



Edexcel International A Level (IAL) Maths: Pure 1



Your notes

Basic Trigonometry

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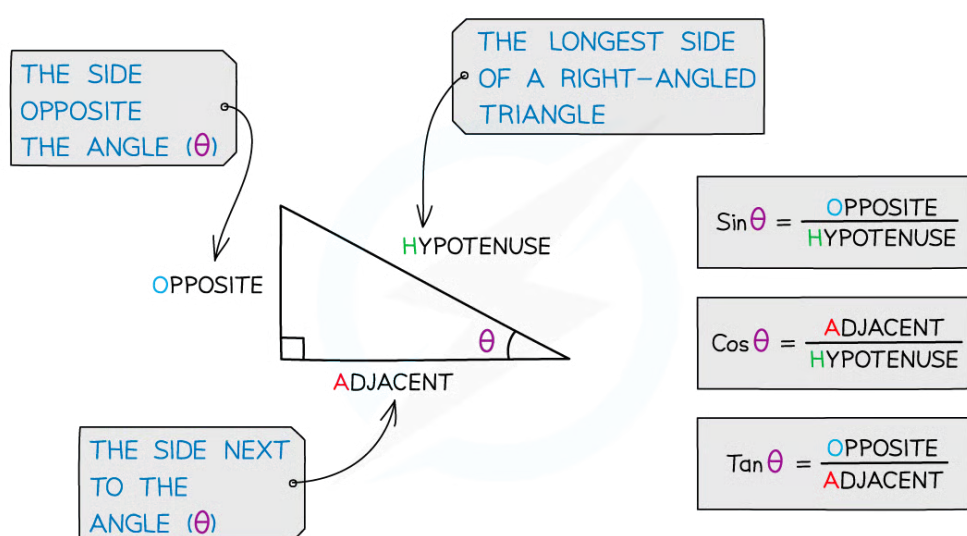
- * Definitions of Trigonometric Functions
- * Right-Angled Triangles
- * Non-Right-Angled Triangles



Definitions of trigonometric functions

What is trigonometry?

- Trigonometry looks at the relationship between side lengths and angles of triangles
- It comes from the Greek words trigonon meaning 'triangle' and metron meaning 'measure'
- The three trigonometric functions Sine, Cosine and Tangent come from ratios of side lengths in right angled triangles



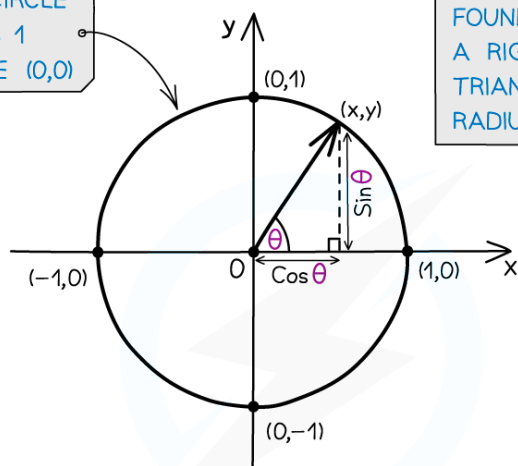
What is the unit circle in trigonometry?

- The unit circle is a circle with radius 1 and centre (0, 0)
- It can be used to calculate trig values as a co-ordinate point (x, y) on the circle
- $(x, y) = (\cos \theta, \sin \theta)$, where θ is the angle measured anticlockwise from the positive x-axis
- It allows us to calculate **sin**, **cos** and **tan** for angles greater than 90°



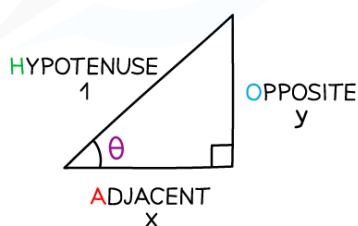
Your notes

THE UNIT CIRCLE HAS RADIUS 1 AND CENTRE (0,0)



