Edexcel International A Level (IAL) Maths: Pure 3

Your notes

Rational Expressions

Contents

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Rational expressions

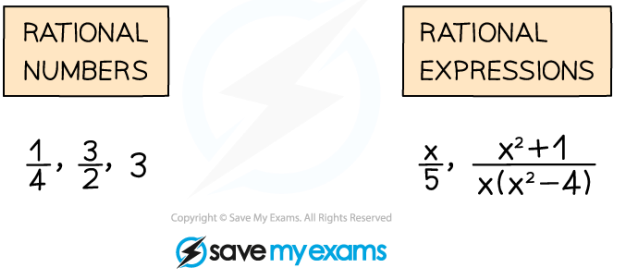
What are rational expressions?

- **Rational** numbers are numbers that can be written as a fraction (**quotient**)

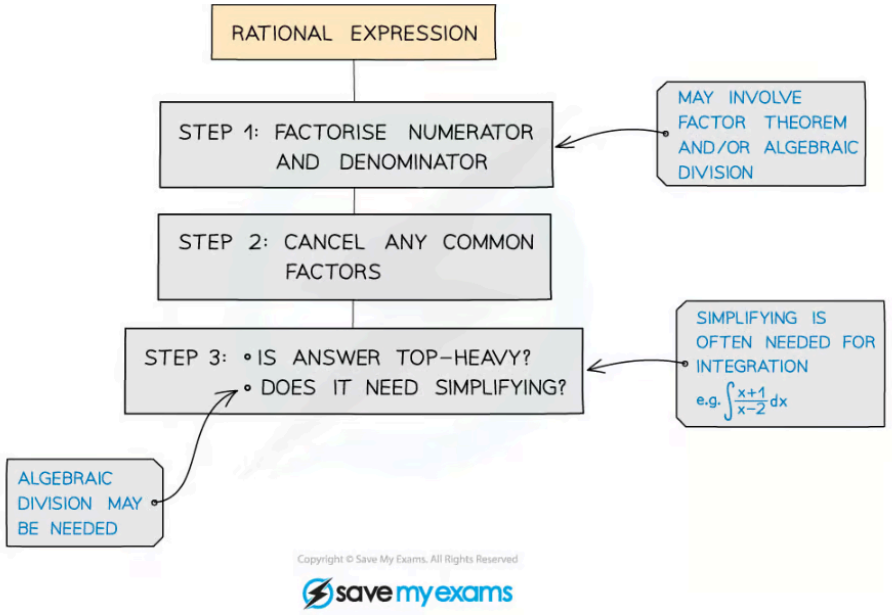
  Rational comes from **ratio** – a number is rational if it can be written as a ratio of two integers – ie a fraction!

- A **rational expression** is an **algebraic fraction**

  The ratio between two algebraic expressions (usually polynomials)



How do I simplify rational expressions?



- You may need to recall the **factor theorem**

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FACTOR THEOREM

IF $f(a)=0$ THEN $(x-a)$ IS A FACTOR OF $f(x)$
i.e. $f(x) = (x-a)(\ldots\ \ldots)$

EXTENDED FACTOR THEOREM

IF $f\left(\frac{b}{a}\right)=0$ THEN $(ax-b)$ IS A FACTOR OF $f(x)$
i.e. $f(x) = (ax-b)(\ldots\ \ldots)$

- Start by **factorising** polynomials using **factor theorem** or **algebraic division**

Simplify $\dfrac{x^3-7x+6}{x^2+2x-3}$

$f(x) = x^3-7x+6$

$f(1) = 1^3-7\times1+6 = 0$

$\therefore (x-1)$ IS A FACTOR OF $f(x)$

FACTOR THEOREM

$$\begin{array}{r|l} & x^2+x-6 \\ \hline x-1 & x^3+0x^2-7x+6 \\ & \underline{x^3-x^2} \\ & \quad x^2-7x \\ & \quad \underline{x^2-x} \\ & \quad\quad -6x+6 \\ & \quad\quad \underline{-6x+6} \\ & \quad\quad\quad 0 \end{array}$$

