Edexcel International A Level (IAL) Maths: Pure 1

Your notes

Graphs of Functions

Contents

- Sketching Polynomials
- Sketching Reciprocal Graphs
- Solving Equations Graphically
- Proportional Relationships
- Modelling with Functions

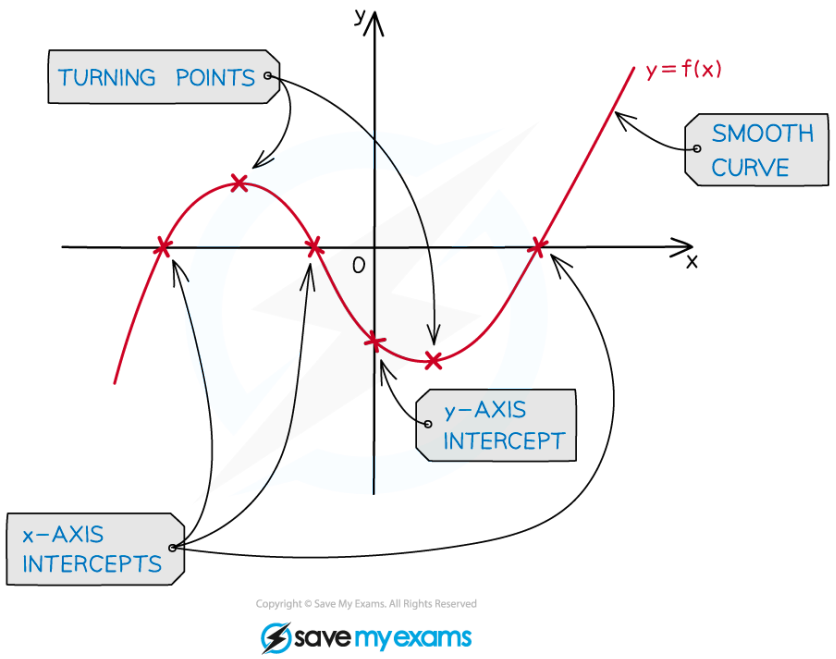
© 2026 Save My Exams, Ltd.

Your notes

Sketching polynomials

How do I sketch the graph of a polynomial?

- A polynomial is any finite function with non-negative indices, that could mean a quadratic, cubic, quartic or higher power



- When asked to sketch a polynomial you'll need to think about the following
  - **y**-axis intercept
  - **x**-axis intercepts (**roots**)
  - turning points (**maximum** and/or **minimum**)
  - a smooth **curve** (this takes practice!)

What steps should I take when sketching a polynomial?

STEP 1 Find the **y**-axis intercept by setting **x = 0**

STEP 2 Find the **x**-axis intercepts (roots) by setting **y = 0**

STEP 3 Consider the shape and "start"/"end" of the graph

© 2026 Save My Exams, Ltd.

eg. a **positive cubic** graph starts in third quadrant ("bottom left") and "ends" in first quadrant ("top right")

STEP 4 Consider where any turning points should go

STEP 5 Draw with a smooth curve

Your notes

© 2026 Save My Exams, Ltd.

$y = x^3 - 4x$

STEP 1: SET x=0 TO FIND y-AXIS INTERCEPT

AT x=0, y=0 (0,0)

STEP 2: SET y=0 TO FIND x-AXIS INTERCEPTS (ROOTS)

AT y=0, $x^3 - 4x = 0$

$x(x^2 - 4) = 0$ ← DIFFERENCE OF TWO SQUARES

$x(x+2)(x-2) = 0$

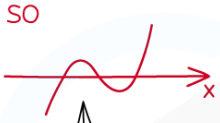
$x = 0, \ x = -2, \ x = 2$

(0,0) (−2,0) (2,0) ← ALSO THE y-AXIS INTERCEPT

STEP 3: CONSIDER SHAPE AND "START/END" OF GRAPH

$y = x^3 - 4x$ POSITIVE CUBIC

SO

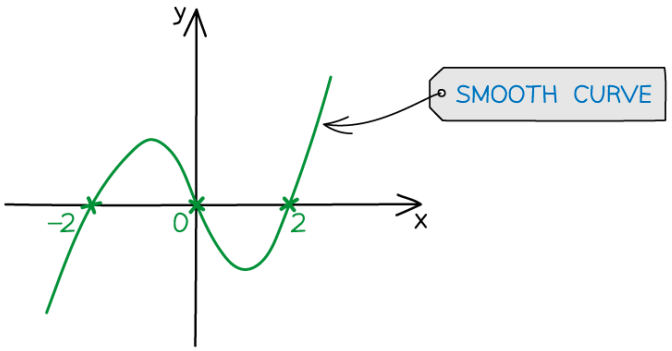


OR

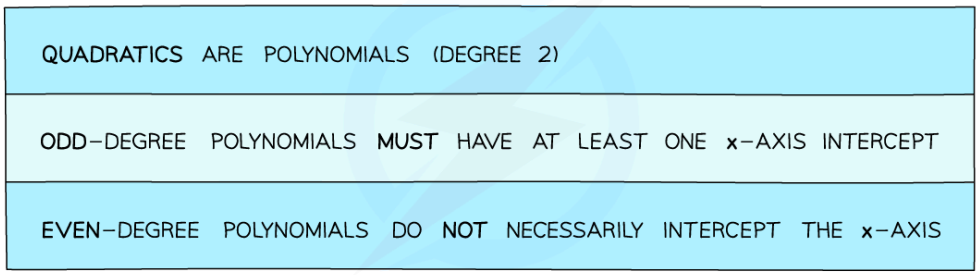
ONLY THIS ONE WILL CROSS x-AXIS THREE TIMES

