Edexcel International A Level (IAL) Maths: Pure 1

Your notes

Transformations of Functions

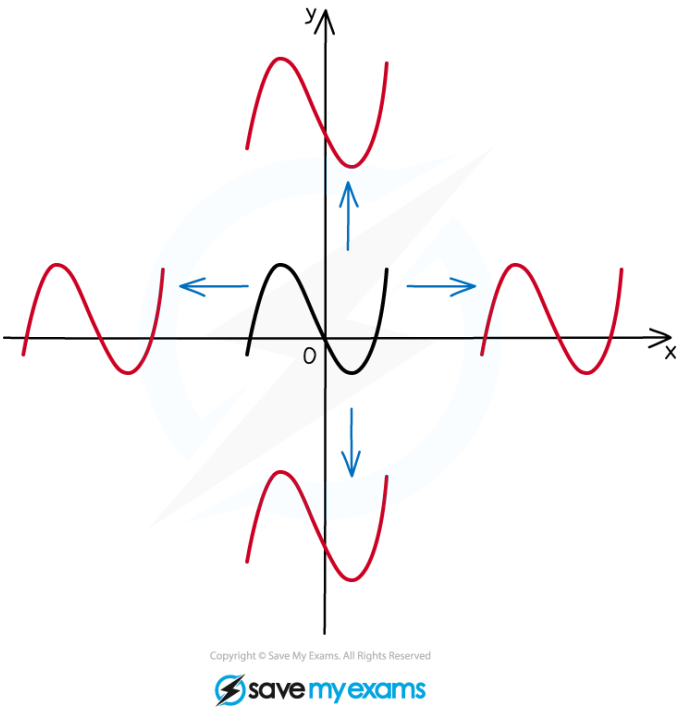
Contents

Your notes

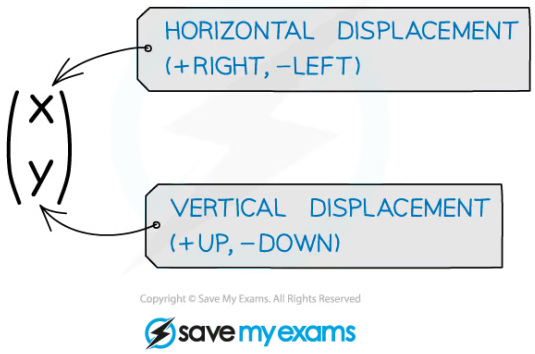
Translations

What are graph translations?

- When you alter a function in certain ways, the effects on the graph of the function can be described by geometrical transformations
- With a **translation** the shape, size, and orientation of the graph remain unchanged – the graph is merely shifted (up or down, left or right) in the *xy* plane



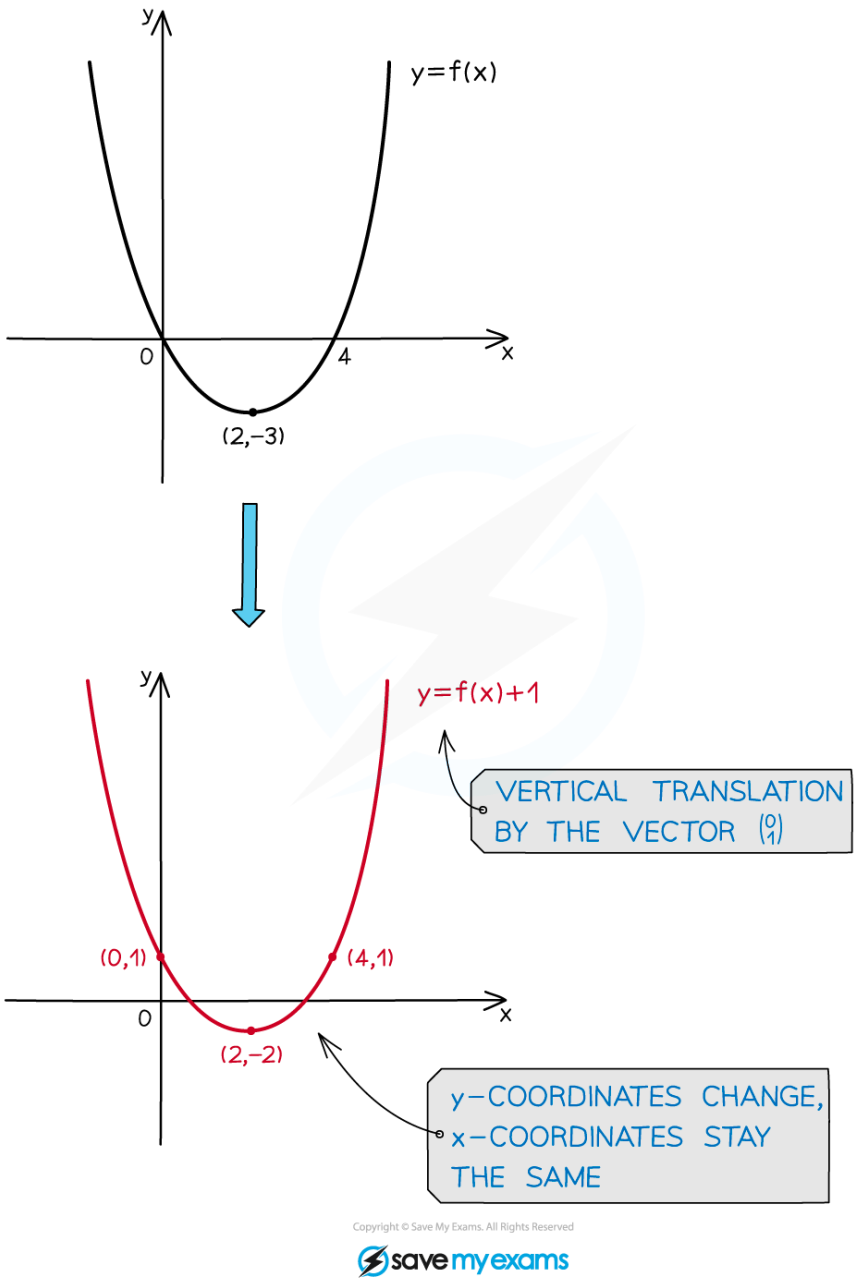
- A particular translation (how far left/right, how far up/down) is specified by a **translation vector**:



Your notes

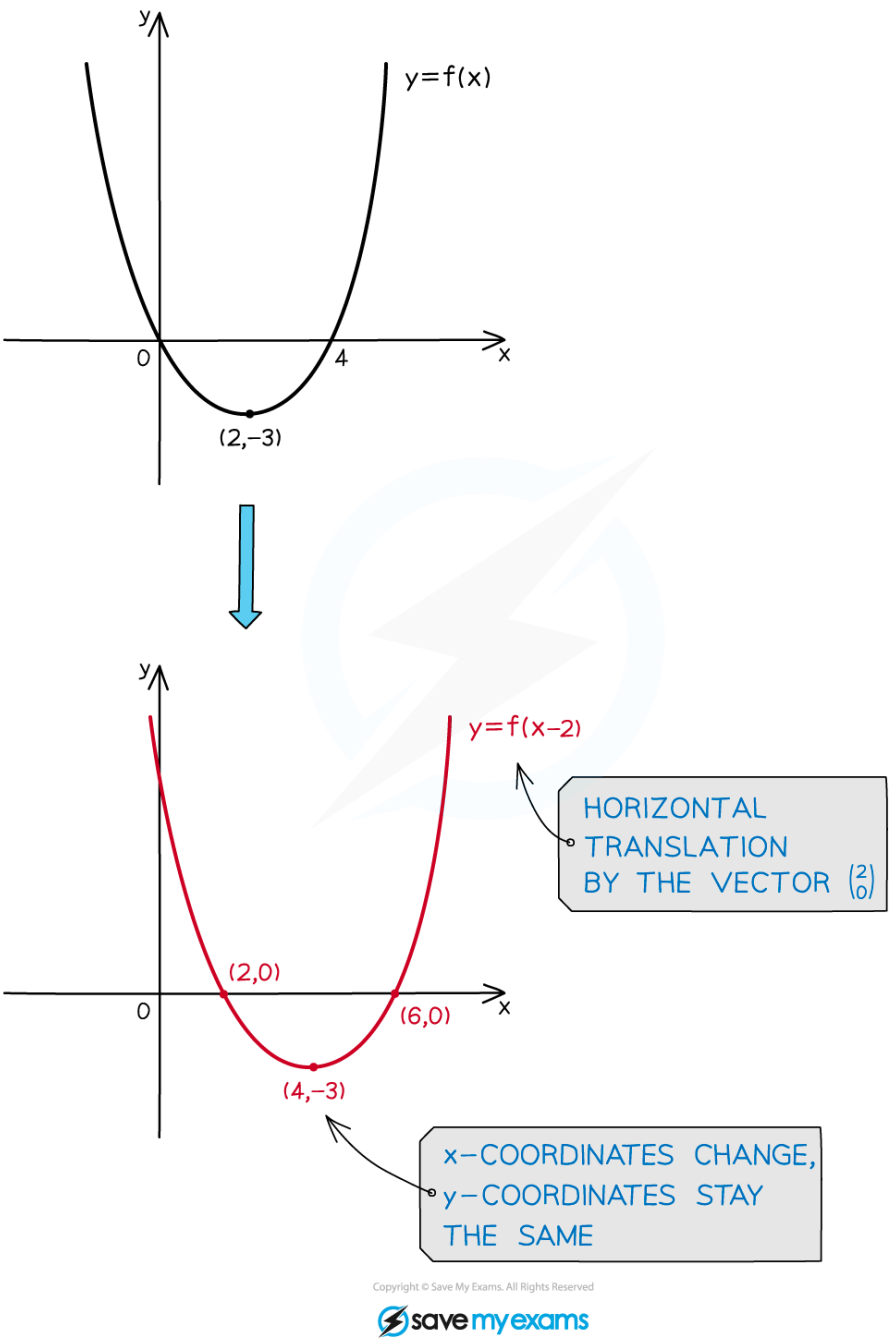
What do I need to know about graph translations?

- The graph of $y = f(x) + a$ is a **vertical** translation of the graph $y = f(x)$ by the vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$
  - The graph moves **up for positive** values of *a* and **down for negative** values of *a*
  - The *x*-coordinates stay the same



