Edexcel International A Level (IAL) Maths: Pure 1

Your notes

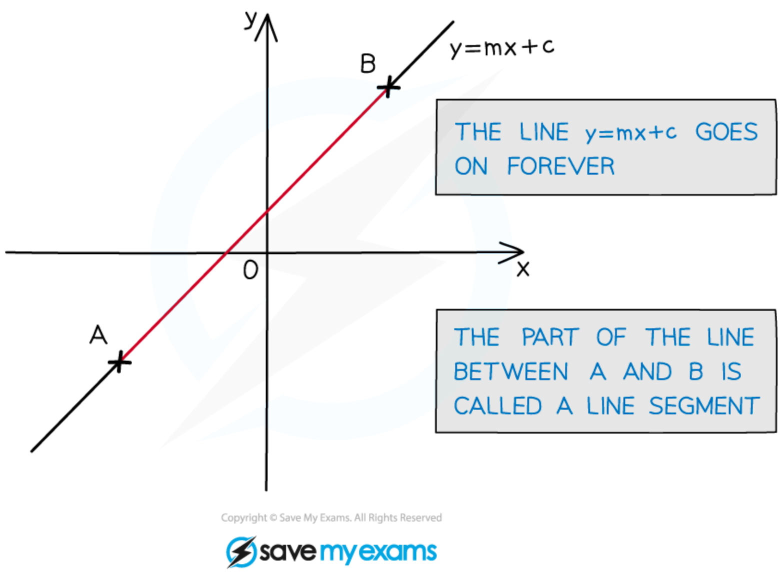
Equation of a Straight Line

Contents

Basic coordinate geometry

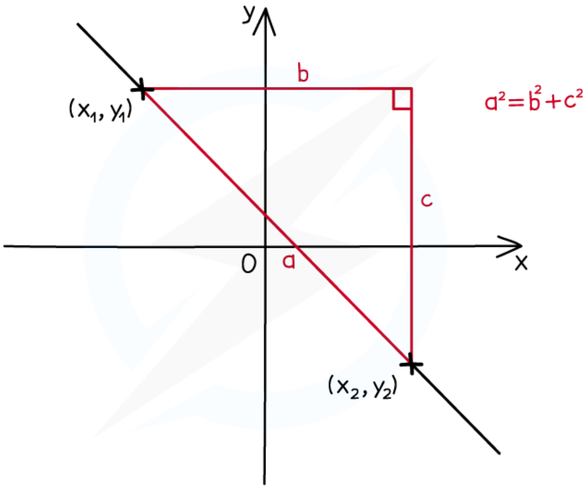
What is basic coordinate geometry?

- Basic coordinate geometry refers to working with points, lines and shapes on the coordinate axis
- **Cartesian** coordinates are simply the **x** -**y** coordinate system
- Using coordinates you can
  - Calculate the distance between two points (length of a line)
  - Divide lines in m:n ratio,
  - Find the mid-point of a line
  - Calculate the area of a triangle (or other shapes)
  - Find missing coordinates using any of the above
- Most of these involve working with straight line graphs
- **Straight line graphs** are of the form **y = mx + c**
  - **m** is the gradient
  - **c** is the **y**-axis intercept
  - Part of a straight line is called a **line segment**

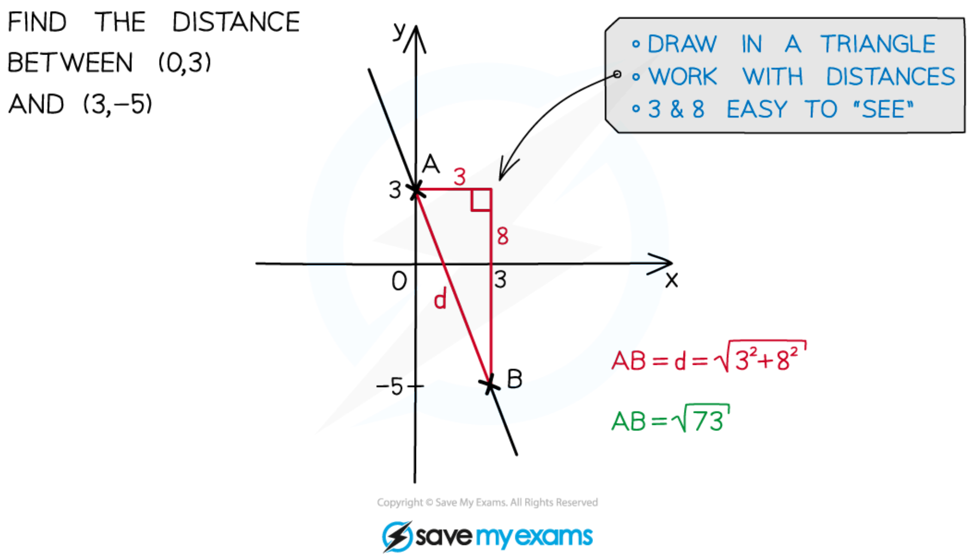


How do I find the length of a line segment?