STEP 4: TURNING POINTS

- ONE MINIMUM
- ONE MAXIMUM



Copyright © Save My Exams. All Rights Reserved
savemyexams



Copyright © Save My Exams. All Rights Reserved
save my exams

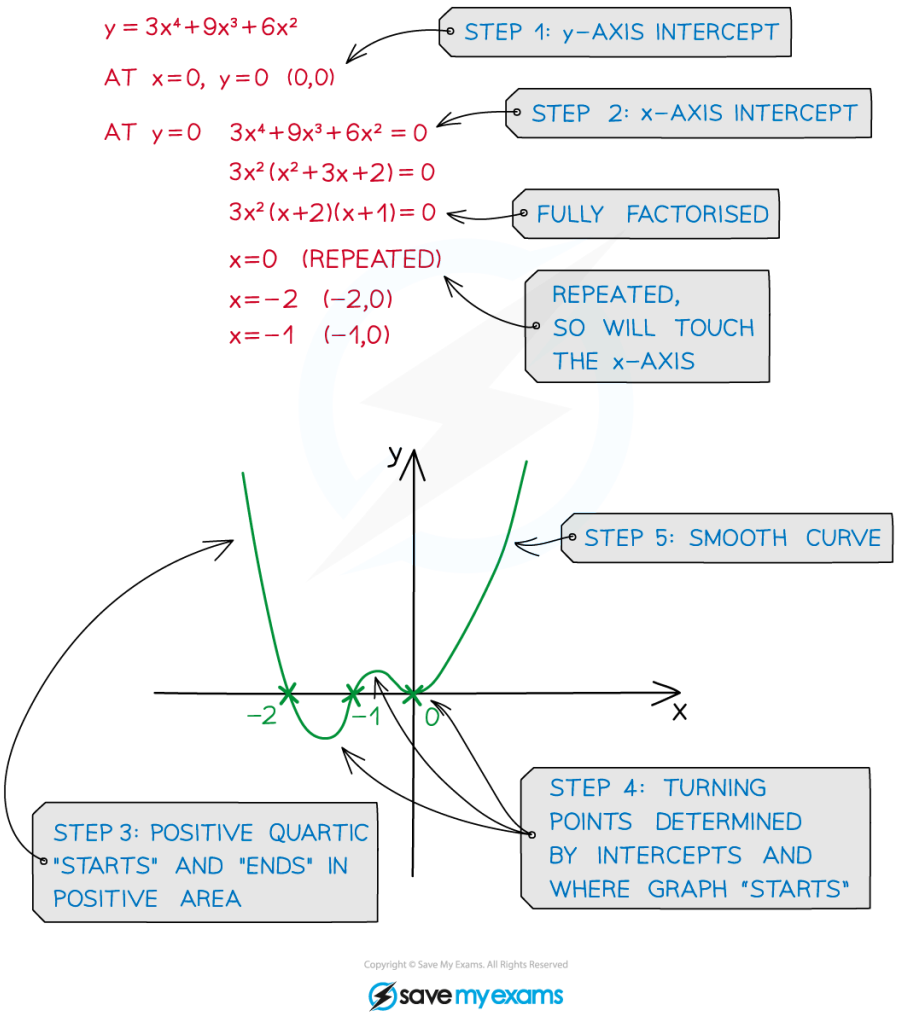
Your notes

- Coordinates of turning points can be found using **differentiation**
- Except with a point of inflection, repeated roots indicate the graph touches the **x**-axis

Worked Example

Your notes

Sketch the graph of $y = 3x^4 + 9x^3 + 6x^2$



Copyright © Save My Exams. All Rights Reserved
savemyexams

Your notes

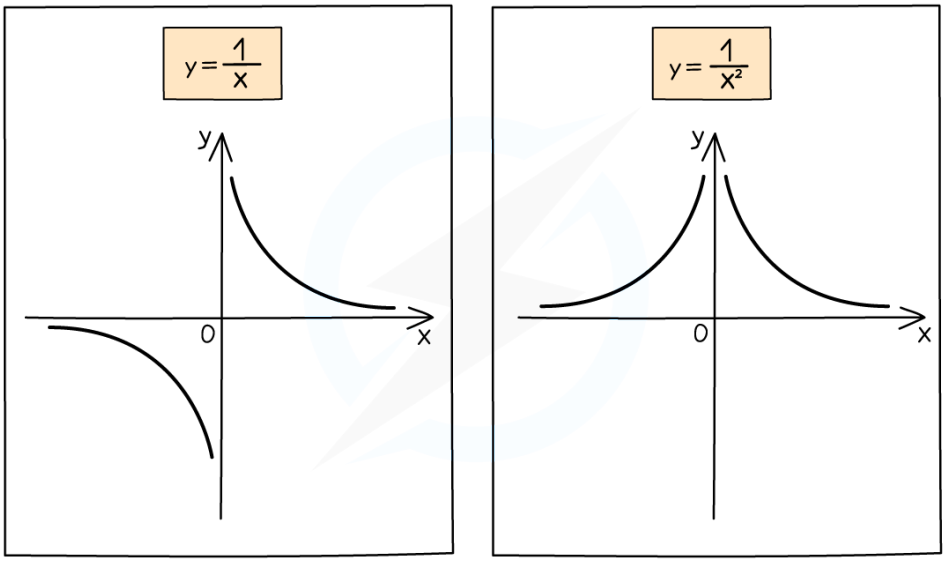
Sketching reciprocal curves

What are reciprocal graphs?