TRIG VALUES CAN BE FOUND BY MAKING A RIGHT ANGLED TRIANGLE WITH THE RADIUS AND x-AXIS

ANY POINT (x,y) ON THE UNIT CIRCLE CAN BE FOUND USING $(\cos \theta, \sin \theta)$



$$\cos \theta = \frac{\text{A}}{\text{H}} = \frac{x}{1} = x$$

$$\sin \theta = \frac{\text{O}}{\text{H}} = \frac{y}{1} = y$$

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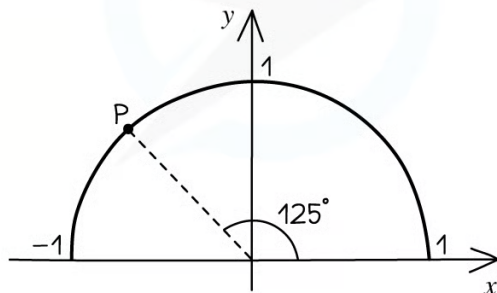
Worked Example



Your notes



- a) The co-ordinates of a point on the unit circle, to 3 s.f., are (0.629, 0.777). Find θ to the nearest degree.
- b) Find the coordinates of point P on the unit circle. Give your answer to 2 d.p.



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WE KNOW $(x,y) = (\cos\theta, \sin\theta)$
SO $\cos\theta = 0.629$ AND $\sin\theta = 0.777$

BOTH WORK BUT
YOU ONLY NEED
TO USE ONE

a) $\cos\theta = 0.629$
 $\theta = \cos^{-1}(0.629) = 51^\circ$ (TO THE NEAREST DEGREE)
OR
 $\sin\theta = 0.777$
 $\theta = \sin^{-1}(0.777) = 51^\circ$ (TO THE NEAREST DEGREE)

AS P IS ON THE
UNIT CIRCLE WE
KNOW $(x,y) = (\cos\theta, \sin\theta)$

b) $\cos(125) = -0.5735764...$
 $\sin(125) = 0.8191520...$
SO $P = (-0.57, 0.82)$

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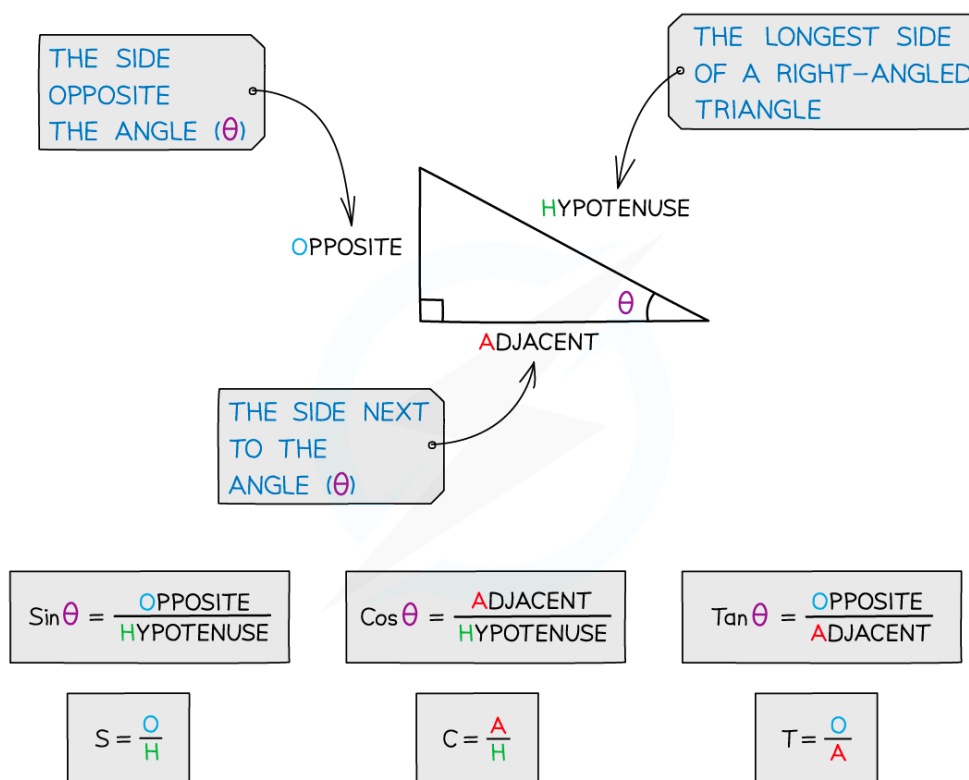




Right-angled triangles

What is SOHCAHTOA?