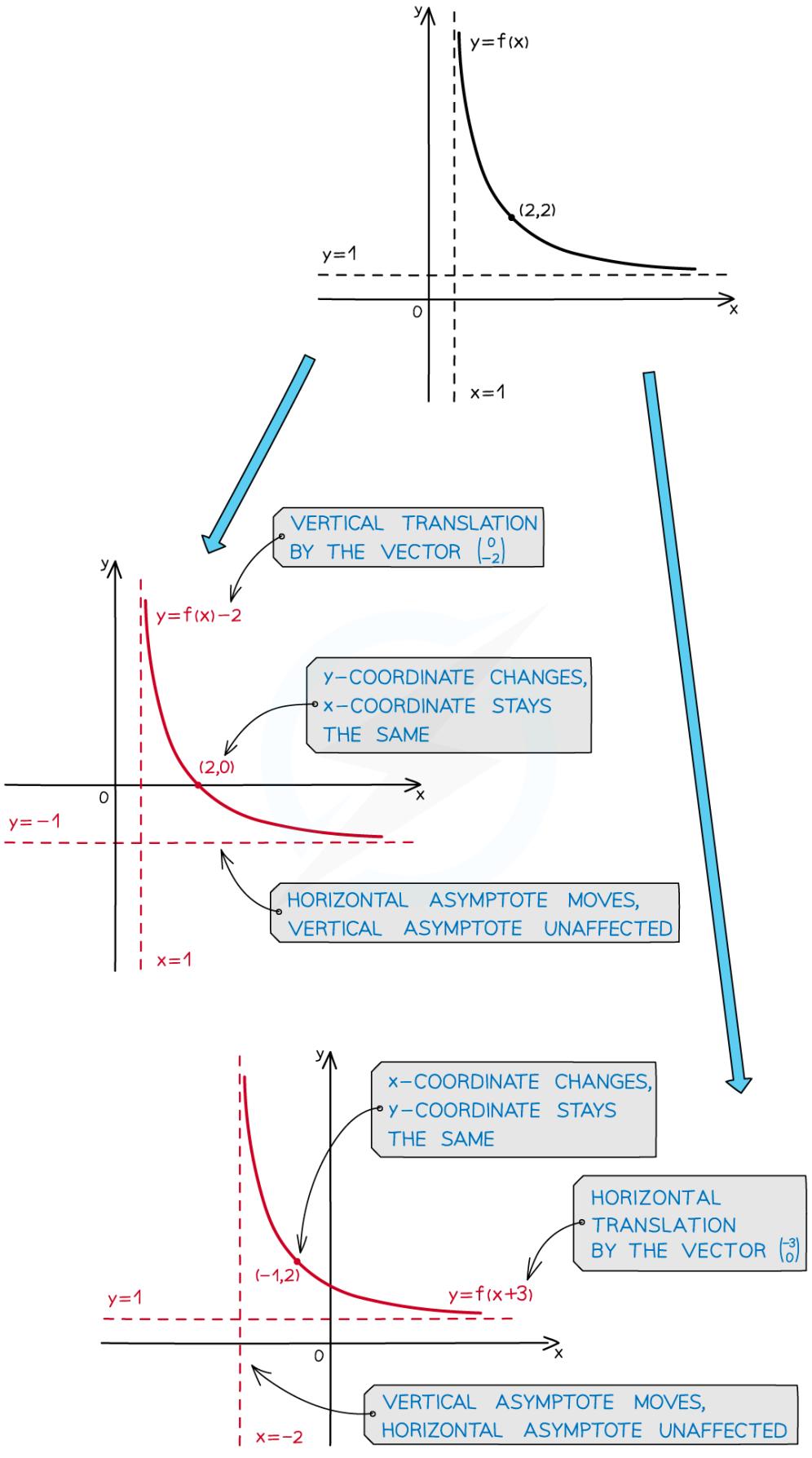
Your notes

- The graph of $y = f(x + a)$ is a **horizontal** translation of the graph $y = f(x)$ by the vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$
  - The graph moves **left for positive** values of *a* and **right for negative** values of *a*
  - The *y*-coordinates stay the same



- Any asymptotes of **f(x)** are also translated. If an asymptote is parallel to the direction of translation, however, it will not be affected

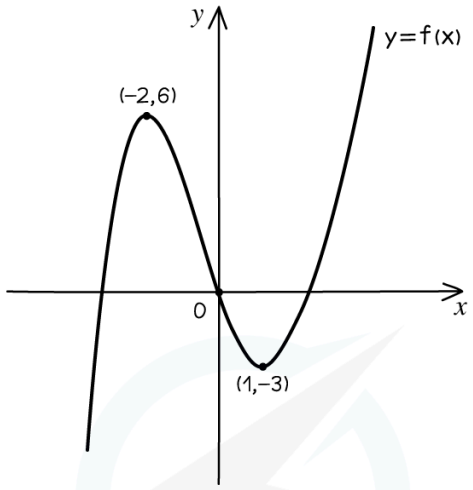
Your notes



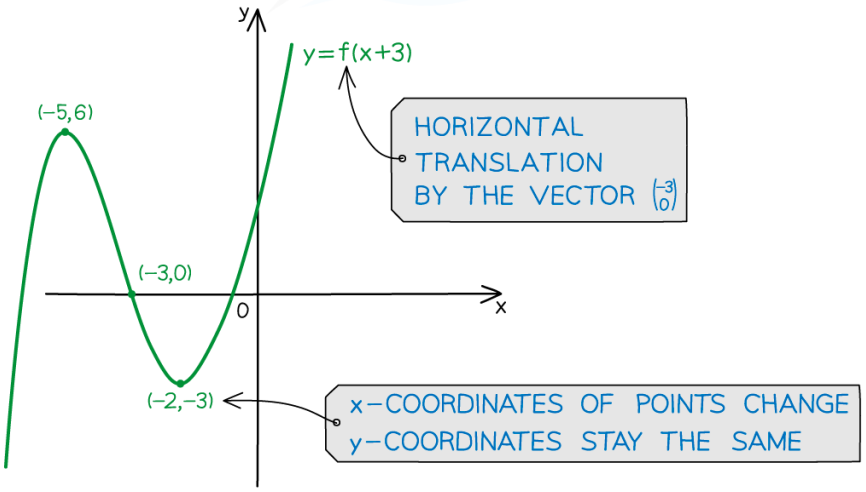
Your notes

Worked Example

The diagram below shows the graph of $y = f(x)$.



Sketch the graph of $y = f(x + 3)$.

