



Edexcel International A Level (IAL) Maths: Pure 3



Your notes

Rational Expressions

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Rational expressions

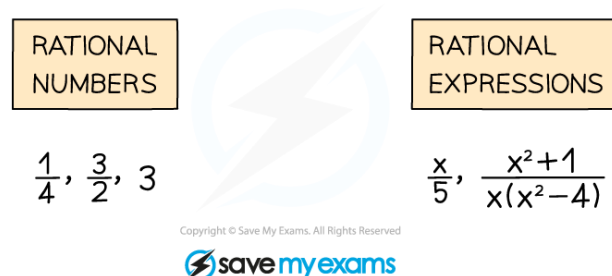
What are rational expressions?

- **Rational** numbers are numbers that can be written as a fraction (**quotient**)

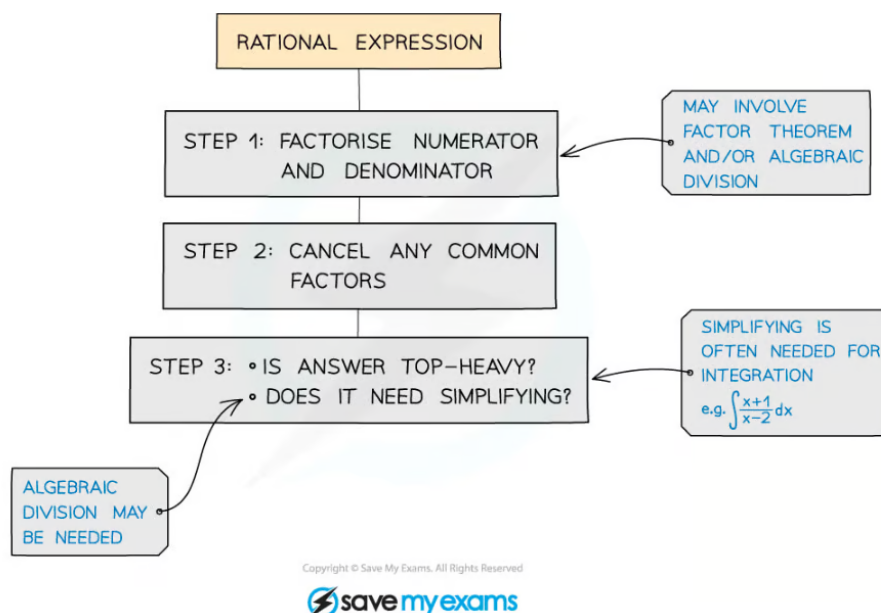
Rational comes from **ratio** – a number is rational if it can be written as a ratio of two integers – ie a fraction!

- A **rational expression** is an **algebraic fraction**

The ratio between two algebraic expressions (usually polynomials)



How do I simplify rational expressions?



- You may need to recall the **factor theorem**



Your notes

FACTOR THEOREM

IF $f(a)=0$ THEN $(x-a)$ IS A FACTOR OF $f(x)$
i.e. $f(x) = (x-a)(\dots \dots)$

EXTENDED FACTOR THEOREM

IF $f(\frac{b}{a})=0$ THEN $(ax-b)$ IS A FACTOR OF $f(x)$
i.e. $f(x) = (ax-b)(\dots \dots)$

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- Start by **factorising** polynomials using **factor theorem** or **algebraic division**

Simplify $\frac{x^3 - 7x + 6}{x^2 + 2x - 3}$

$$f(x) = x^3 - 7x + 6$$

$$f(1) = 1^3 - 7 \times 1 + 6 = 0$$

$\therefore (x-1)$ IS A FACTOR OF $f(x)$

FACTOR
THEOREM

$$\begin{array}{r} x^2 + x - 6 \\ x-1 \overline{) x^3 + 0x^2 - 7x + 6} \\ \underline{x^3 - x^2} \\ x^2 - 7x \\ \underline{x^2 - x} \\ -6x + 6 \\ \underline{-6x + 6} \\ 0 \end{array}$$

