Edexcel International AS Maths: Pure 2

Your notes

Proof

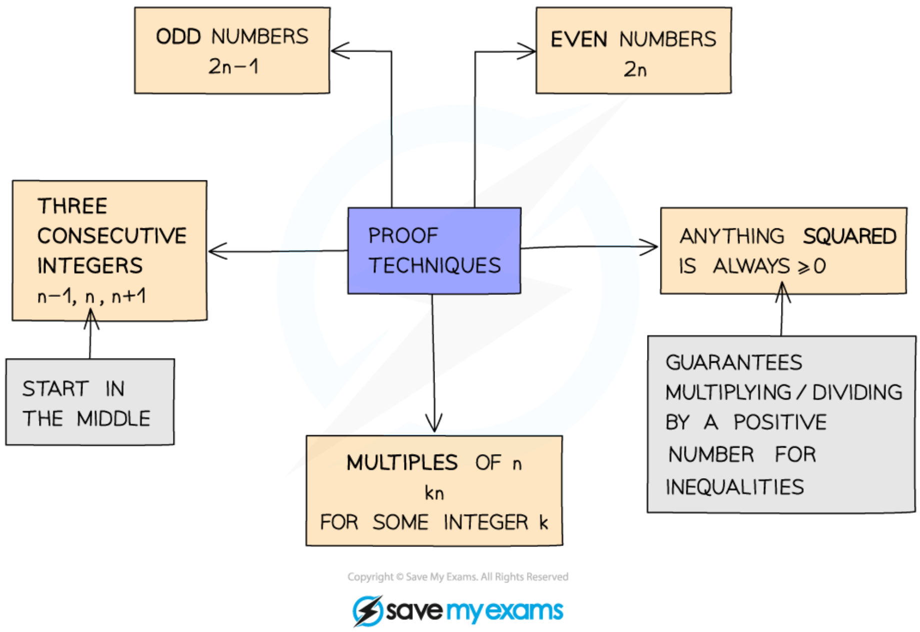
Contents

Language of proof

What is proof?

- Proof is a series of logical steps which show whether a result is **true** or not for a set of specified numbers
  - e.g. All integers, all even numbers etc

Notation in proof



- LHS and RHS are standard abbreviations for left-hand side and right-hand side
- Integers are used frequently in the language of proof
  - The set of integers is denoted by $\mathbb{Z}$

Techniques for proof

Your notes

$P = Q$	P IS EQUAL TO Q (TRUE FOR SOME VALUES)
$P \equiv Q$	P IS IDENTICAL TO Q (TRUE FOR ALL VALUES)
$P \Rightarrow Q$	P IMPLIES Q (IF P IS TRUE THEN Q IS TRUE)
$P \Leftrightarrow Q$	P IS EQUIVALENT TO Q (Q IS TRUE IF AND ONLY IF (IFF) P IS TRUE

Worked Example

Prove that the sum of any three consecutive odd numbers is a multiple of 3.

LET THE THREE CONSECUTIVE ODD NUMBERS BE

$2n-3,\ 2n-1,\ 2n+1$ — START WITH THE MIDDLE VALUE

THEN,

$(2n-3) + (2n-1) + (2n+1)$

$\equiv 6n - 3$

$\equiv 3(2n-1)$ — AIM FOR THE FORM $3k$

WHICH IS A MULTIPLE OF 3

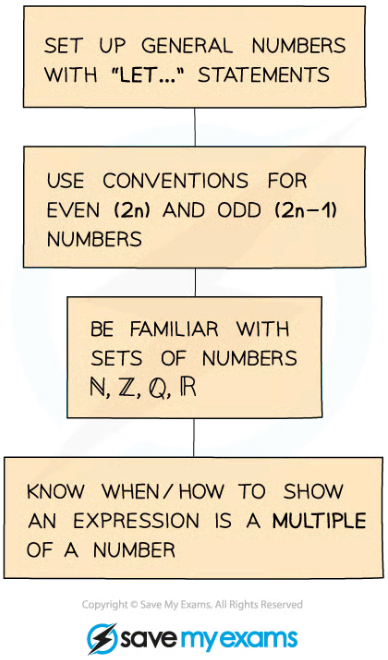
Your notes

Proof by deduction

What is proof by deduction?

Proof by deduction is when a mathematical and logical argument is used to show whether or not a result is true

How do I use proof by deduction?