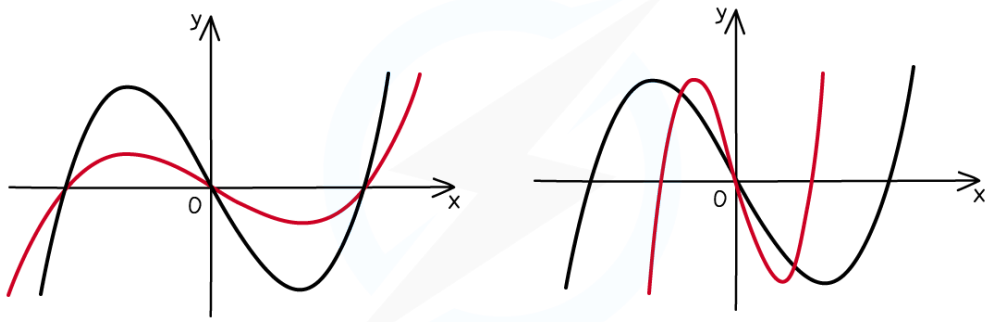


Your notes

Stretches

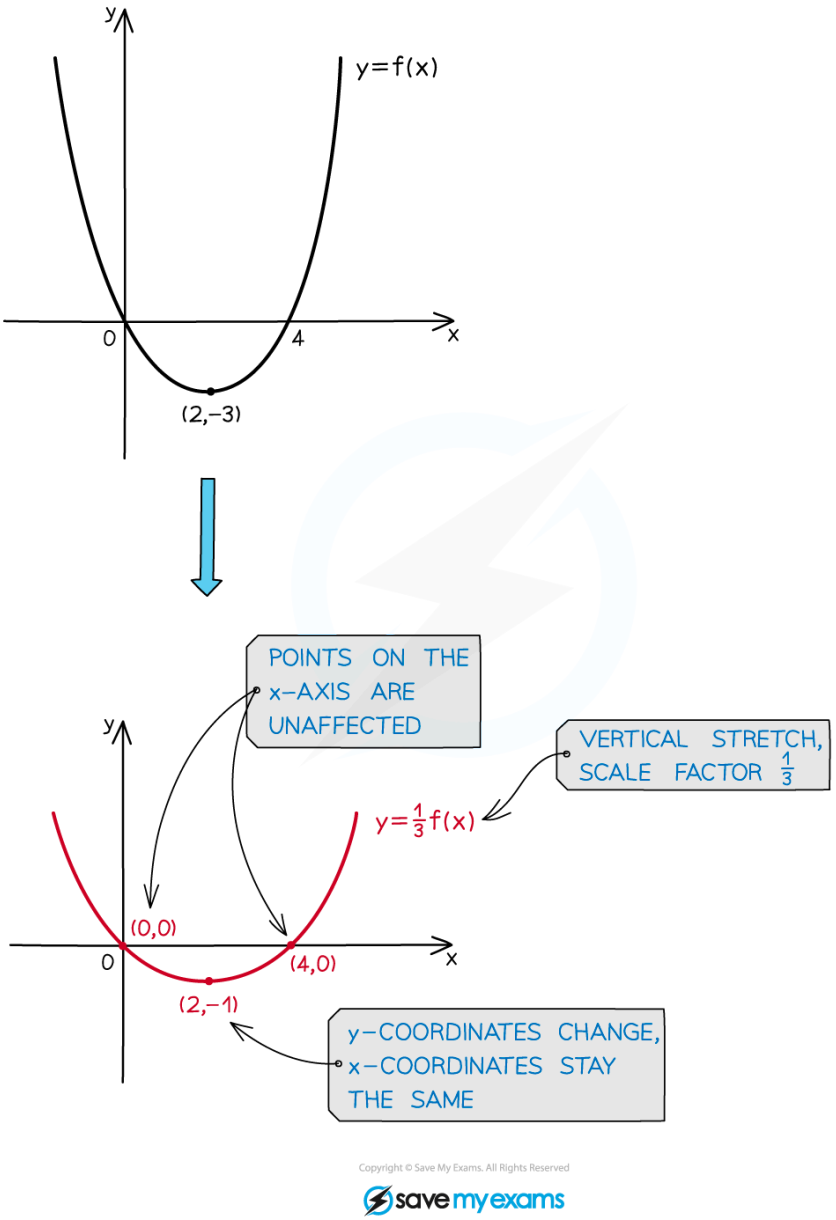
What are graph stretches?

- When you alter a function in certain ways, the effects on the graph of the function can be described by geometrical transformations
- With a **stretch** all the points on the graph are moved towards or away from either the ***x*** or the ***y*** axis by a constant scale factor

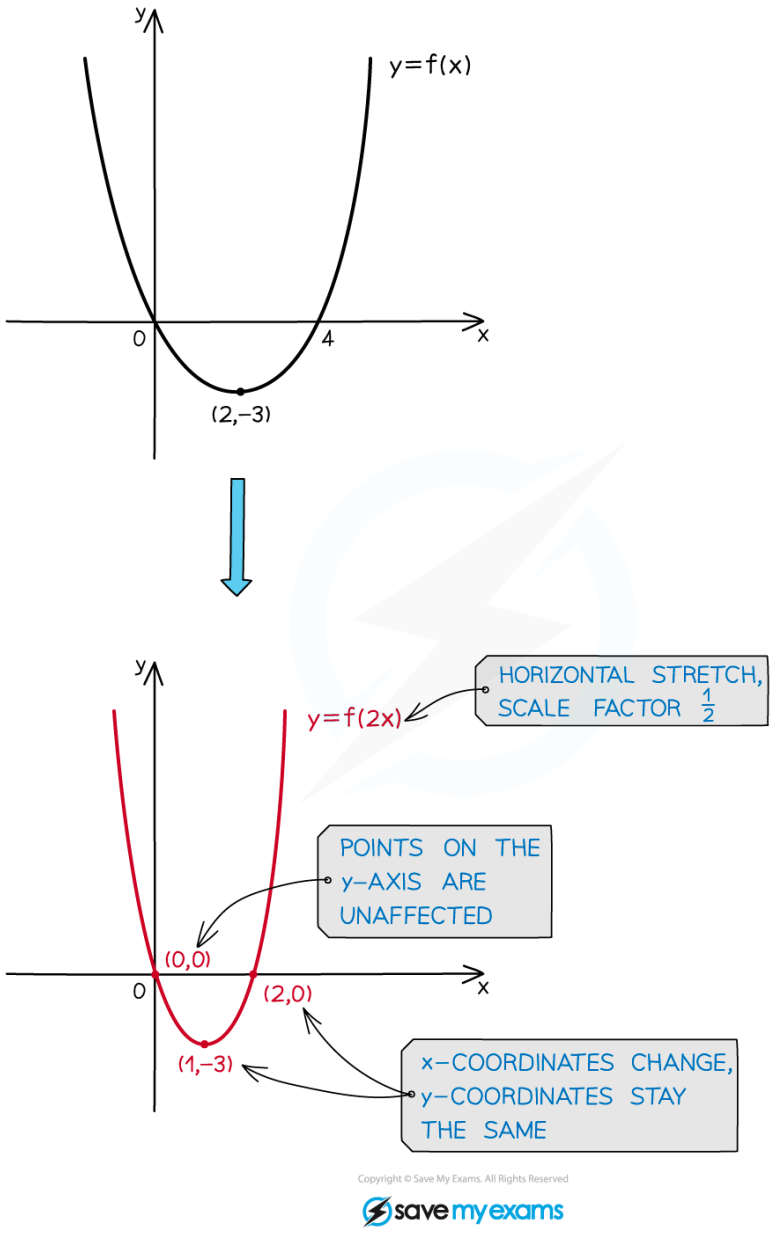


What do I need to know about graph stretches?