- Reciprocal graphs involve equations with an x term on the denominator e.g. $\frac{1}{x}$
- There are two basic reciprocal graphs to know for A level

$$y = \frac{1}{x} \text{ and } y = \frac{1}{x^2}$$

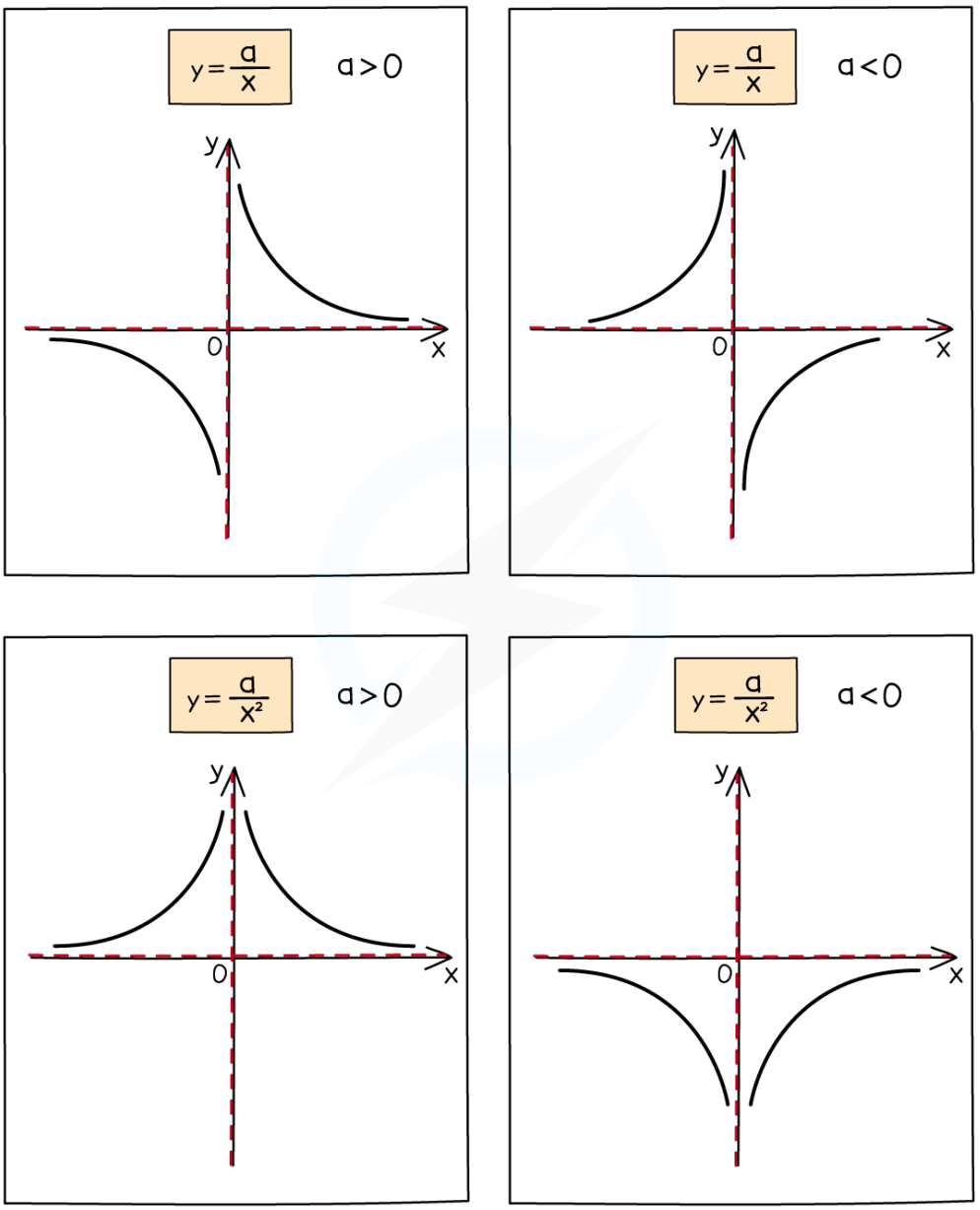
- The second one of these is always positive



