



Edexcel International AS Maths: Pure 2



Your notes

Binomial Expansion

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Binomial expansion

What is the binomial expansion?

$$\begin{aligned}
 (a+b)^n &= {}^nC_0(a)^n(b)^0 \\
 &+ {}^nC_1(a)^{n-1}(b)^1 \\
 &+ {}^nC_2(a)^{n-2}(b)^2 \\
 &+ {}^nC_3(a)^{n-3}(b)^3 \\
 &+ \dots \\
 &+ {}^nC_r(a)^{n-r}(b)^r \\
 &+ \dots \\
 &+ {}^nC_{n-1}(a)^1(b)^{n-1} \\
 &+ {}^nC_n(a)^0(b)^n
 \end{aligned}$$

$${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} \quad n \in \mathbb{N}$$

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- Look at the pattern
 - Start at nC_0 , then nC_1 , nC_2 , etc
 - Powers of **a** start at **n** and decrease by 1
 - Powers of **b** start at 0 and increase by 1
- There are shortcuts but these hide the pattern
 - ${}^nC_0 = {}^nC_n = 1$
 - ${}^nC_1 = {}^nC_{n-1} = n$
 - ${}^nC_r = {}^nC_{n-r}$
 - $(b)^0 = (a)^0 = 1$
- Use the shortcuts once familiar with the pattern
- ! means factorial

$$(a+b)^n = a^n + na^{n-1}b + {}^nC_2a^{n-2}b^2 + \dots + b^n$$

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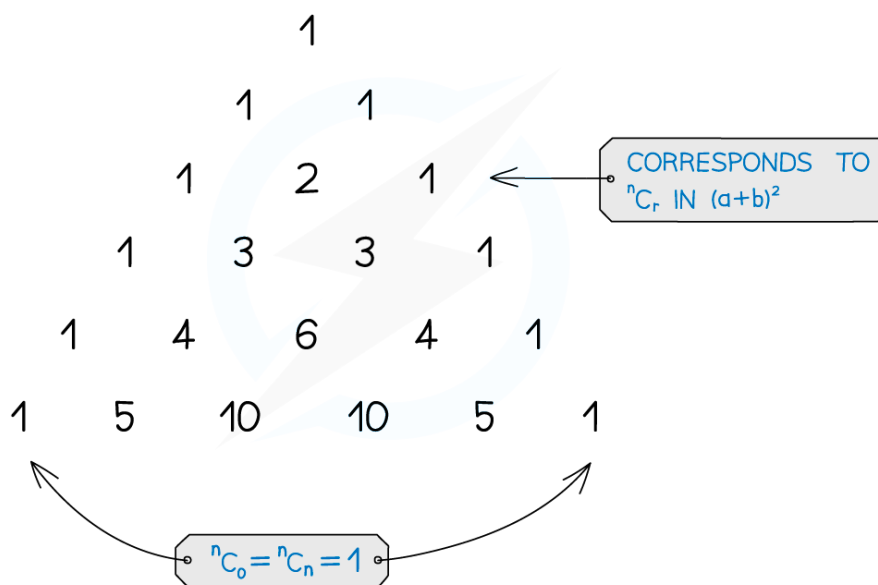


Your notes

- This is given in the formula booklet

What is Pascal's triangle?

PASCAL'S TRIANGLE



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- Pascal's triangle is an alternative to nC_r
- It is useful for lower values of n
- For larger n it is slow and prone to arithmetic errors

How do I expand brackets with the binomial expansion?