ALGEBRAIC
DIVISION

$$\therefore f(x) = (x-1)(x^2 + x - 6)$$

$$f(x) = (x-1)(x+3)(x-2)$$

FACTORISE
THE QUADRATIC

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- cancel** any common (linear) factors



Your notes

$$\frac{x^3 - 7x + 6}{x^2 + 2x - 3} = \frac{\cancel{(x-1)}(x+3)\cancel{(x-2)}}{\cancel{(x+3)}\cancel{(x-1)}} = (x-2)$$

FROM
EARLIER

CANCEL
COMMON
FACTORS

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- recognise a top-heavy (improper) rational expression, simplify if needed



Worked Example

? Simplify $\frac{8x^3 + 40x^2 + 48x}{2x^3 + 4x^2 - 6x}$

$$\frac{8x^3 + 40x^2 + 48x}{2x^3 + 4x^2 - 6x} = \frac{8x(x^2 + 5x + 6)}{2x(x^2 + 2x - 3)}$$

STEP 1
FACTORISE
TOP & BOTTOM

STEP 2
CANCEL COMMON
FACTORS

$$= \frac{4\cancel{8}(x+3)\cancel{(x+2)}}{2\cancel{x}(x+3)\cancel{(x-1)}}$$

$$= \frac{4(x+2)}{x-1}$$

STEP 3
THIS IS TOP HEAVY BUT
NO NEED TO SIMPLIFY
ANY FURTHER

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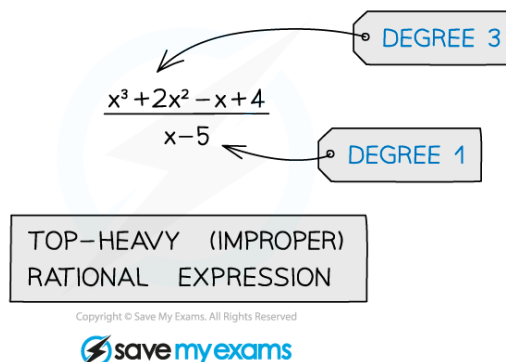




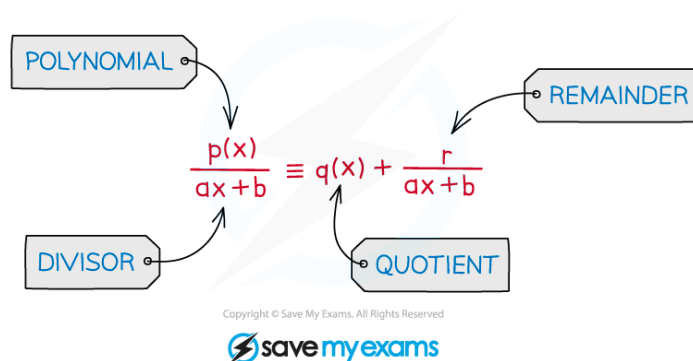
Improper algebraic fractions

What are improper algebraic fractions?

- The **degree** of the numerator is **greater than** or **equal** to the degree of the denominator



How do I simplify improper algebraic fractions?



- Write as a **quotient** and a **remainder**
- The algebraic equivalent of changing a top-heavy fraction to a mixed number



Your notes

$$\frac{x^3 + 3x^2 - 4x + 2}{x - 1}$$

$$\begin{array}{r} x^2 + 4x + 0 \\ x-1 \overline{) x^3 + 3x^2 - 4x + 2} \\ \underline{x^3 - x^2} \\ 4x^2 - 4x \\ \underline{4x^2 - 4x} \\ 0x + 2 \\ \underline{0x + 0} \\ 2 \end{array}$$