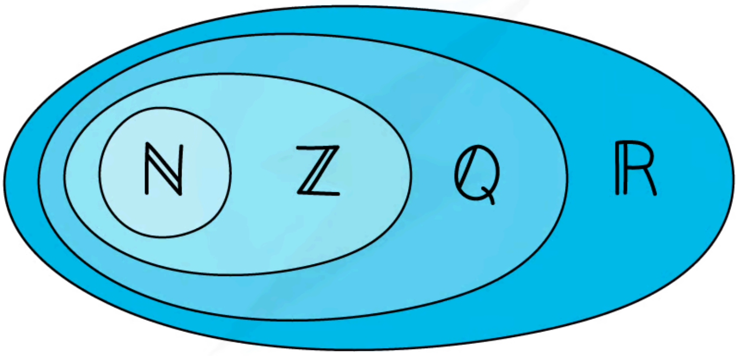
You may also need to:

- Write multiples of **n** in the form **kn** for some integer **k**
- Use algebraic techniques, showing logical steps of simplifying
- Use correct mathematical notation

Sets of numbers

- $\mathbb{N}$ - the set of natural numbers
- $\mathbb{Z}$ - the set of integers
- $\mathbb{Q}$ - the set of quotients/rational numbers
- $\mathbb{R}$ - the set of real numbers

Your notes



Examiner Tips and Tricks

- Try the result you are proving with a few values
  - Use a sequence of them (eg 1, 2, 3)
  - Try different types of numbers (positive, negative, zero)
- This may help you see a pattern and spot what is going on

Worked Example

Prove that the square of an even number is also even.

WRITE EVEN NUMBERS IN THE FORM $2n$

LET AN EVEN NUMBER BE $2n$

$(2n)^2 \equiv 4n^2$

$\equiv 2(2n^2)$

$\equiv 2k$ FOR SOME INTEGER k

"EVEN" IS THE SAME AS "MULTIPLE OF 2"

SO THE SQUARE OF AN EVEN NUMBER IS ALSO EVEN

Your notes

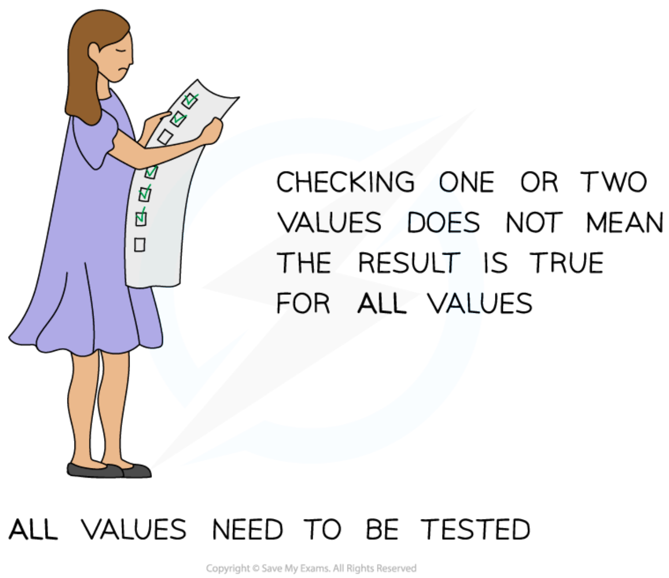
Proof by exhaustion

What is proof by exhaustion?

- Proof by exhaustion is a way to show that the desired result works for every allowed value

How do I prove a result by exhaustion?

- Using proof by exhaustion means testing every allowed value not just showing a few examples



Difficulties with proof by exhaustion

- In many cases proof by exhaustion is not practical, or possible
    - Proving all multiples of 4 are even can't be shown for every multiple of 4
- Aim to minimise the work involved
    - Proving a number is prime only requires testing factors up to the square root

Examiner Tips and Tricks

- Try a simpler case if you are stuck.
- For example, if you are asked to prove that 97 is a prime number you could try thinking about what you would do for smaller primes such as 7 or 11.

Worked Example

Prove that the difference between $n^3 + 7$ and a multiple of 7 is always 1, for all integers in the interval $1 \leq n \leq 4$.