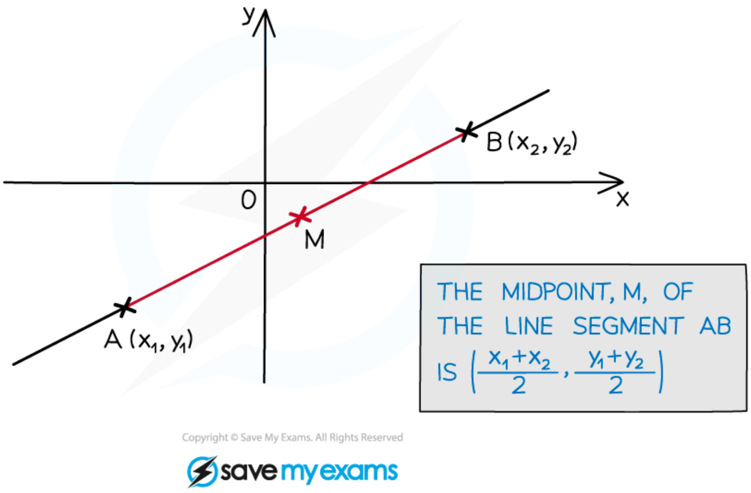
- By finding the distance, **d**, between two points ($\mathbf{x}_1$, $\mathbf{y}_1$) and ($\mathbf{x}_2$, $\mathbf{y}_2$)
- **Pythagoras' Theorem** ($\mathbf{a^2 = b^2 + c^2}$) is used to find the length of a line



$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$



How do I find the midpoint of a line segment?



- M is often used as the midpoint between two points ($\mathbf{x}_1$, $\mathbf{y}_1$) and ($\mathbf{x}_2$, $\mathbf{y}_2$)
- It is the average of both the **x** and **y** coordinates

Examiner Tips and Tricks

- Work with the **square** of a distance for as long as possible as this avoids early rounding errors and surds.

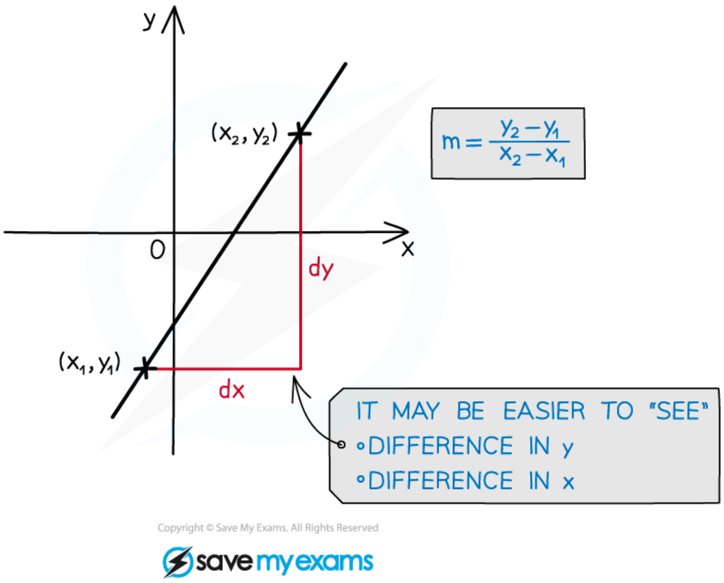
Your notes

- Only square root when forced to or for a final answer, and use the ANS button (and other memory features) on your calculator.

How do I find the gradient of a line segment?

- There are lots of ways to find the gradient, *m*, of a line segment
- Use the **two end points** to find the change in y divided by change in x

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

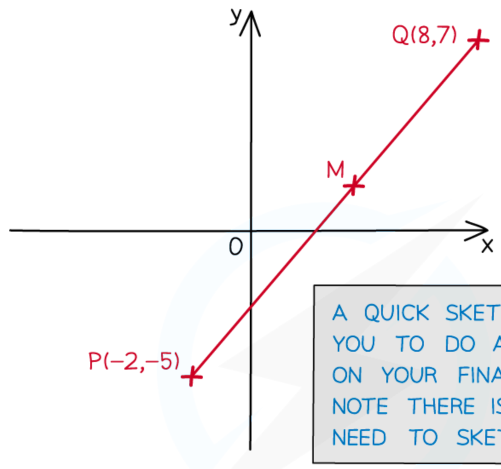


Your notes

Worked Example

Your notes

The line segment PQ is the diameter of a circle with coordinates P(-2, -5) and Q(8, 7).
Find the coordinates of the centre of the circle and the length of the diameter, giving your answer in the form $a\sqrt{b}$, where a and b are integers to be found.