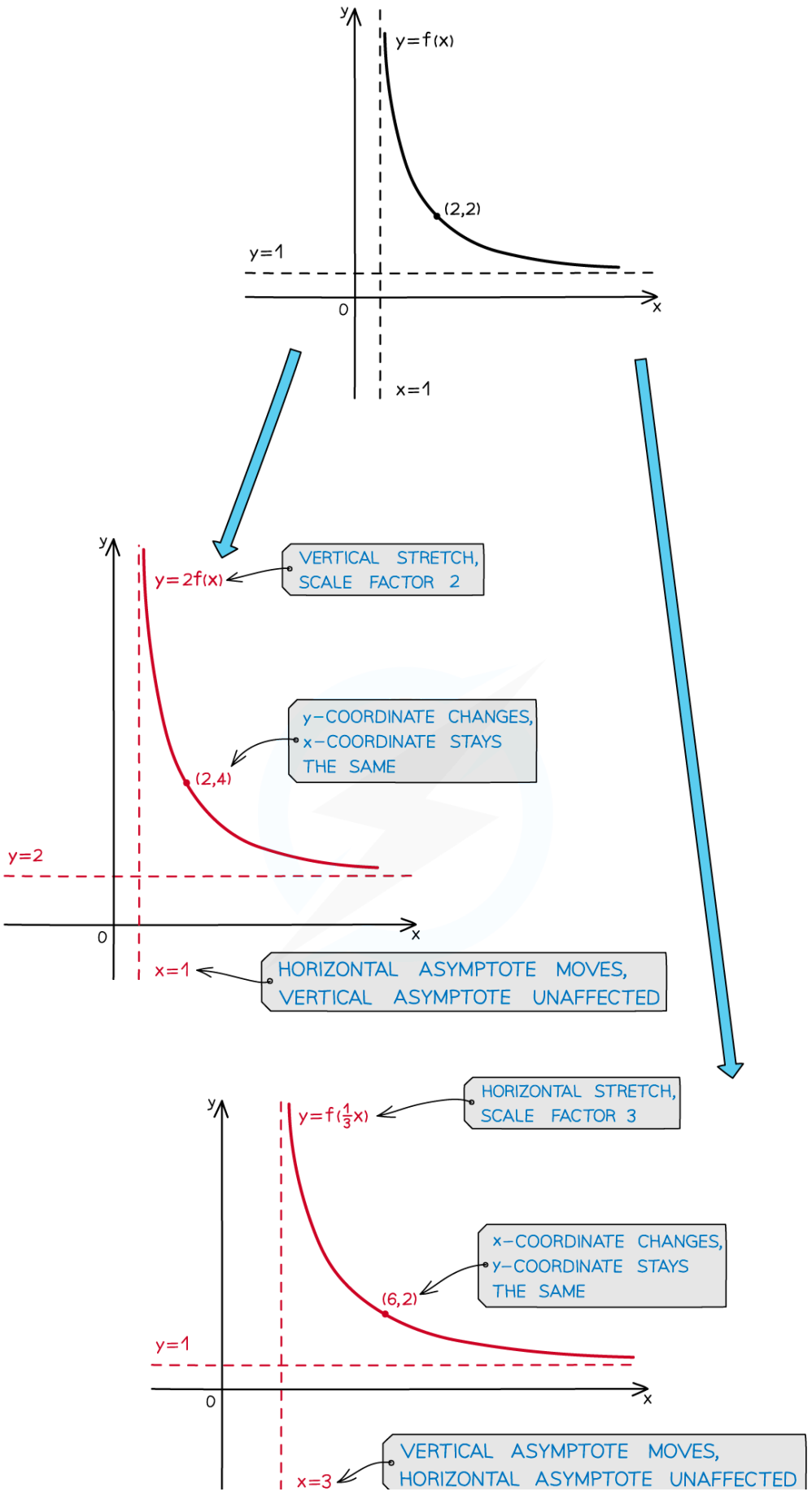
- The graph of $y = af(x)$ is a **vertical** stretch of the graph $y = f(x)$ by a scale factor of a, centred on the x axis
  - The x coordinates of points stay the same; y coordinates are multiplied by a
  - Points **on the *x* axis** stay where they are
  - All other points move **parallel to the *y* axis**, away from ($a > 1$) or towards ($0 < a < 1$) the x axis



- The graph of ***y* = f(*ax*)** is a **horizontal** stretch of the graph ***y* = f(*x*)** by a scale factor of $\frac{1}{a}$, centred on the ***y*** axis
  - The ***y*** coordinates of points stay the same; ***x*** coordinates are multiplied by $\frac{1}{a}$
  - Points **on the *y* axis** stay where they are
  - All other points move **parallel to the *x* axis**, away from (**0 < *a* < 1**) or towards (***a* > 1**) the *y* axis



- Any asymptotes of **f(x)** are also affected by the stretch (stretch them as you would stretch the function of a straight line)
- If an asymptote is one of the coordinate axes, or is parallel to the direction of the stretch, however, it will not be affected