STEP 1
DIVIDE TOP BY
BOTTOM USING
ALGEBRAIC DIVISION

$$\frac{x^3 + 3x^2 - 4x + 2}{x - 1} = x^2 + 4x + \frac{2}{x - 1}$$

STEP 2
WRITE ANSWER AS
A QUOTIENT AND
A REMAINDER

THIS IS THE ALGEBRAIC
EQUIVALENT OF MIXED
NUMBERS, e.g. $\frac{17}{5} = 3\frac{2}{5}$
($17 \div 5 = 3 \text{ r}2$)

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Examiner Tips and Tricks

Remember that simple cases are sometimes the hardest to spot!

$$\frac{x}{x+1} \quad \begin{array}{l} \leftarrow \text{TOP-HEAVY AS NUMERATOR} \\ \leftarrow \text{AND DENOMINATOR BOTH} \\ \leftarrow \text{DEGREE 1} \end{array}$$

$$= 1 - \frac{1}{x+1}$$

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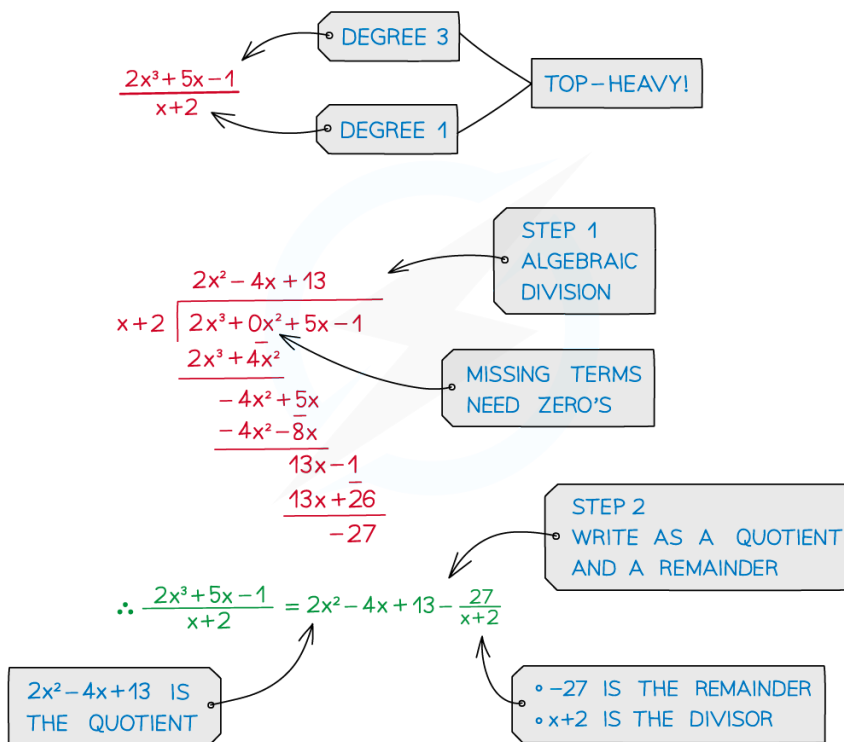
Worked Example



Your notes



Work out $(2x^3 + 5x - 1) \div (x + 2)$



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Quadratic divisor

What is a quadratic divisor?

- A polynomial $f(x)$ of **degree n** divided by a **quadratic divisor** takes the form

$$\frac{f(x)}{ax^2 + bx + c} = q(x) + \frac{r(x)}{ax^2 + bx + c}$$

- The **quotient** q will have degree $n - 2$
- The degree of the **remainder** r will be less than 2
 - It could be degree 1 (linear)
 - Or it could be degree 0 (constant)

How do I divide a polynomial by a quadratic divisor?



Your notes

- You still use **polynomial division!**
- **Step 1**
Divide the **leading term** of the **polynomial** by the **squared term** of the **divisor**
 - This gives the **leading term** of the **quotient**
- **Step 2**
Multiply this term by the **divisor**
- **Step 3**
Subtract this from the polynomial to get a **new polynomial** with a **lower degree**
- **Continue** these steps until you have an expression with a degree lower than 2