



Edexcel International A Level (IAL) Maths: Pure 4



Your notes

Proof by Contradiction

Contents

* Proof by Contradiction



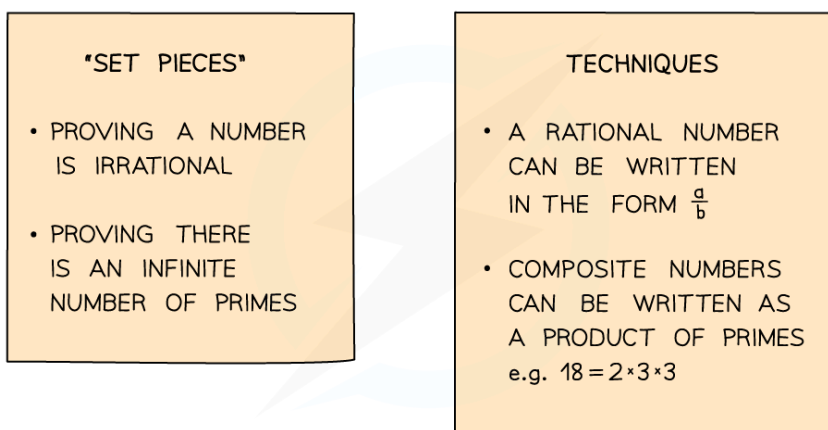
Proof by contradiction

What is proof by contradiction?

- A proof by contradiction assumes the **opposite** result is true
- Then, through a series of logical steps, shows that this cannot be so

How do I use proof by contradiction?

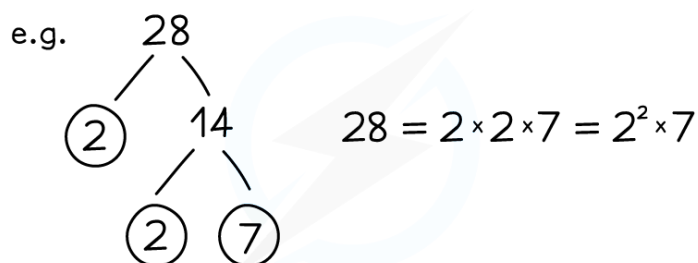
- This type of proof often involves explanation in words alongside mathematical statements
- Start with a statement assuming the opposite is true
 - e.g. 'Assume...'



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- Use **prime factorisation** to write a **composite number** as a product of primes



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Worked Example

? Prove that $\sqrt{5}$ is irrational.

ASSUME $\sqrt{5}$ IS RATIONAL

START WITH THE OPPOSITE

SO $\sqrt{5} = \frac{a}{b}$ WHERE a AND b ARE INTEGERS
WITH NO COMMON FACTORS AND $b \neq 0$

SQUARING BOTH SIDES: $5 = \frac{a^2}{b^2}$
 $\Leftrightarrow 5b^2 = a^2$

THEREFORE a^2 IS A MULTIPLE OF 5
SO a IS A MULTIPLE OF 5

SINCE a IS AN INTEGER

WRITING $a = 5k$ FOR SOME INTEGER k ,

$$a^2 = (5k)^2 = 25k^2$$

$$\text{SO } 5b^2 = 25k^2$$

$$\Leftrightarrow b^2 = 5k^2$$

THEREFORE b^2 IS A MULTIPLE OF 5 AND
SO b IS A MULTIPLE OF 5

SINCE b IS AN INTEGER

IT HAS BEEN SHOWN THAT a AND b ARE MULTIPLES
OF 5 SO THEY SHARE FACTOR OF 5

MAKE IT CLEAR
WHERE THE
CONTRADICTION IS

THIS IS A CONTRADICTION OF THE
ASSUMPTION $\sqrt{5}$ IS RATIONAL, IT CANNOT
BE WRITTEN AS $\frac{a}{b}$, SO, $\sqrt{5}$ IS IRRATIONAL

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