Your notes

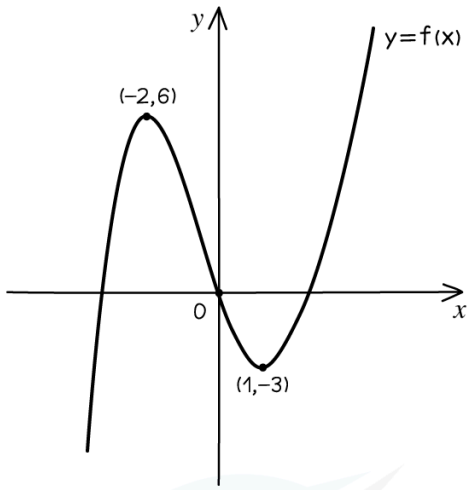
Examiner Tips and Tricks

- When you sketch a stretched graph, be sure to indicate the new coordinates of any points that are marked on the original graph.
- Try to indicate the coordinates of points where the stretched graph intersects the coordinate axes (if you don't have the equation of the original function this may not be possible).
- If the graph has asymptotes, don't forget to sketch the asymptotes of the stretched graph as well.

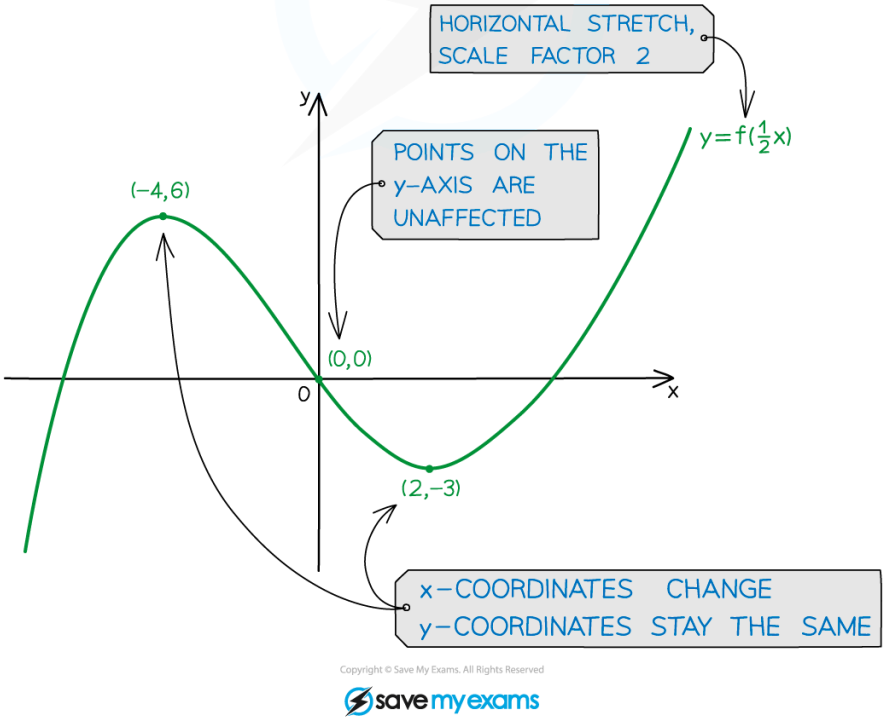
Worked Example

Your notes

The diagram below shows the graph of $y = f(x)$.



Sketch the graph of $y = f(\frac{1}{2}x)$.

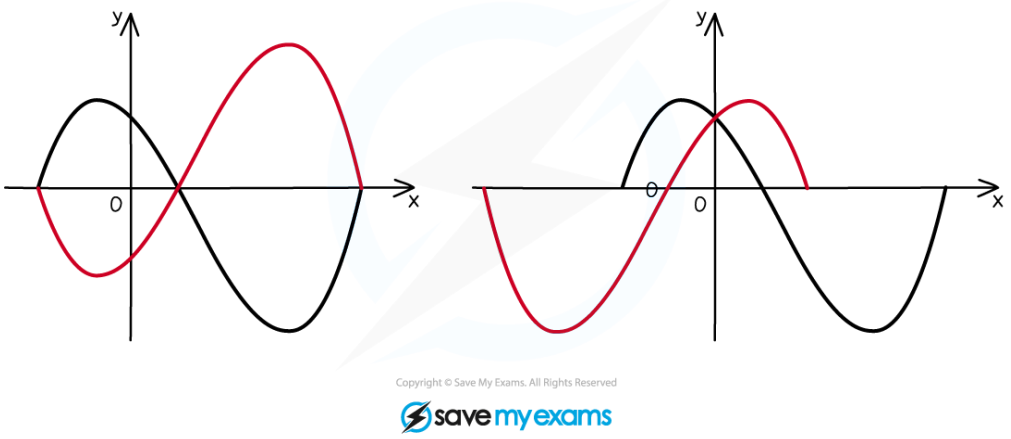


Your notes

Reflections

What are graph reflections?