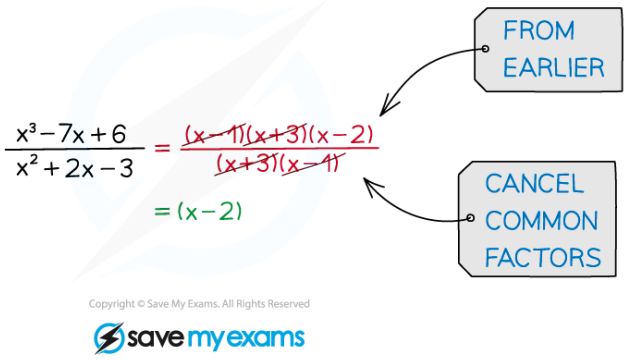
ALGEBRAIC DIVISION

$\therefore f(x) = (x-1)(x^2+x-6)$

$f(x) = (x-1)(x+3)(x-2)$

FACTORISE THE QUADRATIC

- **cancel** any common (linear) factors



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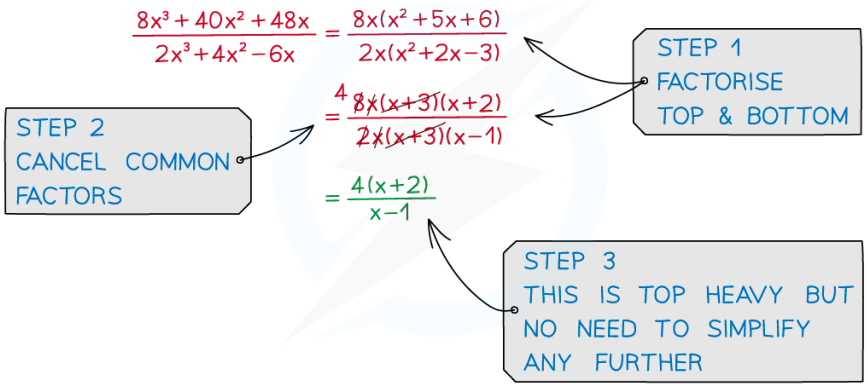
- **recognise** a **top-heavy (improper)** rational expression, simplify if needed

Worked Example

? Simplify $\frac{8x^3 + 40x^2 + 48x}{2x^3 + 4x^2 - 6x}$



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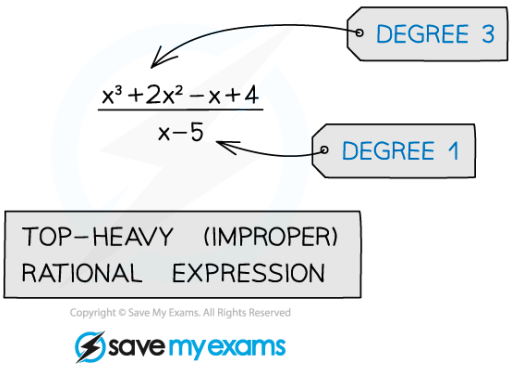
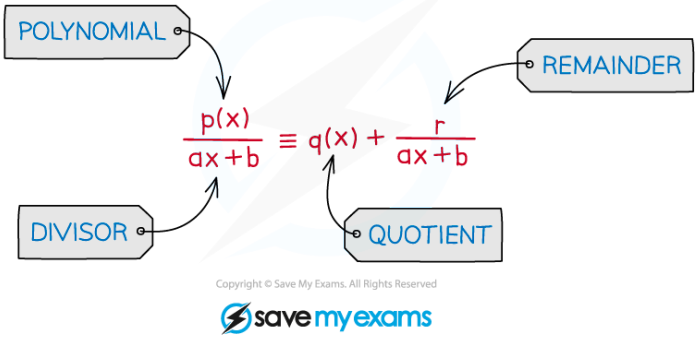
Improper algebraic fractions

What are improper algebraic fractions?

- The **degree** of the numerator is **greater than** or **equal** to the degree of the denominator



How do I simplify improper algebraic fractions?



- Write as a **quotient** and a **remainder**
- The algebraic equivalent of changing a top-heavy fraction to a mixed number

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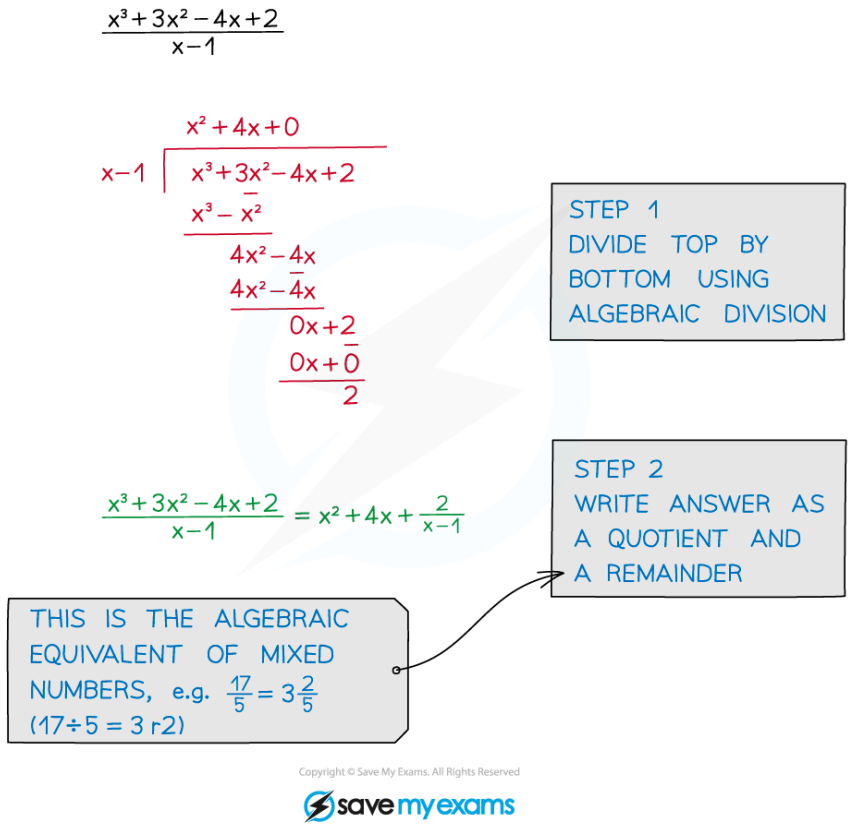
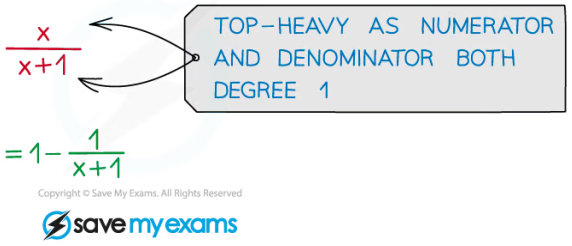