A QUICK SKETCH WILL ALLOW YOU TO DO A VISUAL CHECK ON YOUR FINAL ANSWER.
NOTE THERE IS NO PARTICULAR NEED TO SKETCH THE CIRCLE.

CENTRE OF CIRCLE IS M $\left(\frac{-2+8}{2}, \frac{-5+7}{2}\right)$

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\therefore$ M (3,1)

LENGTH OF DIAMETER IS d, $d=\sqrt{(8-(-2))^2+(7-(-5))^2}$

$d=\sqrt{10^2+12^2}$

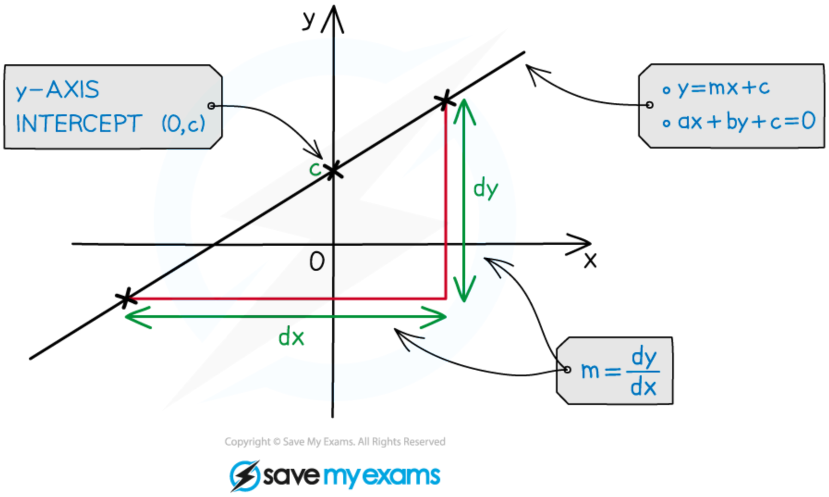
$d=2\sqrt{61}$

AFTER SOME PRACTICE AIM TO GO STRAIGHT TO THIS LINE:
"x HAS INCREASED BY 10"
"y HAS INCREASED BY 12"

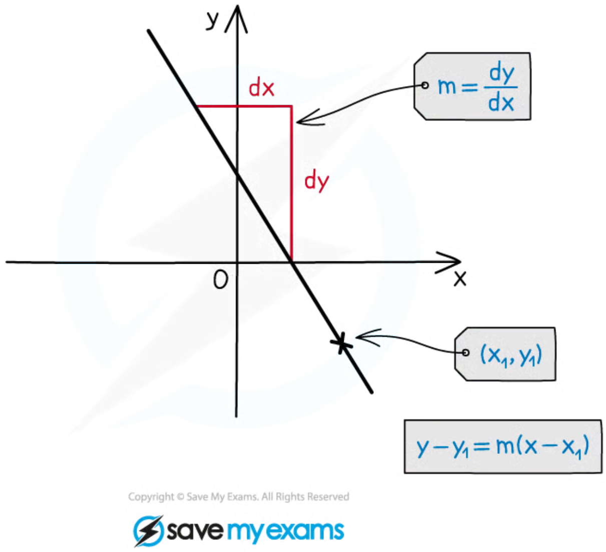
Equation of a straight line

What is the equation of a straight line?

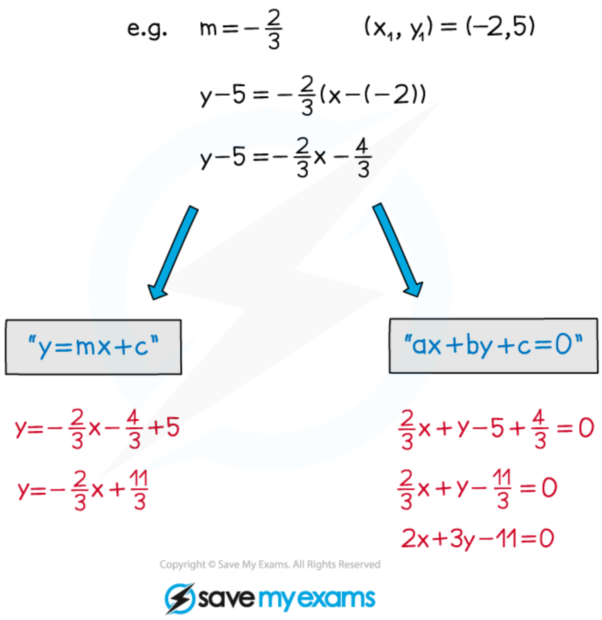
- **y = mx + c** is the equation for any straight line
- **m** is **gradient** given by "difference in **y**" ÷ "difference in **x**" or **dy/dx**
- **c** is the **y**-axis **intercept**
- Alternative form is **ax + by + c = 0** where **a**, **b** and **c** are integers