Copyright © Save My Exams. All Rights Reserved
save my exams

More reciprocal graphs

- You also need to recognise graphs where the numerator is not one

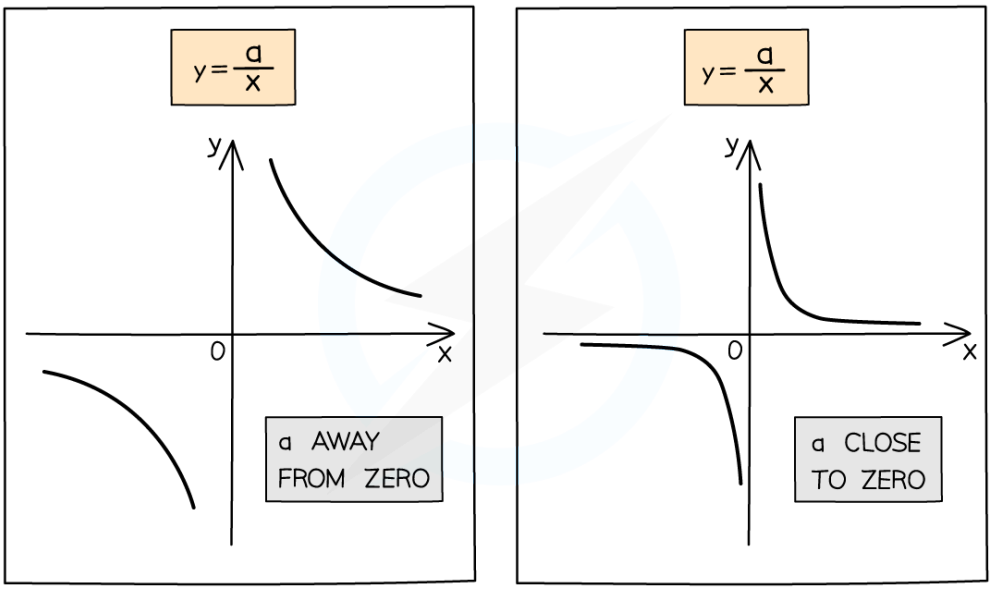


Copyright © Save My Exams. All Rights Reserved

- The **sign** of **a** shows which part of the graph the curves are located

The **size** of **a** shows how steep the curves are

- The closer **a** is to 0 the more **L-shaped** the curves are



Copyright © Save My Exams. All Rights Reserved
save my exams

Your notes

- All have two asymptotes
  - horizontal, **y = 0** (**x**-axis)
  - vertical, **x = 0** (**y**-axis)

How do I sketch a reciprocal graph?

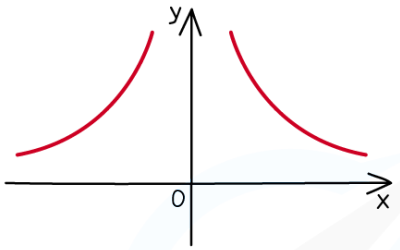
Your notes

e.g. $y = \frac{6}{x^2}$

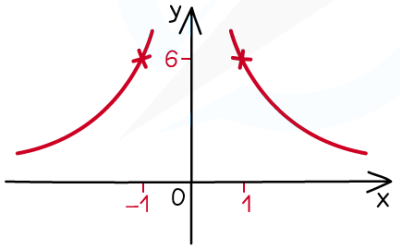
STEP 1: USE SIGN OF "a" TO LOCATE CURVES
USE SIZE OF "a" TO GAUGE STEEPNESS

a>0, CURVES IN POSITIVE AREAS
a=6, LARGE COMPARED TO 1, LESS L-SHAPED

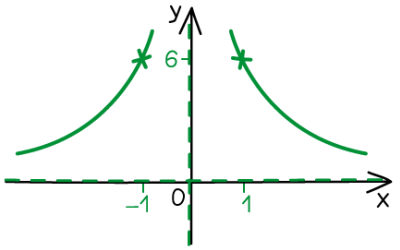
STEP 2: SKETCH THE GRAPH



STEP 3: LABEL x=1 & x=−1 AS A GUIDE



STEP 4: DRAW ASYMPTOTES WITH A DOTTED LINE



Copyright © Save My Exams. All Rights Reserved
save my exams

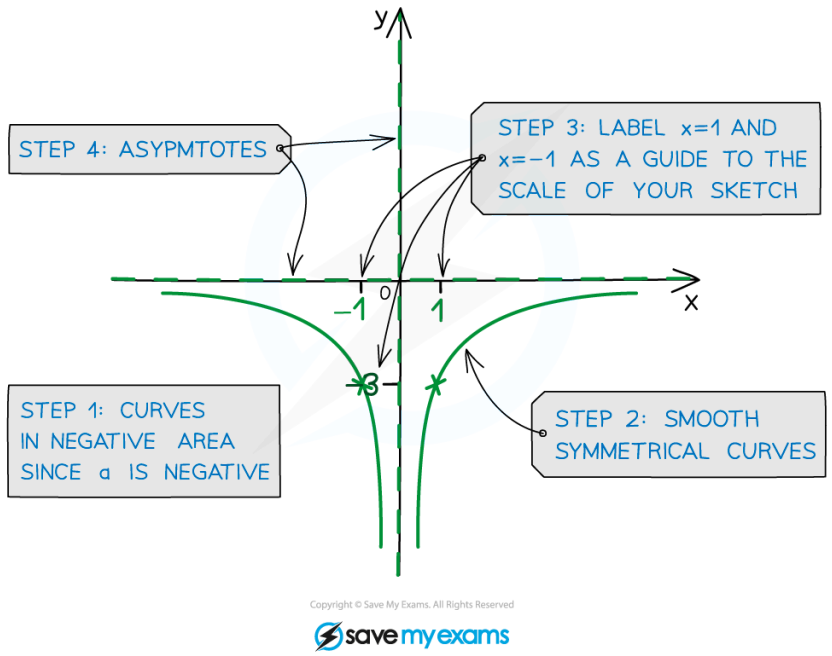
- **STEP 1**
  Use the sign of "a" to locate the curves and use the size of "a" to gauge the steepness of the curve

- **STEP 2**
  Sketch the graph
- **STEP 3**
  Label the points **x = 1** and **x = -1** as a guide to the scale of your graph
- **STEP 4**
  Draw **asymptotes** with **dotted** lines
- These graphs do not intercept either axis
- **Graph transformations** of them could cross the axes (see Translations)

