Edexcel International AS Maths: Pure 2

Your notes

Arithmetic Sequences & Series

Contents

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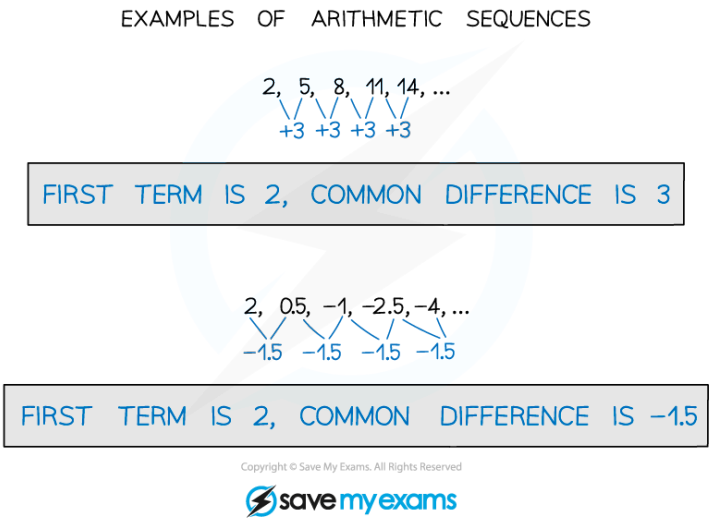
Arithmetic sequences

What do I need to know about arithmetic sequences?

- In an **arithmetic sequence**, the difference between consecutive terms in the sequence is constant



- That constant difference is known as the **common difference** of the sequence
- You need to know the **n^{th} term formula** for an arithmetic sequence

$$u_n = a + (n-1)d$$

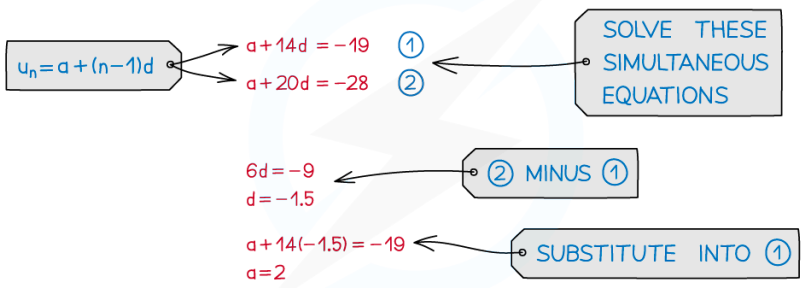
  - a is the first term
  - d is the common difference

Examiner Tips and Tricks

If you know two terms in an arithmetic sequence you can find ***a*** and ***d*** using simultaneous equations.

Your notes

e.g. THE 15th TERM OF AN ARITHMETIC SEQUENCE IS −19.
THE 21st TERM IS −28.
FIND THE FIRST TERM AND THE COMMON DIFFERENCE.



THE FIRST TERM IS 2 AND
THE COMMON DIFFERENCE IS −1.5

Worked Example

? The first three terms in an arithmetic sequence are $12, k^2, 5k, \ldots$

Find two possible values of k.

THIS IS AN ARITHMETIC SEQUENCE SO THERE'S A SINGLE COMMON DIFFERENCE

$d = k^2 - 12$ ← 2nd TERM MINUS 1st TERM

AND

$d = 5k - k^2$ ← 3rd TERM MINUS 2nd TERM

SO $k^2 - 12 = 5k - k^2$

$2k^2 - 5k - 12 = 0$

$(2k+3)(k-4) = 0$ ← SOLVE THE QUADRATIC

$k = -\frac{3}{2}$ OR $k = 4$

Your notes

Arithmetic series

How do I find the sum of an arithmetic series?

- An **arithmetic series** is the sum of the terms of an **arithmetic sequence**

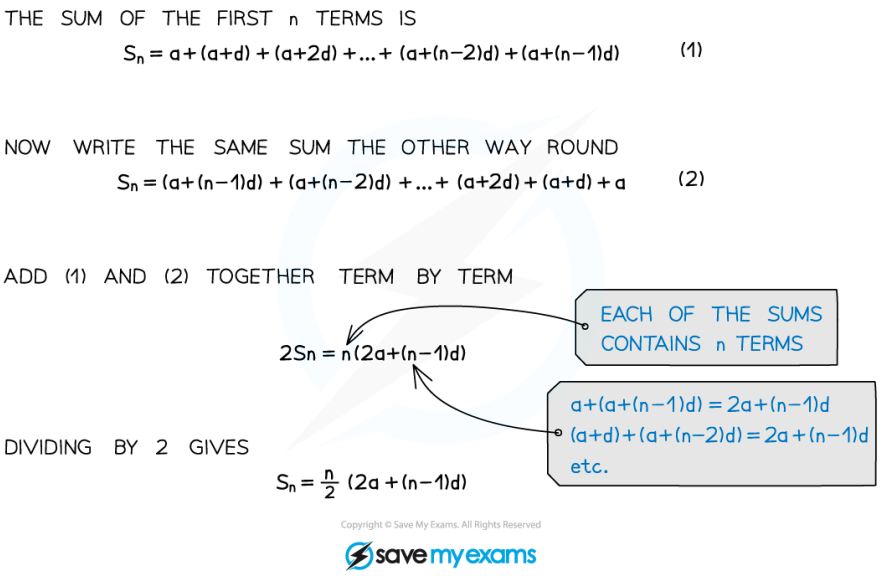
- The following formulae will let you find the sum of the first ***n*** terms of an arithmetic series:

$$S_n = \frac{n}{2}(2a + (n-1)d) \text{ or } S_n = \frac{n}{2}(a + l)$$

  - *a* is the first term
  - *d* is the common difference
  - *l* is the last term
- You can use whichever formula is more convenient for a given question
- The ***a*** and the ***d*** in those formulae are exactly the same as the ones used with **arithmetic sequences**

How do I prove the arithmetic series formula?

- Learn this proof of the arithmetic series formula – you can be asked to give it on the exam:
  - Write the terms out once in order
  - Write the terms out again in reverse order
  - Add the two sums together
    - The terms will pair up to give the same sum $2a + (n-1)d$
    - There will be *n* of these terms
  - Divide by two as two of the sums have been added together



Your notes

Examiner Tips and Tricks

The arithmetic series formulae are in the formulae booklet – you don't need to memorise them.

Worked Example

Your notes

The ninth term of an arithmetic series is 28. The fifteenth term is 46. The sum of the first n terms is 2500.

a) Show that $3n^2 + 5n - 5000 = 0$.

b) Hence find the value of n.

a) $a+(9-1)d = a+8d = 28$ ①
$a+(15-1)d = a+14d = 46$ ②

SIMULTANEOUS EQUATIONS

$6d = 18$
$d = 3$

② MINUS ①

$a = 28-8(3)$
$a = 4$

SUBSTITUTE INTO ①

SO $\frac{n}{2}(2(4)+(n-1)(3)) = 2500$

S_n FORMULA

$n(3n+5) = 5000$
$3n^2+5n-5000 = 0$

b) $(3n+125)(n-40) = 0$
$n = -\frac{125}{3}$ OR $n = 40$

SOLVE THE QUADRATIC — YOU COULD ALSO USE THE QUADRATIC FORMULA OR YOUR CALCULATOR

BUT n MUST BE A POSITIVE INTEGER
$n = 40$