- SOH, CAH, and TOA are an easy way to remember the three basic trigonometry ratios
- They show the relationship between an angle and the different sides of a right-angled triangle which we can label **Hypotenuse**, **Adjacent** and **Opposite**
- We use them to find missing angles and sides for right-angled triangles
 - Hypotenuse** is the longest side of a right-angled triangle
 - Adjacent** is the side next to the angle you are working with (in between the angle and the right angle)
 - Opposite** is the side opposite the angle you are working with



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- We use the inverse trig functions to find missing angles

$$\theta = \sin^{-1}\left(\frac{O}{H}\right)$$

$$\theta = \cos^{-1}\left(\frac{A}{H}\right)$$

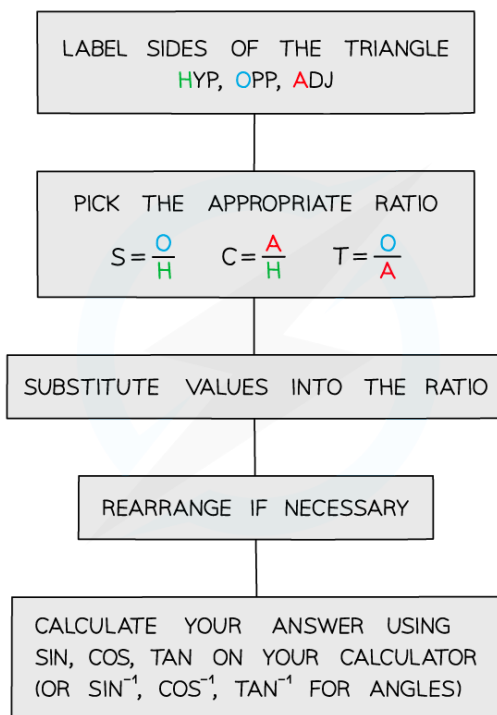
$$\theta = \tan^{-1}\left(\frac{O}{A}\right)$$

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Your notes

How do I use SOHCAHTOA?



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Examiner Tips and Tricks

- Remember SOHCAHTOA are only for calculating missing sides and angles for right-angled triangles
 - For non-right-angled triangles you will need to use the Sine or Cosine rules.
- Make sure your calculator is in degree mode (D).
- Always check your answers are sensible...
 - Should my answer be longer or shorter than the sides I have already i.e. is it the hypotenuse?
 - Is my angle sensible...
 - Have I remembered to use $^{-1}$?

- Check if the question asks you to round your answer.



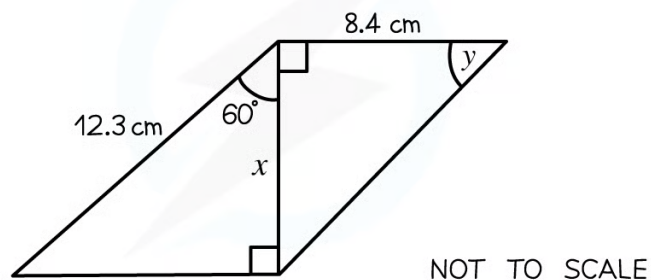
Your notes



Worked Example



Find the values of x and y in the following diagram.
Give your answers to 3 significant figures.



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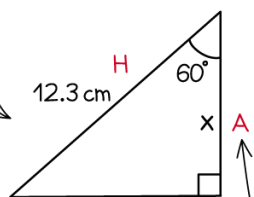




