
STEP 3
FIND THE MAGNITUDE (LENGTH) OF EACH SIDE OF THE TRIANGLE USING PYTHAGORAS
FIND THE MAGNITUDE (LENGTH) OF EACH SIDE OF THE TRIANGLE USING PYTHAGORAS
STEP 4
USE COSINE RULE TO FIND THE ANGLE θ
$\cos\theta = \frac{b^2 + c^2 - a^2}{2bc}$
USE COSINE RULE TO FIND THE ANGLE θ (IF ANGLE NOT GIVEN)
$\cos\theta = \frac{b^2 + c^2 - a^2}{2bc}$
THE FIRST FOUR STEPS OF BOTH PROBLEMS ARE THE SAME AS BOTH REQUIRE SIDE LENGTHS AND ANGLE TO BE CALCULATED
STEP 5
USE VECTOR AREA FORMULA
AREA $= \frac{1}{2}ab\sin\theta$
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Your notes

Examiner Tips and Tricks

- Think of vectors like a journey from one place to another – you may have to take a detour eg. A to B might be A to O then O to B.
- Diagrams can help, if there isn't one, draw one. For a given diagram labelling all known vectors and quantities will help.

Worked Example

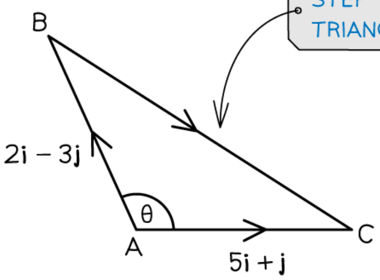
Your notes

The vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are 2i – 3j and 5i + j respectively.

Find the area of triangle ABC.

Give your answer to 3 significant figures.

STEP 1: DRAW TRIANGLE



STEP 2: FIND $\overrightarrow{BC}$

$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$$

$$\begin{pmatrix} 5 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

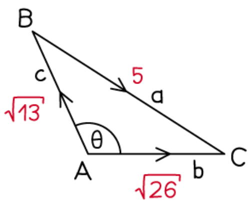
STEP 3: FIND **MAGNITUDE** OF EACH SIDE–LEAVE ANSWERS IN **SURD FORM**

$$|\overrightarrow{AB}| = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$|\overrightarrow{AC}| = \sqrt{5^2 + 1^2} = \sqrt{26}$$

$$|\overrightarrow{BC}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

STEP 4: USE **COSINE RULE** TO FIND θ BE CAREFUL TO **LABEL TRIANGLE CORRECTLY** WITH a OPPOSITE θ



$$\cos\theta = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{\sqrt{26}^2 + \sqrt{13}^2 - 5^2}{2 \times \sqrt{26} \times \sqrt{13}}$$

$$\cos\theta = \frac{7\sqrt{2}}{26}$$

$$\theta = \cos^{-1}\left(\frac{7\sqrt{2}}{26}\right) = 67.62 \quad (2 \text{ dp})$$

STEP 5: USE AREA FORMULA. AGAIN BE CAREFUL TO LABEL TRIANGLE CORRECTLY WITH θ BETWEEN SIDES a AND b

$$\text{AREA} = \frac{1}{2} ab\sin\theta$$

$$= \frac{1}{2}\sqrt{26}\sqrt{13}\sin 67.62$$

$$= 8.50000825\ldots$$

$$= 8.50 \ (3 \text{ s.f.})$$

IF GIVEN θ JUST WORK OUT **MAGNITUDE** OF **a** AND **b** FIRST