How do I find the equation of a straight line?



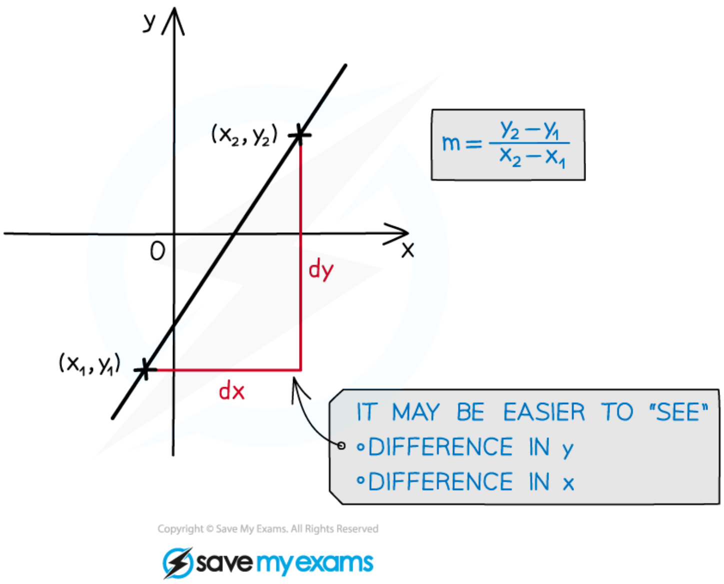
- **Two** features of a straight line are needed
  - gradient, **m**
  - a point the line passes through, $(\mathbf{x_1}, \mathbf{y_1})$
- The equation can then be found using $\mathbf{y - y_1 = m(x - x_1)}$
- This can be rearranged into either $\mathbf{y = mx + c}$ or $\mathbf{ax + by + c = 0}$



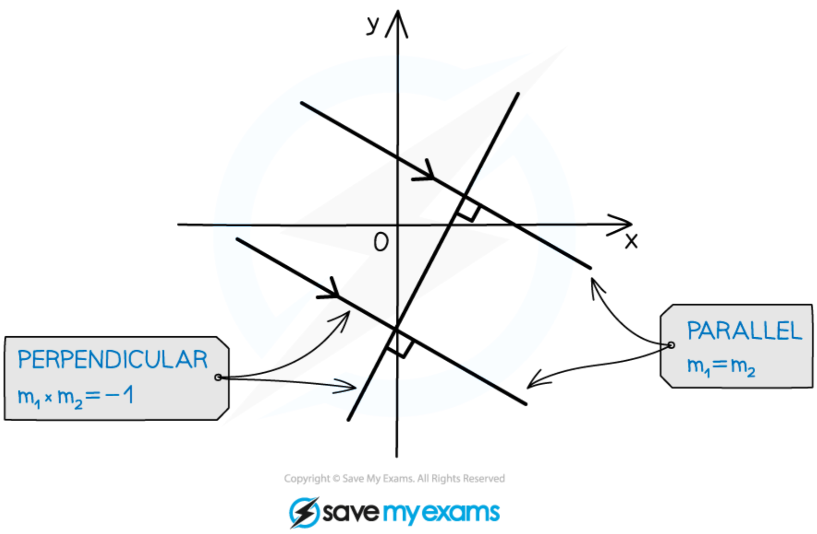
How do I find the gradient of a straight line?