Examiner Tips and Tricks

Remember that simple cases are sometimes the hardest to spot!



Worked Example

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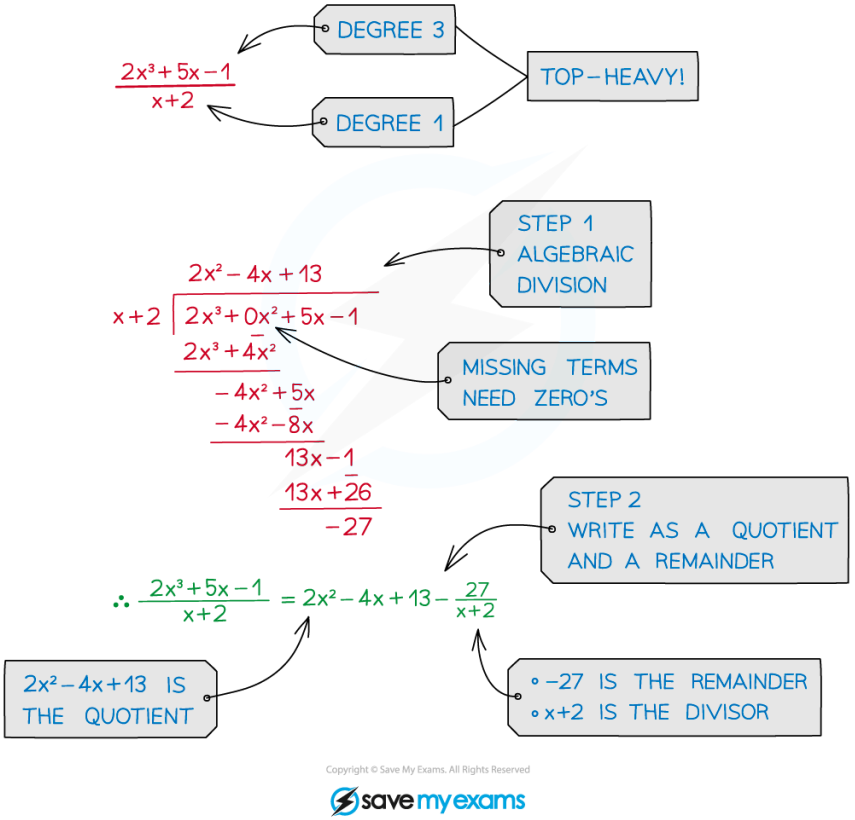
Work out $(2x^3 + 5x - 1) \div (x + 2)$



Quadratic divisor

What is a quadratic divisor?

- A polynomial f(x) of **degree** n divided by a **quadratic divisor** takes the form
  - $$\frac{f(x)}{ax^2 + bx + c} = q(x) + \frac{r(x)}{ax^2 + bx + c}$$
- The **quotient** q will have degree $n - 2$
- The degree of the **remainder** *r* will be less than 2
  - It could be degree 1 (linear)
  - Or it could be degree 0 (constant)

How do I divide a polynomial by a quadratic divisor?

- You still use **polynomial division!**
- **Step 1**
  Divide the **leading term** of the **polynomial** by the **squared term** of the **divisor**
    - This gives the **leading term** of the **quotient**
- **Step 2**
  Multiply this term by the **divisor**
- **Step 3**
  Subtract this from the polynomial to get a **new polynomial** with a **lower degree**
- **Continue** these steps until you have an expression with a degree lower than 2

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