Your notes

TO FIND x

SKETCH AND LABEL THE TRIANGLE



$$\cos \theta = \frac{A}{H}$$

WE KNOW H AND WE WANT TO FIND A SO MUST NEED CAH

$$\cos 60 = \frac{x}{12.3}$$

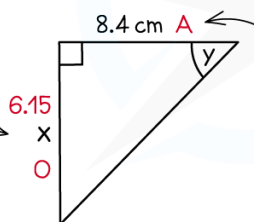
SUBSTITUTE VALUES AND REARRANGE

$$x = 12.3 \times \cos 60$$

$$x = 6.15 \text{ cm}$$

TO FIND y

NOW WE'VE FOUND $x=6.15$, SKETCH AND LABEL THE TRIANGLE



$$\tan \theta = \frac{O}{A}$$

WE HAVE A AND O SO MUST NEED TOA

$$\tan y = \frac{6.15}{8.4}$$

USE $\tan^{-1}$ TO CALCULATE THE ANGLE

$$y = \tan^{-1}\left(\frac{6.15}{8.4}\right)$$

$$y = 36.209...$$

$$y = 36.2$$

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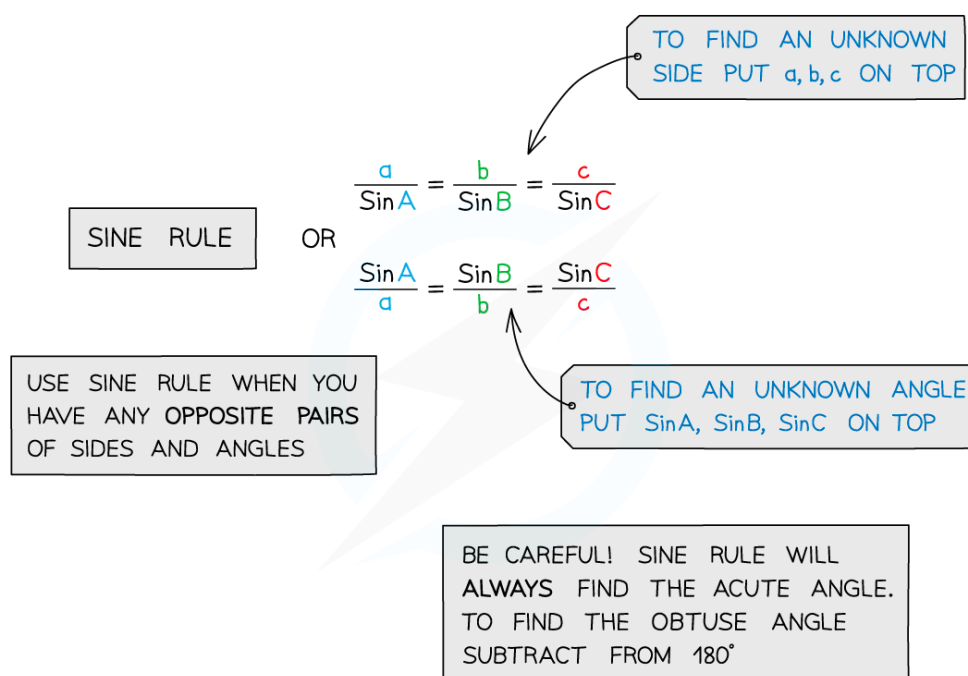
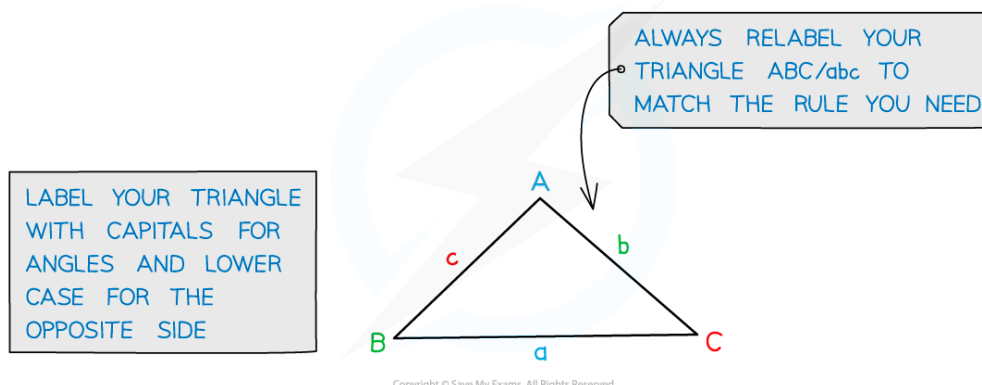




Non-right-angled triangles

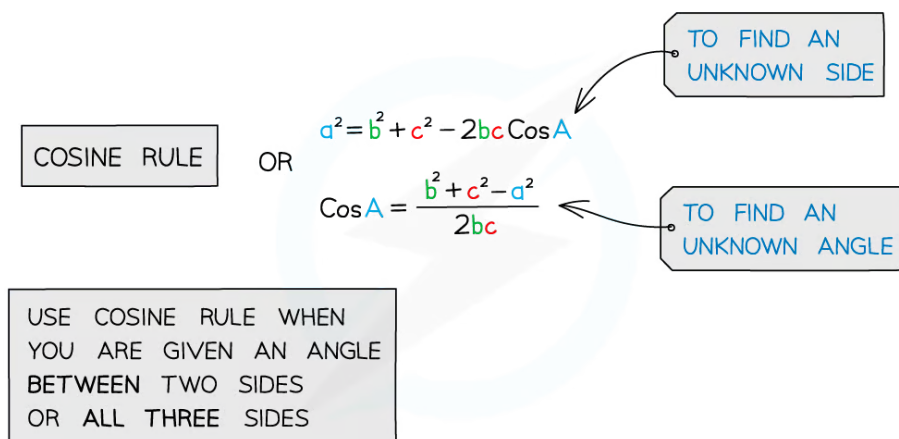
What is the sine rule, cosine rule and area rule for non-right-angled triangles?