- There are lots of ways to find the gradient of a line
- Using t**wo points** on a line to find the change in y divided by change in x

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

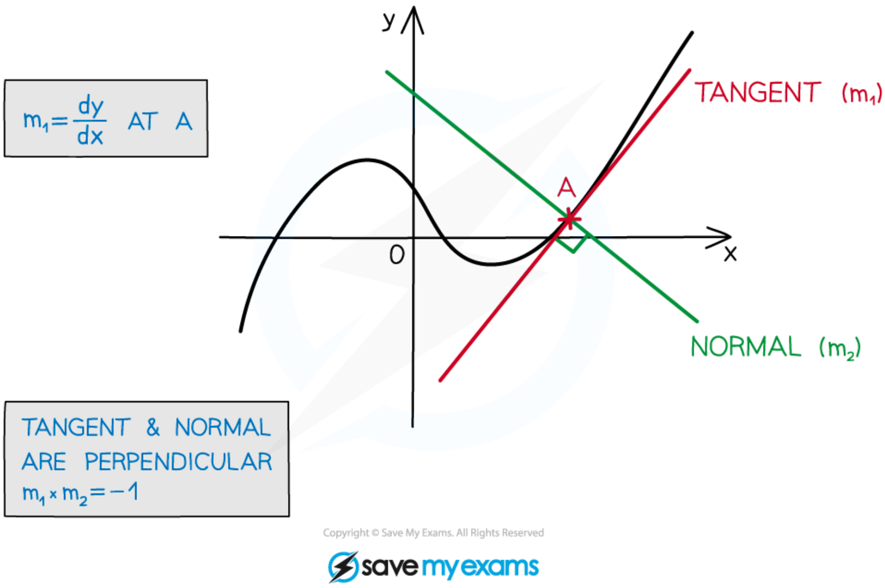


- Using the fact that lines are **parallel** or **perpendicular** to another line (see Parallel and Perpendicular Gradients)



Your notes

- Using **Tangents** and **Normals** - **Differentiation** (see Gradients, Tangents & Normals)



- Other ways
  - **Collinear** lines are the same straight line so gradients are equal
  - Angle facts and **circle theorems**

    eg. a **radius** and **tangent** are **perpendicular**

Examiner Tips and Tricks

- Working with straight lines can involve lots of algebra, but **sketching** a diagram will always help.
- Use a **sketch** to check if answers seem about right.

Worked Example

Your notes

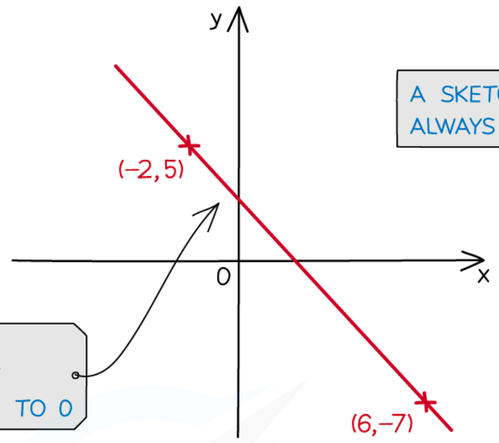
Your notes

The line l_1 passes through the points $(-2, 5)$ and $(6, -7)$.

Find the equation l_1, giving your answer in the form

$ax + by + c = 0$ where a, b and c are integers to be found.

A SKETCH WILL ALWAYS HELP



EXPECT:
- A NEGATIVE GRADIENT
- POSITIVE c BUT CLOSE TO 0

"$y - y_1 = m(x - x_1)$"

