Edexcel International AS Maths: Pure 2

Your notes

Geometric Sequences & Series

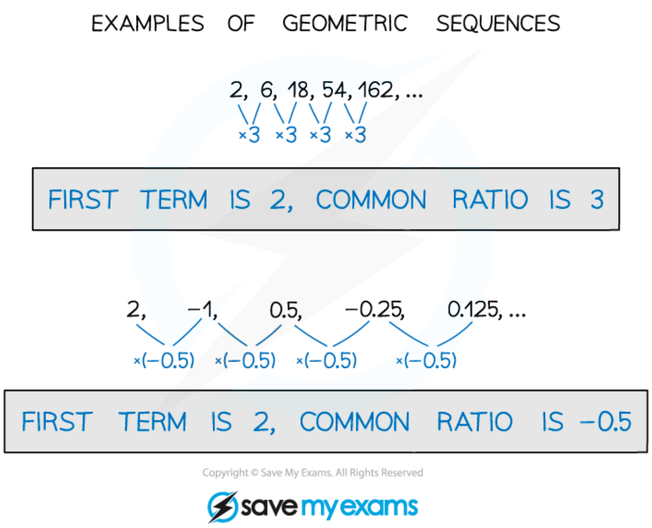
Contents

Your notes

Geometric sequences

What do I need to know about geometric sequences?

- In a **geometric sequence**, there is a **common ratio** between consecutive terms in the sequence



- You need to know the ***n*th term formula** for a geometric sequence

$$u_n = ar^{n-1}$$

  - a is the first term
  - *r* is the common ratio

Examiner Tips and Tricks

If you know two terms in a geometric sequence you can find ***a*** and ***r*** using simultaneous equations.

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e.g. THE 6^{th} TERM OF A GEOMETRIC SEQUENCE IS 486.
THE 10^{th} TERM IS 39 366.
FIND THE FIRST TERM AND THE COMMON DIFFERENCE.

$u_n = ar^{n-1}$

$ar^5 = 486$ ①

$ar^9 = 39\,366$ ②

SOLVE THESE SIMULTANEOUS EQUATIONS

$r^4 = \dfrac{39\,366}{486} = 81$

② DIVIDED BY ①

$r = 81^{\frac{1}{4}} = 3$

$a(3)^5 = 486$

$243a = 486$

$a = 2$

SUBSTITUTE INTO ①

THE FIRST TERM IS 2 AND THE COMMON RATIO IS 3

Worked Example

Your notes

The first three terms in a geometric sequence are $2 - x$, $3x$, and x^2 respectively where $x < 0$.

a) Show that $x^3 + 7x^2 = 0$.

b) Find the 6th term of the sequence.

a) THIS IS A GEOMETRIC SEQUENCE SO THERE'S A SINGLE COMMON RATIO

$r = \frac{3x}{2-x}$ 2nd TERM DIVIDED BY 1st TERM

AND

$r = \frac{x^2}{3x}$ 3rd TERM DIVIDED BY 2nd TERM

SO $\frac{3x}{2-x} = \frac{x^2}{3x}$

$(3x)^2 = x^2(2-x)$

$9x^2 = 2x^2 - x^3$

$x^3 + 7x^2 = 0$

b) $x^2(x+7) = 0$ SOLVE THE EQUATION FROM a)

$x = 0$ OR $x = -7$

BUT $x < 0$ SO $x = -7$

$a = 2-(-7) = 9$ FIND a AND r

$r = \frac{3(-7)}{2-(-7)} = \frac{-21}{9} = \frac{-7}{3}$

$u_6 = 9\left(\frac{-7}{3}\right)^5 = -\frac{16\,807}{27}$ $u_n = ar^{n-1}$

THE SIXTH TERM IS $-\frac{16\,807}{27}$

Your notes

Geometric series

How do I find the sum of a geometric series?

- A **geometric series** is the sum of the terms of a **geometric sequence**

1, 3, 9, 27, 81, ... IS A GEOMETRIC SEQUENCE

1 + 3 + 9 + 27 + 81 + ... IS A GEOMETRIC SERIES

- The following formulae will let you find the sum of the first ***n*** terms of a geometric series:

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{or} \quad S_n = \frac{a(r^n-1)}{r-1}$$

  - a is the first term
  - r is the common ratio
- The one on the left is more convenient if $r < 1$, the one on the right is more convenient if $r > 1$
- The ***a*** and the ***r*** in those formulae are exactly the same as the ones used with **geometric sequences**

How do I prove the geometric series formula?

- Learn this proof of the geometric series formula – you can be asked to give it in the exam:
  - Write out the sum once
  - Write out the sum again but multiply each term by r
  - Subtract the second sum from the first
    - All the terms except two should cancel out
  - Factorise and rearrange to make S the subject

THE SUM OF THE FIRST n TERMS IS

$$S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-2} + ar^{n-1} \qquad (1)$$

MULTIPLYING BY r GIVES

$$rS_n = ar + ar^2 + ar^3 + ar^4 + \ldots + ar^{n-1} + ar^n \qquad (2)$$

SUBTRACT (2) FROM (1) AND REARRANGE

$$S_n - rS_n = a - ar^n$$

$$S_n(1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

What is the sum to infinity of a geometric series?

- If (and only if!) $|r| < 1$, then the geometric series **converges** to a finite value given by the formula

$$S_\infty = \frac{a}{1-r}$$

- S_∞ is known as the **sum to infinity**
- If $|r| \geq 1$ the geometric series is **divergent** and the sum to infinity does not exist

Examiner Tips and Tricks

- The geometric series formulae are in the formulae booklet – you don't need to memorise them
- You will sometimes need to use logarithms to answer geometric series questions (see **Exponential Equations**)

Worked Example

Your notes

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a) Find the least value of n such that the sum

$10 + 9 + 8.1 + \ldots$ to n terms exceeds 99.

b) What is the sum to infinity of the series?

a) THIS IS A GEOMETRIC SERIES WITH

$a = 10 \quad r = 0.9$

WE WANT

$$S_n = \frac{10(1-0.9^n)}{1-0.9} > 99$$

$$100(1-0.9^n) > 99$$

$$100 - 100(0.9^n) > 99$$

$$100(0.9^n) < 1$$

$$0.9^n < 0.01$$

TAKE LOGARITHMS

$$\log(0.9^n) < \log 0.01$$

$$n \log 0.9 < \log 0.01$$

$\log 0.9$ IS NEGATIVE, SO INEQUALITY SIGN FLIPS

$$n > \frac{\log 0.01}{\log 0.9}$$

$$n > 43.7\ldots$$

n HAS TO BE AN INTEGER

$$n = 44$$

b) WITH $a = 10$ AND $r = 0.9$

$$S_\infty = \frac{10}{1-0.9} = 100$$

THE SUM TO INFINITY IS 100