Your notes

Worked Example

Sketch the graph of $y = \frac{-3}{x^2}$

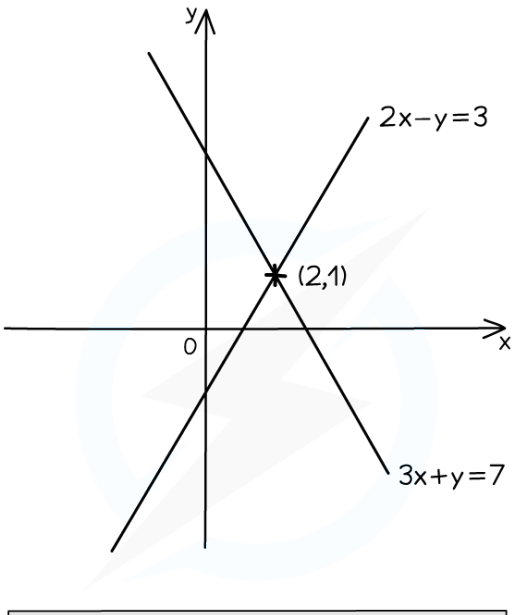


Copyright © Save My Exams. All Rights Reserved
savemyexams

Your notes

Solving equations graphically

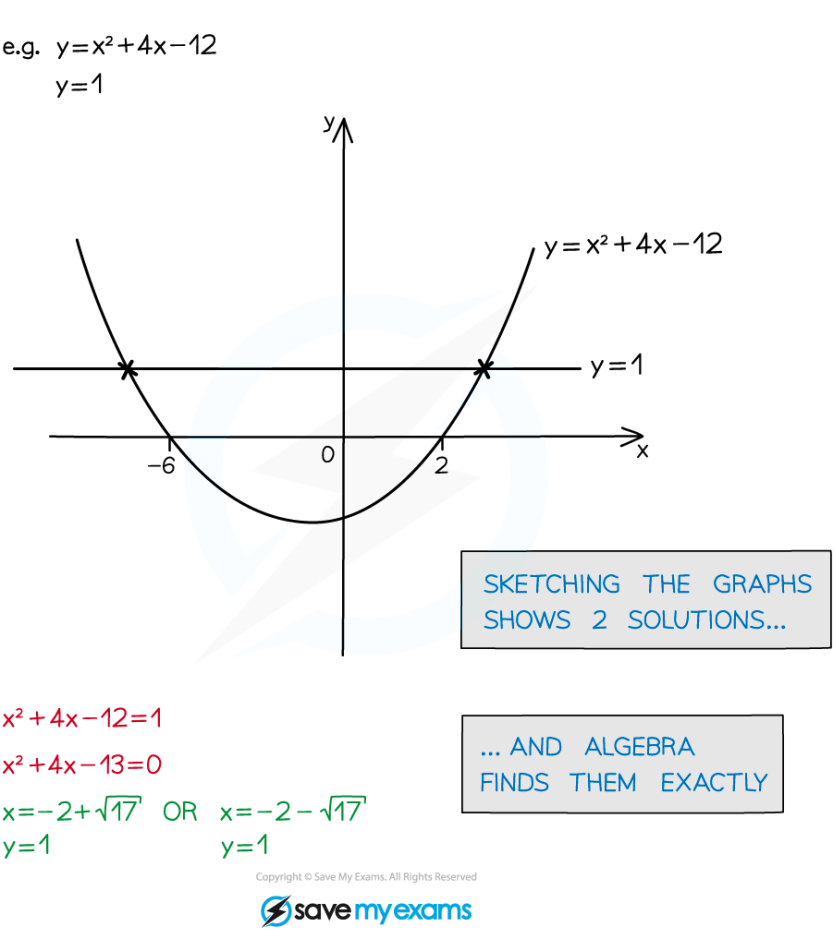
What does solving simultaneous equations graphically mean?



Copyright © Save My Exams. All Rights Reserved
save my exams

- This is a way to solve **simultaneous equations** (see Simultaneous Equations)
- Coordinates of the intersections are the **solutions**

How do I solve simultaneous equations using a graph?

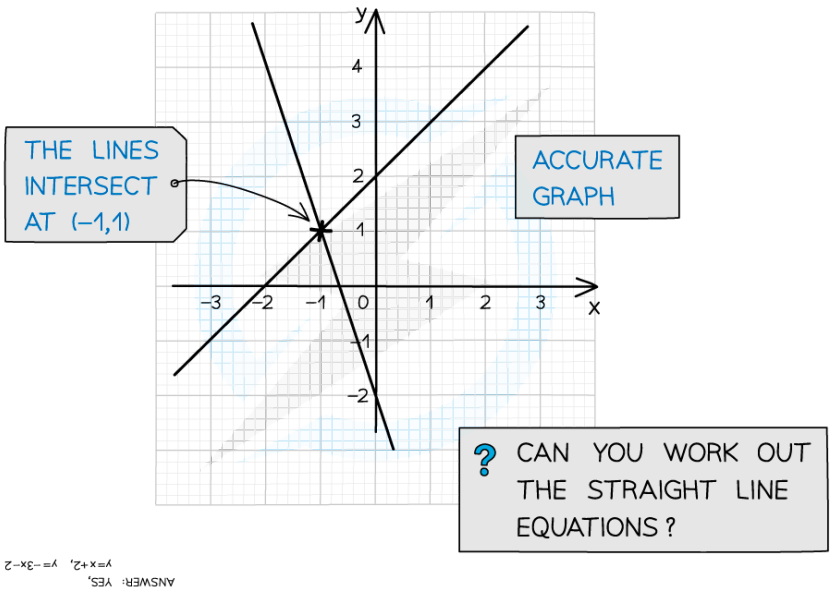


Copyright © Save My Exams. All Rights Reserved
save my exams

Your notes

- Use **graphs** and **algebra** together
- **Sketch** a graph if it has not been given
- **Read/interpret** a graph if it has been given