$(x_1, y_1) = (-2, 5)$

COULD'VE USED (6,−7) INSTEAD

$m = \frac{(-7)-5}{6-(-2)} = -\frac{12}{8} = -\frac{3}{2}$

COULD START HERE USING $\frac{dy}{dx}$

TWO POINTS TO FIND m

$y - 5 = -\frac{3}{2}(x - (-2))$

$y - 5 = -\frac{3}{2}x - 3$

$\frac{3}{2}x + y - 2 = 0$

SUBSTITUTE AND REARRANGE

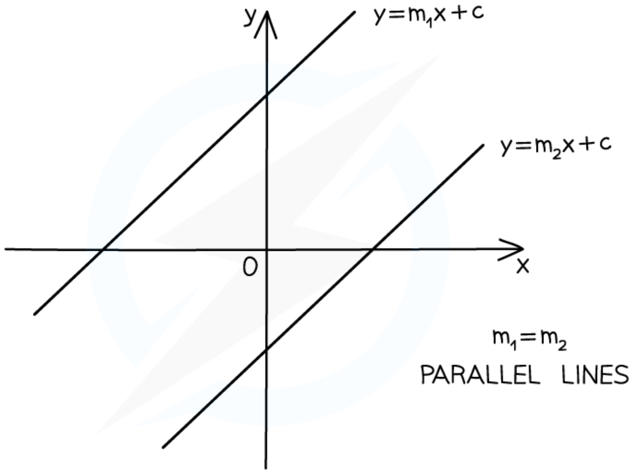
$3x + 2y - 4 = 0$

$a=3$, $b=2$, $c=-4$
INTEGERS

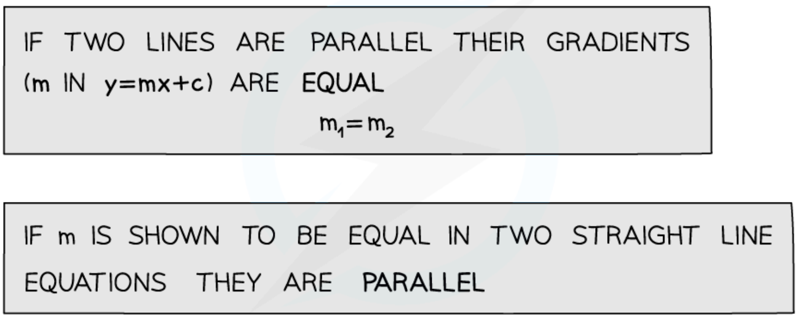
Your notes

Parallel & perpendicular gradients

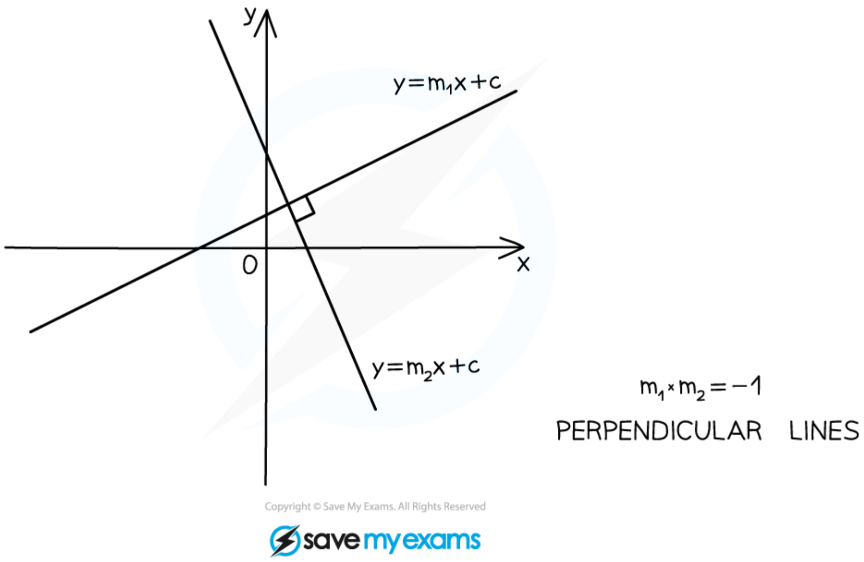
What are parallel lines?



- **Parallel** lines are equidistant meaning they never meet
- **Parallel** lines have **equal gradients**



What are perpendicular lines?



- **Perpendicular** lines meet at **right angles**
- The **product** of their **gradients** is -1

THE PRODUCT OF THE GRADIENTS OF **PERPENDICULAR** LINES IS -1
$m_1 \times m_2 = -1$

IF THE **PRODUCT** OF TWO GRADIENTS IS -1 THE GRAPHS ARE **PERPENDICULAR**

What does collinear mean?

- **Collinear** line segments are two line segments that are part of the **same** line
- They have **equal gradients**
  - They may **pass** through, or **meet** at, a common **point**
  - When they are extended, they become the same line
- **Rearrange** their equations into the same form and they will be identical

Your notes

e.g. DETERMINE IF THE LINES $3y+2x=4$ AND THE LINE PASSING THROUGH $(5,-2)$ AND $(-4,4)$ ARE COLLINEAR OR NOT

$y-y_1=m(x-x_1)$

$3y+2x=4$

$3y=-2x+4$

$y=-\frac{2}{3}x+\frac{4}{3}$

$m=\frac{y_2-y_1}{x_2-x_1}$

$y-(-2)=m(x-5)$

$m=\frac{4-(-2)}{-4-5}=\frac{6}{-9}$

$y+2=-\frac{2}{3}(x-5)$

$y=-\frac{2}{3}x+\frac{4}{3}$

REARRANGE BOTH TO $y=mx+c$

EQUATIONS ARE THE SAME WHEN REARRANGED SO THE TWO LINES ARE COLLINEAR

How do I tell if lines are parallel or perpendicular?