- The Sine Rule, Cosine Rule and Area Formula can be used for ANY type of triangle
- They can help us calculate missing side lengths, angles and area

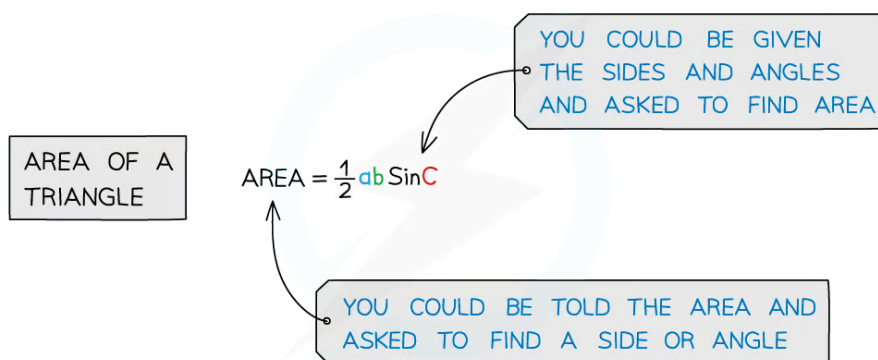




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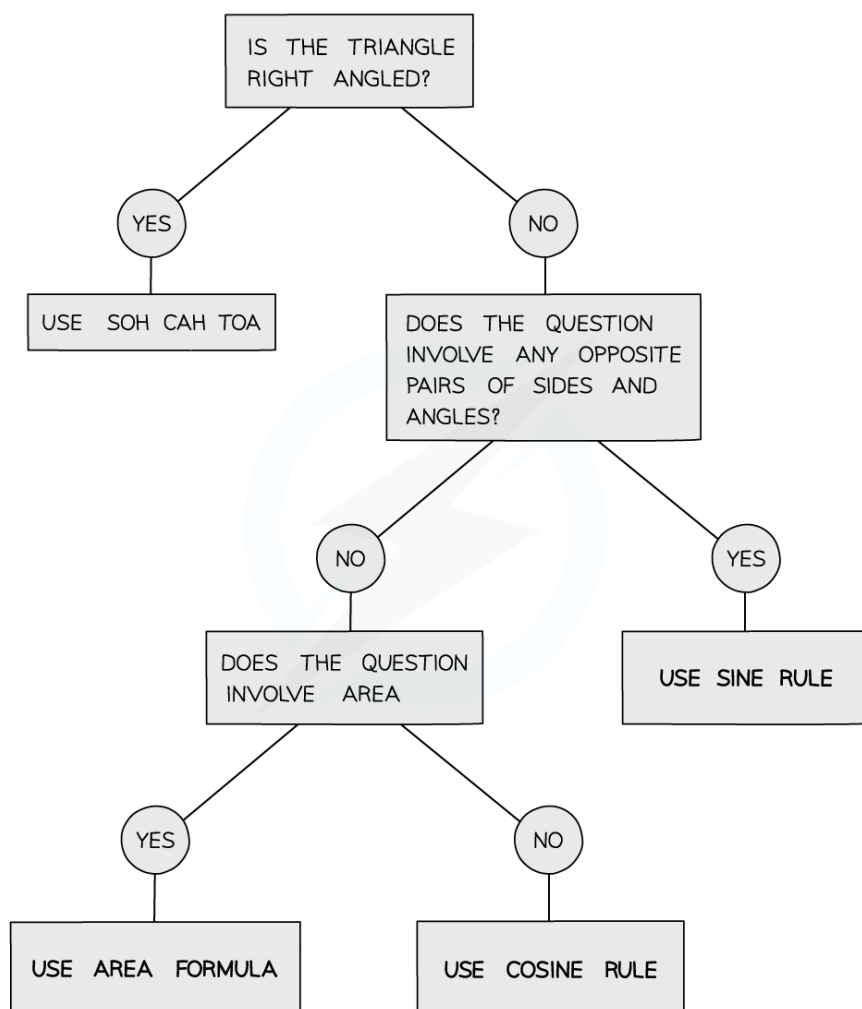


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How do I know which rule to use?