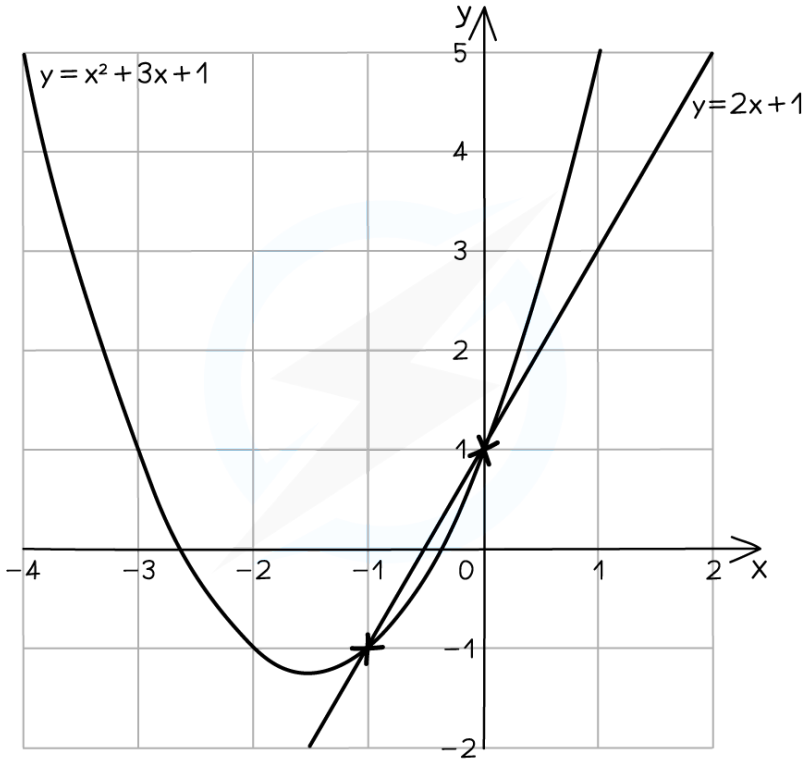
Your notes



ANSWER: YES, $y=x+2$, $y=-3x-2$

Copyright © Save My Exams. All Rights Reserved
save my exams

- It can be difficult to tell from a sketch if graphs intersect once, more, or not at all



Copyright © Save My Exams. All Rights Reserved
save my exams

Your notes

Examiner Tips and Tricks

When writing out your solutions to simultaneous equations, always pair the correct x solution with the correct y solution

Worked Example

(a) Find all points of intersection between the curve with equation $y = 2x^3 + 5x^2 + 4$ and the straight line $2x + y = 4$

(b) Sketch the graphs on the same diagram clearly showing the points of intersection.

a) $y = 2x^3 + 5x^2 + 4$
$2x + y = 4$

THIS IS UNUSUAL BUT IT IS EASIER TO SUBSTITUTE THE CUBIC EQUATION INTO THE LINEAR ONE

$2x + 2x^3 + 5x^2 + 4 = 4$
$2x^3 + 5x^2 + 2x = 0$
$x(2x^2 + 5x + 2) = 0$
$x(2x + 1)(x + 2) = 0$

FACTORISE...

...AND SOLVE

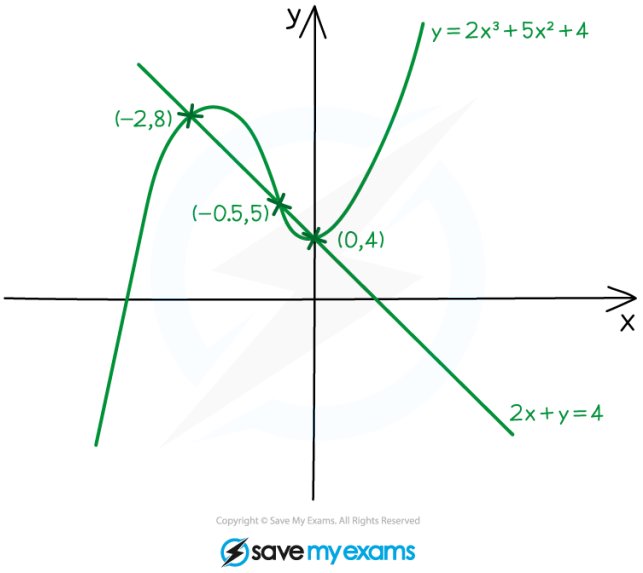
$x = 0, \quad y = 4$
$x = -\frac{1}{2}, \quad y = 5$
$x = -2, \quad y = 8$

REMEMBER THE y-COORDINATES

$\therefore$ INTERSECTIONS ARE $(0,4)$, $(-\frac{1}{2},5)$ AND $(-2,8)$

Copyright © Save My Exams. All Rights Reserved
save my exams

b)