- Rearrange equations into the form **y = mx + c**
- **m** is the **gradient**

1. $2x-4y+3=0$

   $4y=2x+3$

   $y=\frac{1}{2}x+\frac{3}{4}$

2. $3x-2=6y$

   $y=\frac{3}{6}x-\frac{2}{6}$

   $y=\frac{1}{2}x-\frac{1}{3}$

3. $4x+2y=1$

   $2y=-4x+1$

   $y=-2x+\frac{1}{2}$

EASY TO COMPARE IN $y=mx+c$ FORM

GRADIENTS OF EQUATIONS 1 AND 2 ARE EQUAL SO LINES ARE PARALLEL.

$\frac{1}{2}\times -2=-1$ SO EQUATION 3 IS PERPENDICULAR TO THE OTHER TWO

Examiner Tips and Tricks

- Exam questions are good at “hiding” parallel and perpendicular lines.

- For example a **tangent** and a **radius** are **perpendicular.**
- **Parallel lines** could be implied by phrases like "… at the same rate …"

Your notes

Worked Example

Determine whether the following pairs of straight-line graphs are parallel, perpendicular or neither:

(a) $y = 3x + 2$ and $3y = 27 - x$

(b) $2x + 3y = 4$ and $4x - 5y = 10$

(c) $x + y = 2$ and $3x + 3y = 6$

a) $y=3x+2$

$3y=27-x$

REARRANGE, IF NECESSARY, BOTH EQUATIONS INTO THE FORM $y=mx+c$

$y=9-\frac{1}{3}x$

$y=-\frac{1}{3}x+9$

$m_1=3,\ m_2=-\frac{1}{3}$

$m_1 \times m_2=-1$

$\therefore$ LINES ARE PERPENDICULAR

b) $2x+3y=4$

$3y=-2x+4$

$y=-\frac{2}{3}x+\frac{4}{3}$

$m_1=-\frac{2}{3},\ m_2=\frac{4}{5}$

$4x-5y=10$

$5y=4x-10$

$y=\frac{4}{5}x-2$

$m_1 \neq m_2$

$m_1 \times m_2 \neq -1$

$\therefore$ LINES ARE NEITHER PARALLEL NOR PERPENDICULAR

THIS WILL MEAN THEY INTERSECT

c) $x+y=2$

$y=-x+2$

$m_1=-1,\ m_2=-1,$

$3x+3y=6$

$3y=-3x+6$

$y=-x+2$

YOU MAY SPOT THE FACTOR OF 3 BEFORE REARRANGING

$m_1=m_2$

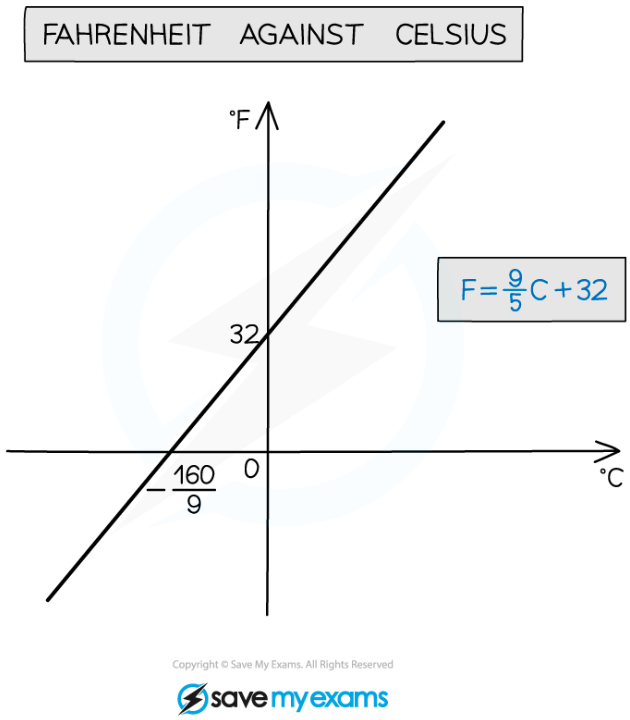
$\therefore$ LINES ARE PARALLEL

THESE "LINES" ARE COLLINEAR — THE SAME STRAIGHT LINE

Modelling with straight lines

