



# Edexcel International AS Maths: Pure 2



Your notes

## Arithmetic Sequences & Series

### Contents

- \* Arithmetic Sequences
- \* Arithmetic Series



## Arithmetic sequences

### What do I need to know about arithmetic sequences?

- In an **arithmetic sequence**, the difference between consecutive terms in the sequence is constant

#### EXAMPLES OF ARITHMETIC SEQUENCES

$$2, 5, 8, 11, 14, \dots$$

$\swarrow \quad \swarrow \quad \swarrow \quad \swarrow$   
 $+3 \quad +3 \quad +3 \quad +3$

FIRST TERM IS 2, COMMON DIFFERENCE IS 3

$$2, 0.5, -1, -2.5, -4, \dots$$

$\swarrow \quad \swarrow \quad \swarrow \quad \swarrow$   
 $-1.5 \quad -1.5 \quad -1.5 \quad -1.5$

FIRST TERM IS 2, COMMON DIFFERENCE IS -1.5

Copyright © Save My Exams. All Rights Reserved



- That constant difference is known as the **common difference** of the sequence
- You need to know the  **$n^{\text{th}}$  term formula** for an arithmetic sequence

$$u_n = a + (n - 1)d$$

- $a$  is the first term
- $d$  is the common difference



#### Examiner Tips and Tricks

If you know two terms in an arithmetic sequence you can find  **$a$**  and  **$d$**  using simultaneous equations.



Your notes

e.g. THE 15<sup>th</sup> TERM OF AN ARITHMETIC SEQUENCE IS -19.  
THE 21<sup>st</sup> TERM IS -28.  
FIND THE FIRST TERM AND THE COMMON DIFFERENCE.

$$u_n = a + (n-1)d$$

$$\begin{aligned} a + 14d &= -19 & \textcircled{1} \\ a + 20d &= -28 & \textcircled{2} \end{aligned}$$

SOLVE THESE SIMULTANEOUS EQUATIONS

$$\begin{aligned} 6d &= -9 \\ d &= -1.5 \end{aligned}$$

② MINUS ①

$$\begin{aligned} a + 14(-1.5) &= -19 \\ a &= 2 \end{aligned}$$

SUBSTITUTE INTO ①

THE FIRST TERM IS 2 AND  
THE COMMON DIFFERENCE IS -1.5

Copyright © Save My Exams. All Rights Reserved



## Worked Example



The first three terms in an arithmetic sequence are 12,  $k^2$ ,  $5k$ , ...

Find two possible values of  $k$ .

THIS IS AN ARITHMETIC SEQUENCE SO THERE'S A SINGLE COMMON DIFFERENCE

$$\begin{aligned} d &= k^2 - 12 & \leftarrow 2^{\text{nd}} \text{ TERM MINUS } 1^{\text{st}} \text{ TERM} \\ \text{AND} \\ d &= 5k - k^2 & \leftarrow 3^{\text{rd}} \text{ TERM MINUS } 2^{\text{nd}} \text{ TERM} \end{aligned}$$

$$\text{SO } k^2 - 12 = 5k - k^2$$

$$2k^2 - 5k - 12 = 0$$

$$(2k+3)(k-4) = 0$$

SOLVE THE QUADRATIC

$$k = -\frac{3}{2} \quad \text{OR} \quad k = 4$$

Copyright © Save My Exams. All Rights Reserved





# Arithmetic series

## How do I find the sum of an arithmetic series?

- An **arithmetic series** is the sum of the terms of an **arithmetic sequence**

1, 3, 5, 7, 9, ... IS AN ARITHMETIC SEQUENCE

1 + 3 + 5 + 7 + 9 + ... IS AN ARITHMETIC SERIES

Copyright © Save My Exams. All Rights Reserved



- The following formulae will let you find the sum of the first  $n$  terms of an arithmetic series:

$$S_n = \frac{n}{2} (2a + (n-1)d) \text{ or } S_n = \frac{n}{2} (a + l)$$

- $a$  is the first term
- $d$  is the common difference
- $l$  is the last term
- You can use whichever formula is more convenient for a given question
- The  $a$  and the  $d$  in those formulae are exactly the same as the ones used with **arithmetic sequences**

## How do I prove the arithmetic series formula?

- Learn this proof of the arithmetic series formula – you can be asked to give it on the exam:
  - Write the terms out once in order
  - Write the terms out again in reverse order
  - Add the two sums together
    - The terms will pair up to give the same sum  $2a + (n-1)d$
    - There will be  $n$  of these terms
  - Divide by two as two of the sums have been added together



Your notes

THE SUM OF THE FIRST  $n$  TERMS IS

$$S_n = a + (a+d) + (a+2d) + \dots + (a+(n-2)d) + (a+(n-1)d) \quad (1)$$

NOW WRITE THE SAME SUM THE OTHER WAY ROUND

$$S_n = (a+(n-1)d) + (a+(n-2)d) + \dots + (a+2d) + (a+d) + a \quad (2)$$

ADD (1) AND (2) TOGETHER TERM BY TERM

$$2S_n = n(2a + (n-1)d)$$

EACH OF THE SUMS  
CONTAINS  $n$  TERMS

DIVIDING BY 2 GIVES

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$a + (a+(n-1)d) = 2a + (n-1)d$   
 $(a+d) + (a+(n-2)d) = 2a + (n-1)d$   
etc.

Copyright © Save My Exams. All Rights Reserved



### Examiner Tips and Tricks

The arithmetic series formulae are in the formulae booklet – you don't need to memorise them.



### Worked Example



Your notes



The ninth term of an arithmetic series is 28.  
The fifteenth term is 46. The sum of the first  $n$  terms is 2500.

- a) Show that  $3n^2 + 5n - 5000 = 0$ .  
b) Hence find the value of  $n$ .

a)

$$\begin{aligned} a + (9-1)d &= a + 8d = 28 & \textcircled{1} \\ a + (15-1)d &= a + 14d = 46 & \textcircled{2} \end{aligned}$$

SIMULTANEOUS EQUATIONS

$$\begin{aligned} 6d &= 18 \\ d &= 3 \end{aligned}$$

② MINUS ①

$$\begin{aligned} a &= 28 - 8(3) \\ a &= 4 \end{aligned}$$

SUBSTITUTE INTO ①

$$\begin{aligned} \text{SO } \frac{n}{2} (2(4) + (n-1)(3)) &= 2500 & S_n \text{ FORMULA} \\ n(3n+5) &= 5000 \\ 3n^2 + 5n - 5000 &= 0 \end{aligned}$$

b)

$$\begin{aligned} (3n+125)(n-40) &= 0 \\ n &= -\frac{125}{3} \quad \text{OR} \quad n = 40 \end{aligned}$$

SOLVE THE QUADRATIC — YOU COULD ALSO USE THE QUADRATIC FORMULA OR YOUR CALCULATOR

BUT  $n$  MUST BE A POSITIVE INTEGER  
 $n = 40$

Copyright © Save My Exams. All Rights Reserved