Your notes



- Once you know which rule to use, simply label your triangle, substitute in the values and rearrange if necessary



Examiner Tips and Tricks

- If you are given two angles in a triangle, finding the missing angle by subtracting from 180° might help.
- Make sure you are careful to pick the correct rule to use, you'll need to remember all the rules as they aren't in the formula booklet.
- Make sure your calculator is in degree mode (D).
- Always check your answers are sensible...
 - Have I remembered to square root at the end?
 - Have I remembered to use $^{-1}$?
- Check if the question asks you to round your answer.



Worked Example



Your notes



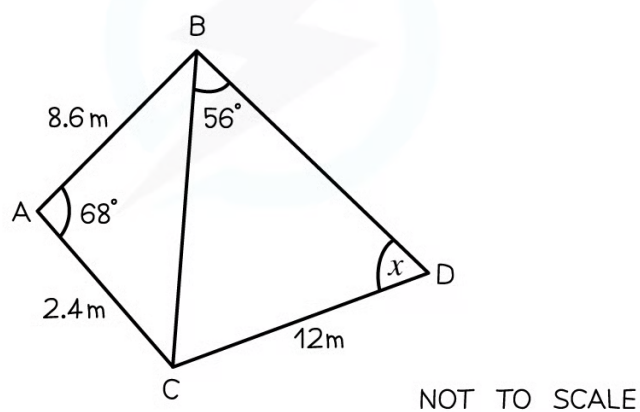
ABCD is a quadrilateral as shown in the diagram below.

a) Find the area of ABC.

Angle x is an acute angle.

b) Work out the size of angle x .

Give your answers to 3 significant figures.



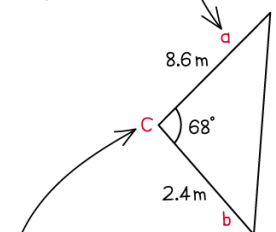
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a)



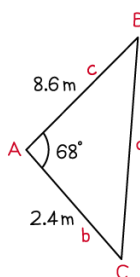
RELABEL DIAGRAM
TO MAKE THE
GIVEN ANGLE C

IT DOESN'T MATTER
WHICH WAY AROUND
YOU LABEL a AND b

SUBSTITUTE
AND CALCULATE

$$\begin{aligned}\text{AREA} &= \frac{1}{2}ab\sin C \\ &= \frac{1}{2} \times 8.6 \times 2.4 \times \sin 68 \\ &= 9.568... \\ &= 9.57\text{m}^2 \text{ (3SF)}\end{aligned}$$

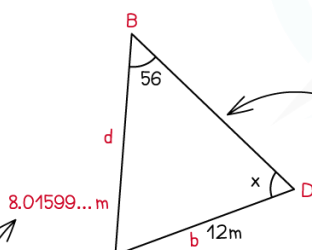
b)



WE HAVE AN ANGLE BETWEEN
TWO SIDES SO WE CAN START
BY FINDING THE LENGTH OF BC
USING COSINE RULE

$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc\cos A \\ a^2 &= 2.4^2 + 8.6^2 - 2 \times 2.4 \times 8.6 \cos 68 \\ a^2 &= 64.256... \\ a &= \sqrt{64.256...} = 8.01599...\end{aligned}$$

DON'T FORGET
TO $\sqrt{\text{SQUARE ROOT}}$



WE NOW HAVE OPPOSITE PAIRS
OF SIDES AND ANGLES SO CAN
USE SINE RULE TO FIND x

$$\begin{aligned}\frac{\sin D}{d} &= \frac{\sin B}{b} \\ \frac{\sin x}{8.01599} &= \frac{\sin 56}{12}\end{aligned}$$

THIS IS WHAT
WE HAVE JUST
FOUND

USE **ANS** BUTTON ON
YOUR CALCULATOR TO
PRESERVE ACCURACY

$$\sin x = 8.01599... \times \frac{\sin 56}{12}$$

$$\sin x = 0.55379...$$

$$x = \sin^{-1}(0.55379...)$$

$$x = 33.627...$$

USE **INVERSE⁻¹**
TO FIND x

$$x = 33.6^\circ \text{ (3SF)}$$

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