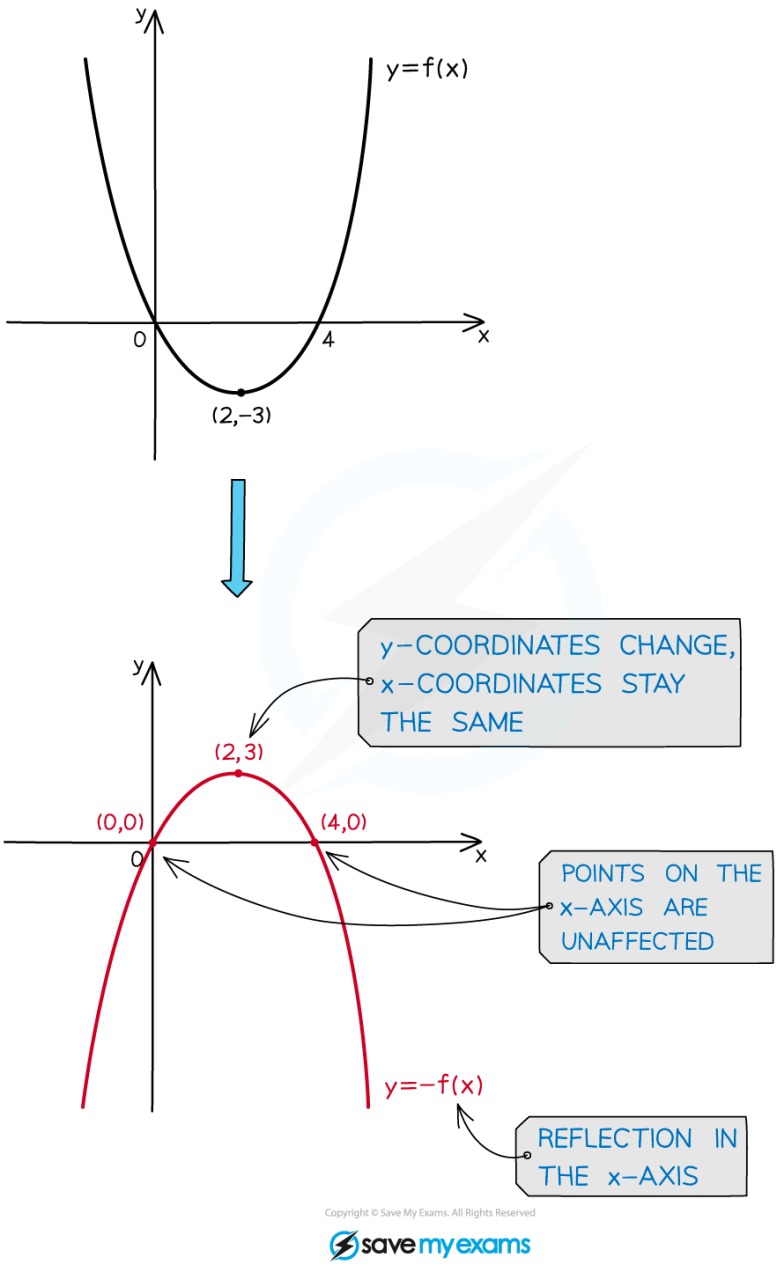
- When you alter a function in certain ways, the effects on the graph of the function can be described by geometrical transformations
- With a **reflection** all the points on the graph are reflected in either the ***x*** or the ***y* axis**



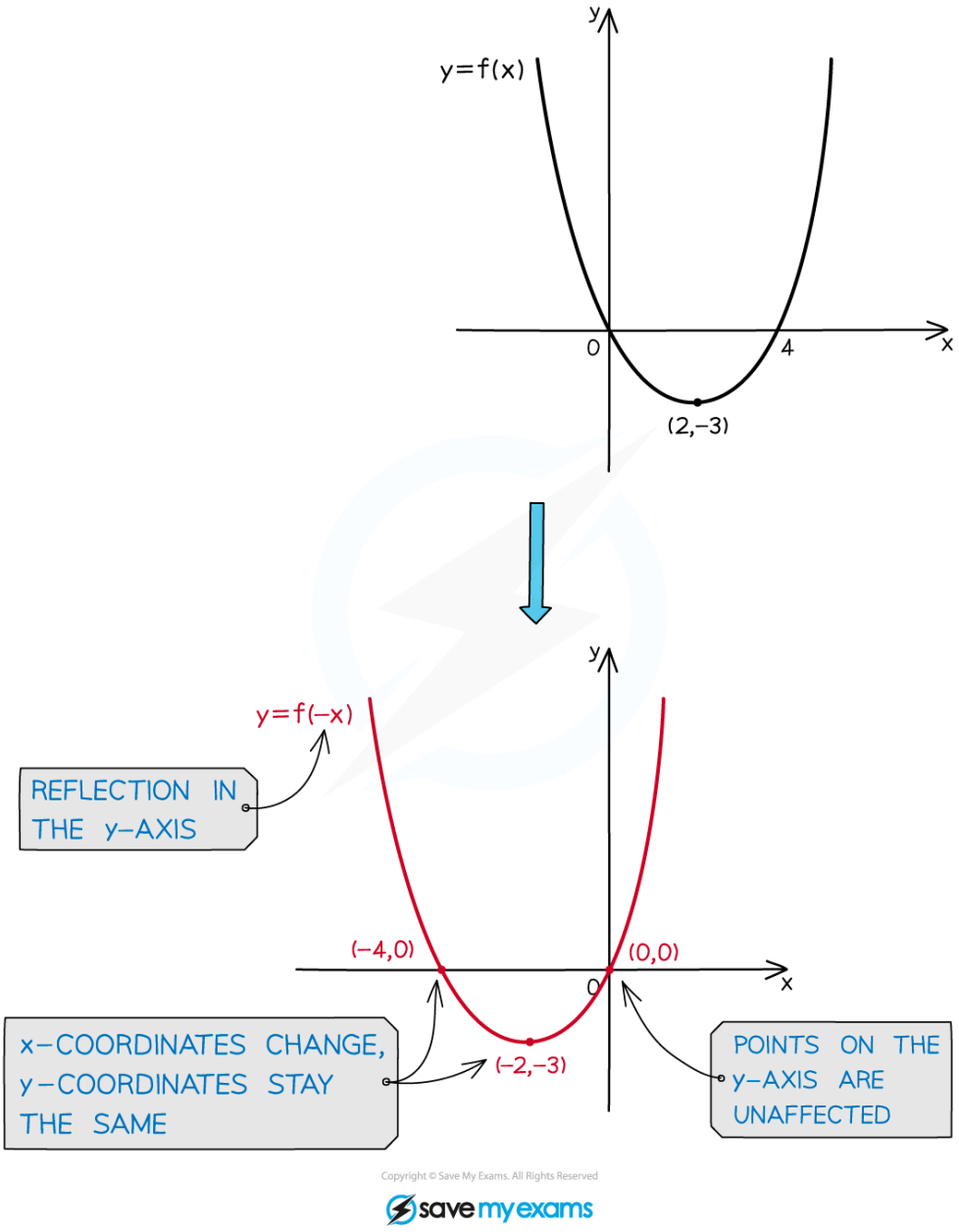
- Any asymptotes of **f(*x*)** are also affected by the reflection (reflect them as you would reflect the function of a straight line)
- If an asymptote is one of the coordinate axes, or is perpendicular to the coordinate axis in which the graph is reflected, it will not be affected

What do I need to know about graph reflections?