Copyright © Save My Exams. All Rights Reserved
save my exams

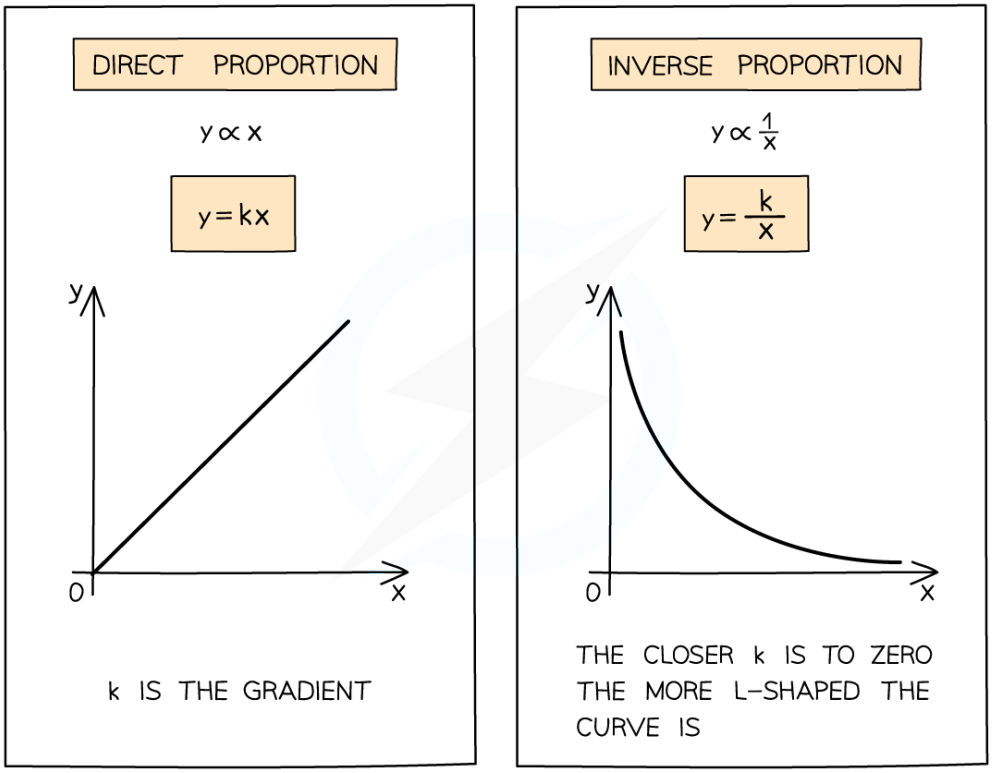
Your notes

Your notes

Proportional relationships

What are proportional relationships?

- Proportional relationships describe a proportional connection between two variables
- This can happen in two ways
  - Direct proportion $y = kx$
    - one variable increases or decreases the other does the same
  - Inverse proportion $y = \frac{k}{x}$
    - one variable increases the other decreases and vice versa
- Proportional relationships use the symbol $\propto$ which means is proportional to



Copyright © Save My Exams. All Rights Reserved
save my exams

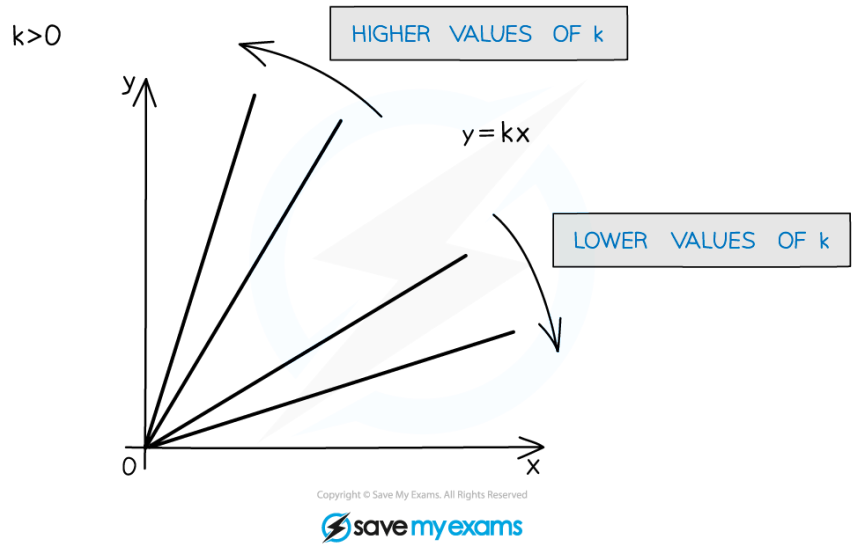
- Both direct and inverse proportion can be represented graphically

- Direct proportion creates a linear graph where k is the gradient
- Inverse proportion creates a reciprocal graph

Your notes

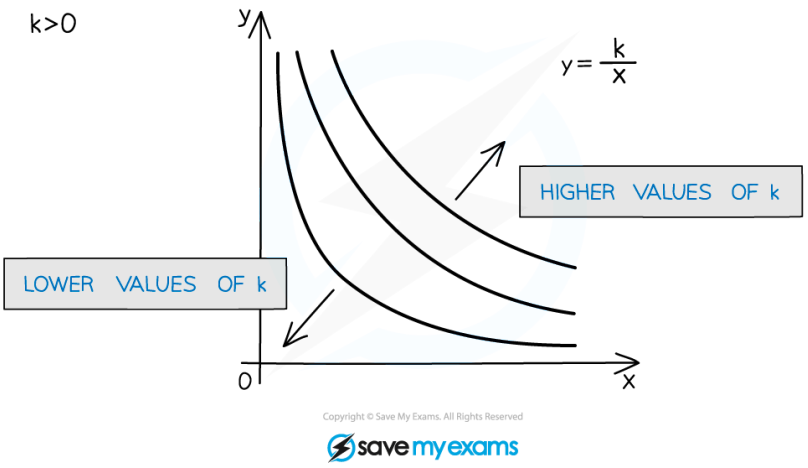
What is direct proportion?