Your notes

$a=3, b=2x$

e.g. $(3+2x)^4 = 3^4$
 $+ 4(3)^3(2x)$
 $+ {}^4C_2(3)^2(2x)^2$
 $+ {}^4C_3(3)(2x)^3$
 $+ (2x)^4$
 $= 81 + 216x + 216x^2 + 96x^3 + 16x^4$

THIS 2 WILL GET CUBED

USE A LINE FOR EACH TERM

USE BRACKETS AND CALCULATOR FOR ACCURACY

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- Use a line for each term to make things easier to read and follow
- Use brackets, particularly helpful when negatives involved
- Use a calculator for nC_r

SHIFT

nC_r

$4C2 \times (3)^2 \times (2)^2$

216

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Worked Example



Your notes



Find the first four terms, in ascending powers of x , in the expansion of $(2 - 3x)^6$

USE SEPARATE LINES AND BRACKETS TO KEEP WORKING CLEAR

$$(2-3x)^6 \approx (2)^6$$

$$+ 6(2)^5(-3x)$$

$$+ {}^6C_2(2)^4(-3x)^2$$

$$+ {}^6C_3(2)^3(-3x)^3$$

TYPE

$${}^6C_3 \times (2)^3 \times (-3)^3$$

INTO CALCULATOR

$$= 64 - 576x + 2160x^2 - 4320x^3$$

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Applications of binomial expansion

What are binomial expansions used for?

- Expanding brackets, it is usual to be asked for either ...
 - ... the first few terms only

e.g. FIND, IN ASCENDING POWERS OF x ,
THE FIRST THREE TERMS OF $(5+2x)^9$

$$(5+2x)^9 \approx (5)^9 + 9(5)^8(2x) + {}^9C_2(5)^7(2x)^2$$

$$= 1\,953\,125 + 7\,031\,250x + 11\,250\,000x^2$$

APPROXIMATELY EQUAL
TO AS 3 TERMS ONLY

ASCENDING POWERS
OF x SO START
WITH CONSTANT
TERM (x^0)

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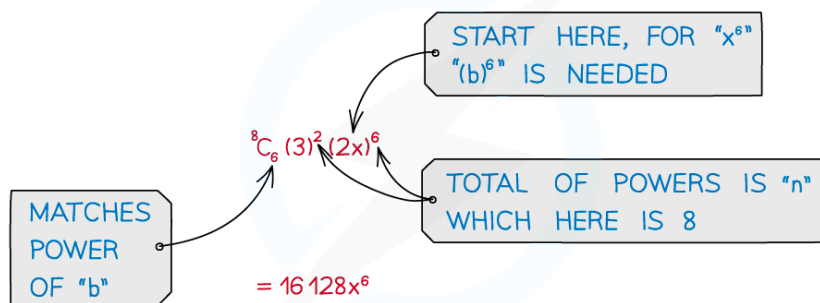


- or (the coefficient of) a particular term



Your notes

e.g. FIND THE COEFFICIENT OF x^6 IN THE EXPANSION OF $(3+2x)^8$



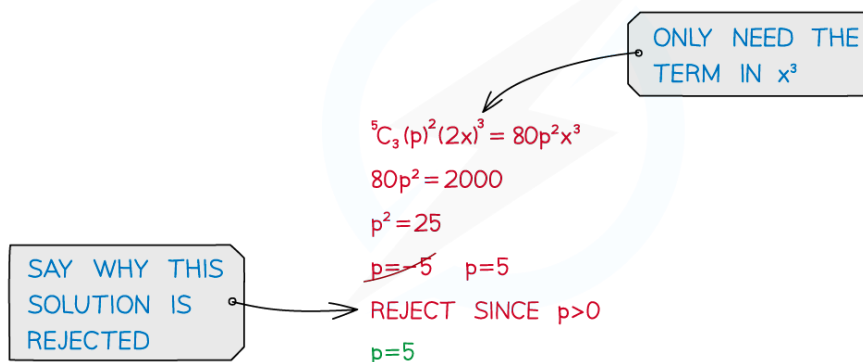
SO THE COEFFICIENT OF x^6 IS 16 128

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■ Solving problems in unknowns

e.g. THE COEFFICIENT OF THE x^3 TERM IN THE EXPANSION OF $(p+2x)^5$ IS 2000.
FIND THE VALUE OF p , GIVEN $p > 0$



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