NOTE: THIS RESULT MAY BE TRUE FOR ALL VALUES OF n BUT YOU ARE ONLY ASKED TO PROVE IT FOR INTEGERS IN THE INTERVAL $1 \leqslant n \leqslant 4$, i.e. $n=1$, $n=2$, $n=3$, $n=4$ SO 'EXHAUST' ALL POSSIBILITIES

$n = 1$, $n^3 + 7 = 1^3 + 7 = 8$ DIFFERENCE OF 1 FROM 7

$n = 2$, $n^3 + 7 = 2^3 + 7 = 15$ DIFFERENCE OF 1 FROM 14

$n = 3$, $n^3 + 7 = 3^3 + 7 = 34$ DIFFERENCE OF 1 FROM 35

$n = 4$, $n^3 + 7 = 4^3 + 7 = 71$ DIFFERENCE OF 1 FROM 70

BY EXHAUSTION, n^3+7 HAS A DIFFERENCE OF 1 FROM A MULTIPLE OF 7 FOR ALL INTEGERS $1 \leqslant n \leqslant 4$

DO SHOW THE APPROPRIATE MULTIPLES OF 7

Your notes

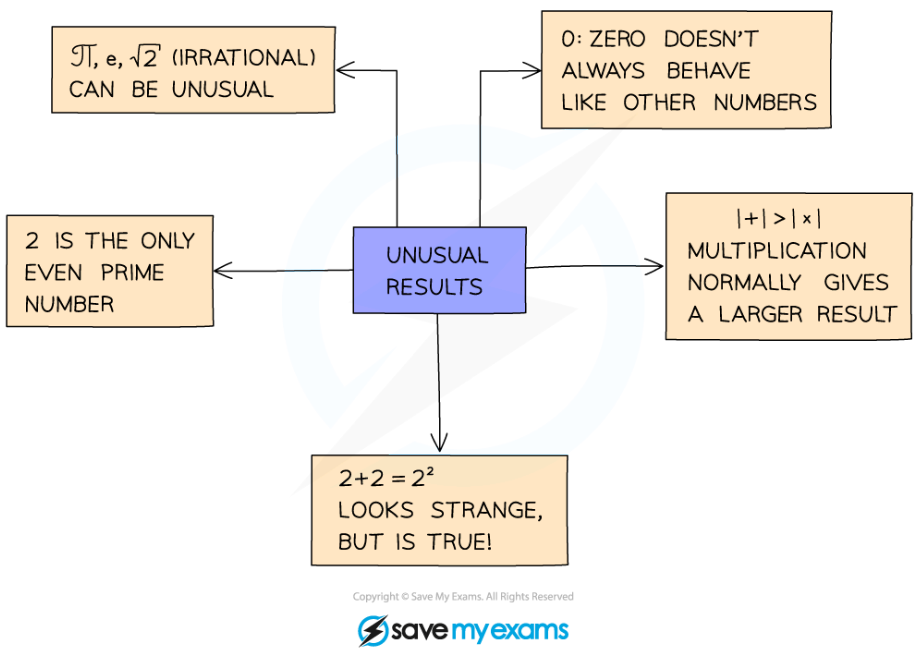
Disproof by counter example

What is disproof by counter example?

- Disproof by counter examples involves finding a value that does **not** work for the given statement
    - That value is called a **counter example**

How do I disprove a result?

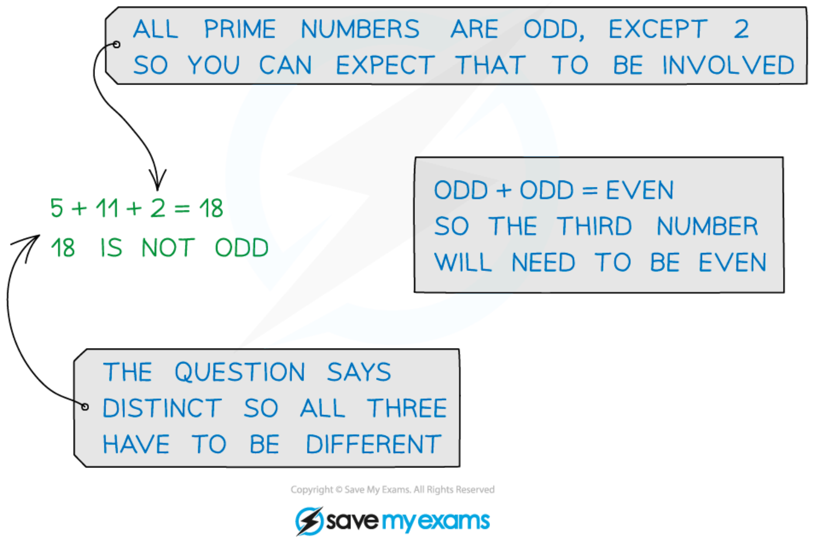
- You only need to find **one** value that does "not work"
- Numbers that have unusual results are often helpful to try



- Look carefully to see what types of number the result is claimed for – it may just be integers rather than real numbers
- Irrational numbers can be unusual
- Think about areas of maths where unusual things happen – such as in trigonometry where there are several angles with the same sine value

Worked Example

Use a counter-example to show that the sum of three distinct prime numbers is not always odd.



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