- The graph of ***y* = −f(*x*)** is a reflection in the *x* axis
  - The ***x*** coordinates of points stay the same; ***y*** coordinates have their signs flipped (positive to negative, negative to positive)
  - Points **on the *x* axis** stay where they are
  - All other points are reflected to the other side of the ***x*** axis



- The graph of ***y* = f(−*x*)** is a reflection in the *y* axis
  - The ***y*** coordinates of points stay the same; ***x*** coordinates have their signs flipped (positive to negative, negative to positive)
  - Points **on the *y* axis** stay where they are
  - All other points are reflected to the other side of the ***y*** axis



- Any asymptotes of **f(x)** are also affected by the reflection (reflect them as you would reflect the function of a straight line)
- If an asymptote is one of the coordinate axes, or is perpendicular to the coordinate axis in which the graph is reflected, it will not be affected

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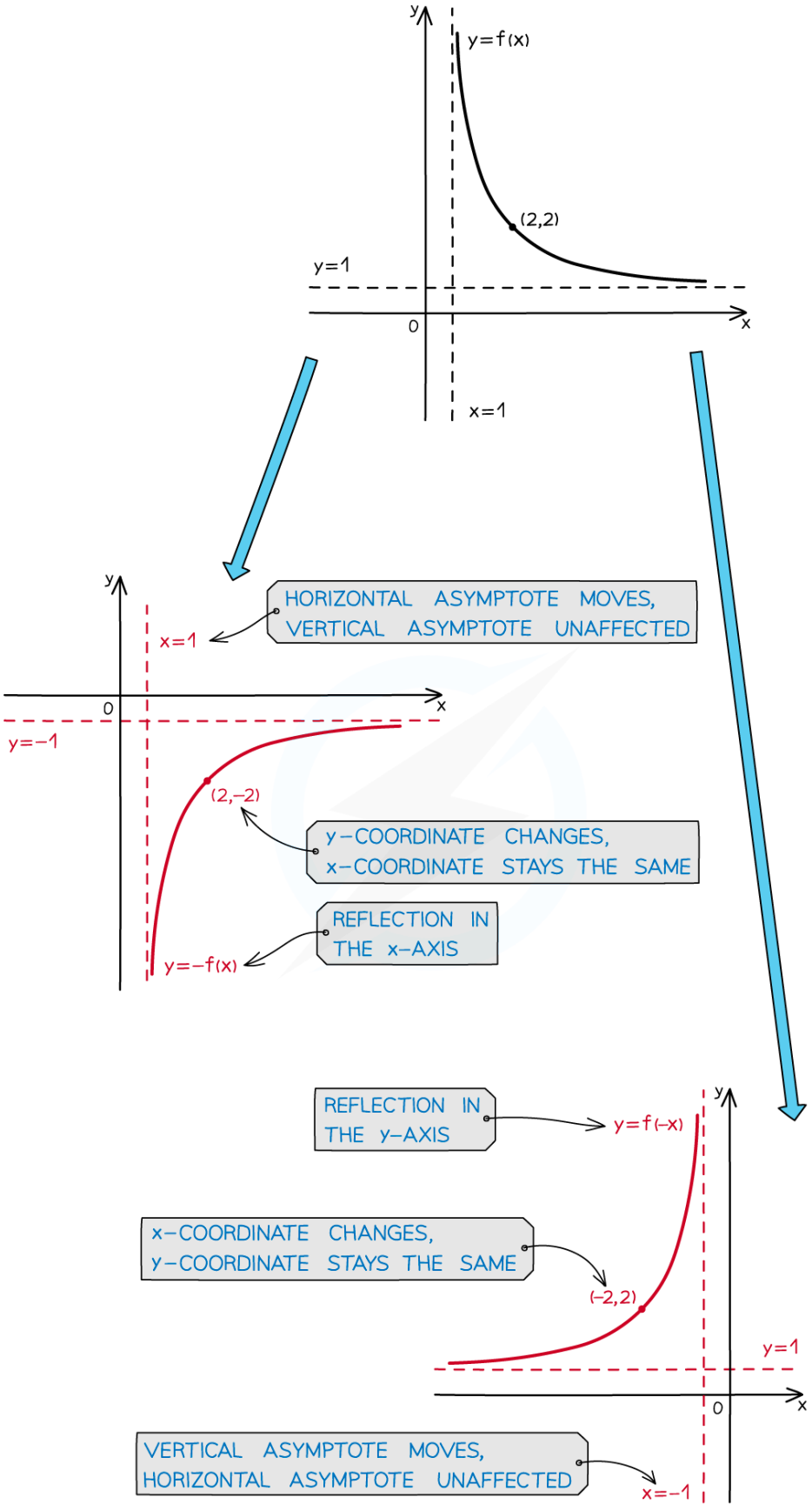

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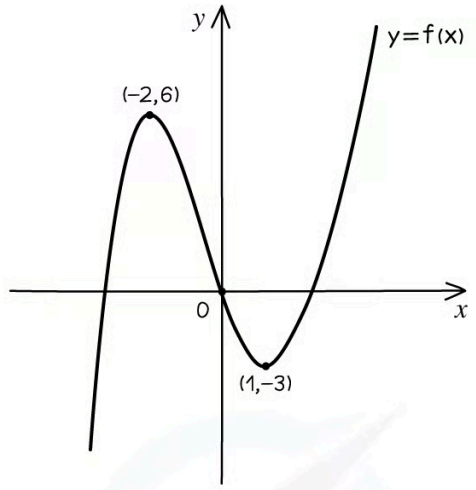
Worked Example

The diagram below shows the graph of $y = f(x)$.



Sketch the graph of $y = -f(x)$.