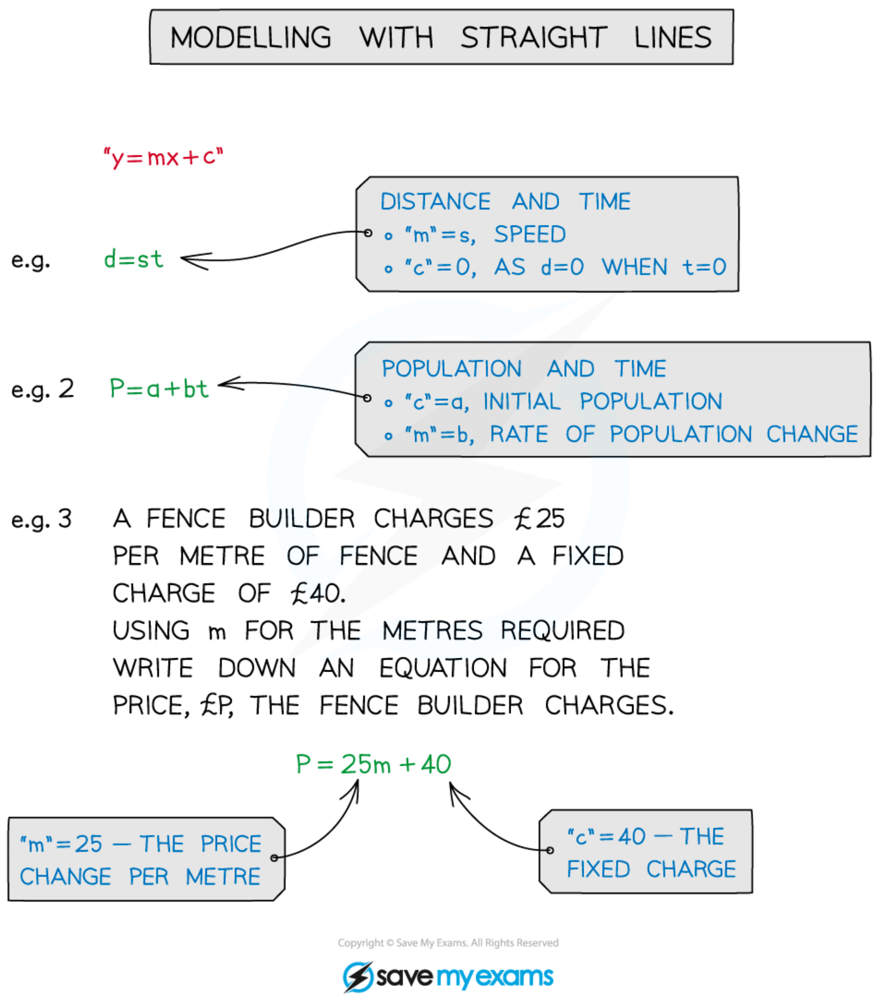
What is modelling with straight lines?

- **Modelling** means applying the mathematics to a **real-life** situation
- The model is used to **find** or **predict** future values
- Models will often need **refining** to improve their accuracy



How do I work with straight line models?

- Turn the words from the question into mathematical equations
- In **y = mx + c** constants "**m**" and "**c**" will have a meaning
  - gradient "**m**" will be the **rate of change**
  - eg extension in a spring per 1 kg of mass added
  - **y**-axis intercept "**c**" will be the **initial value**
  - eg resting length of a spring



- The skills needed are covered in the other pages of the section Equation of a Straight Line

Worked Example

Your notes

An endangered animal is introduced to a new area with an initial population of 20. Scientists use the linear model $P = a + bt$ to predict the future population of the animal where P is the population and t is the time in years since the animal was first introduced to the area.

(a) Write down the value of the constant a.

(b) After 5 years scientists observe the population has tripled since the animal was first introduced to the area. Work out the value of b.

(c) Sketch the graph of population for the first 10 years

(d) Suggest one criticism of this model

$P=a+bt$ — OF THE FORM "$y=mx+c$"

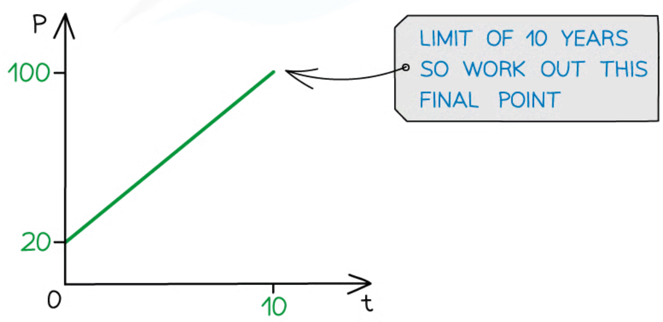
a) $a=20$ — a IS THE INITIAL POPULATION, WHEN $t=0$

b) AT $t=5$, $P=3\times20=60$

$60=20+5b$ — $a=20$ FROM a), $P=60$ WHEN $t=5$

$b=8$

c)



d) POPULATION INCREASES INDEFINITELY

POPULATION GROWTH IS USUALLY MODELLED EXPONENTIALLY — SLOW AT FIRST THEN MUCH QUICKER