- $y \propto x$ means y is proportional to x
- **y** increases as **x** does, **k** determines the **rate** (gradient)
- by changing this to the equation $y = kx$ we can substitute in given values and solve to find k
  - Note that this means the **ratio** of x and y is constant **k = y / x**



What is inverse proportion?

- $y \propto \frac{1}{x}$ means y is proportional to $\frac{1}{x}$ or y is inversely proportional to x
- **y** decreases as **x** increases and vice versa, **k** determines the rate
- by changing this to the equation $y = \frac{k}{x}$ we can substitute in given values and solve to find k
  - Note that this means the **product** of x and y is constant **k = xy**



Your notes

- Set up your proportional relationship using $\propto$ then change to = k
- Be clear about what **y** is proportional to ...
  - "... the square of **x**" ($\mathbf{x^2}$)
  - "... **x** plus four" (**x + 4**)
- **Calculate** or **deduce** the value of **k** from the information given or a graph
- Once you've found k sub it back in to your original proportion equation
- You can now find any values using this proportional relationship
- **y = mx + c** rearranges to **y – c = mx** so (**y – c**) is **directly** proportional to **x**
- **Proportional relationships** are often used in modelling

Worked Example

Your notes

(a) y is inversely proportional to the cube of x.

Given that $y = 3$ when $x = 2$ find the value of y when $x = 4$.

(b) Sketch the graph of y against x

a) $y \propto \frac{1}{x^3}$ ← INVERSE PROPORTION "... THE CUBE OF x"

$\therefore y = \frac{k}{x^3}$

AT (2,3): $3 = \frac{k}{2^3}$ ← USE THE GIVEN VALUES OF x AND y TO FIND k

$k = 3 \times 8$

$k = 24$

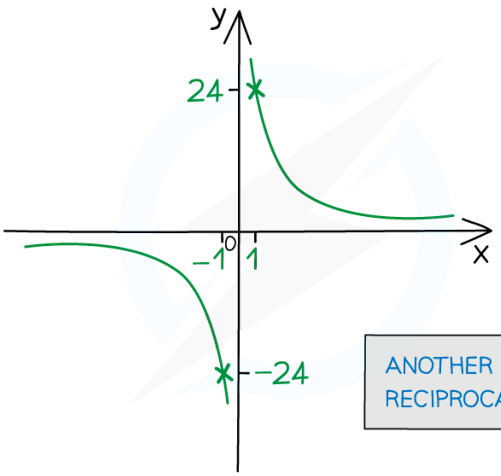
$\therefore y = \frac{24}{x^3}$

AT $x=4$, $y = \frac{24}{4^3}$ ← ONCE YOU KNOW THE EQUATION OF y YOU CAN SUBSTITUTE x=4 INTO IT

$y = \frac{3}{8}$

Copyright © Save My Exams. All Rights Reserved
save my exams

b)



ANOTHER TYPE OF RECIPROCAL GRAPH

Copyright © Save My Exams. All Rights Reserved
save my exams

Your notes

Modelling with functions

What is a mathematical model?

- A **mathematical model** simplifies a real-world situation so it can be described using mathematics which can then be used to make predictions
  - The path a stone will take if thrown from the top of a cliff
  - The number of toys a factory can produce in a day
- **Assumptions** about the situation are made in order to simplify the mathematics
  - Air resistance on the stone can be ignored
  - The machines/people at the factory produce toys at a constant rate
- Models can be **refined** (improved) if further information is available or the model is compared to real-world data
  - The mass of the stone needs to be considered
  - 30-minutes downtime per day is allowed for machine repairs/maintenance

How do I use mathematical models?

- There will be no one-size-fits-all step-by-step guide to solving modelling questions
- A combination of skills and **problem-solving** skills will be needed

© 2026 Save My Exams, Ltd.

Your notes

BOYLE'S IDEAL GAS LAW STATES THAT, FOR A CONSTANT TEMPERATURE, THE PRESSURE OF A GAS IS INVERSELY PROPORTIONAL TO THE VOLUME OF ITS CONTAINER.
A COMPANY IS TO USE THIS AS A MODEL TO FIND A SUITABLE CONTAINER FOR A PARTICULAR GAS THEY NEED TO STORE.

a) USING P FOR PRESSURE (MEASURED IN Pa, PASCALS) AND V FOR VOLUME (MEASURED IN m^3, CUBIC METRES) WRITE DOWN AN EQUATION FOR THE MODEL TO BE USED.

b) IT IS KNOWN THAT A CONTAINER WITH A VOLUME OF $5m^3$ CREATES A PRESSURE OF 350 000 Pa. FIND THE VALUE OF THE CONSTANT OF PROPORTIONALITY

c) FIND THE PRESSURE IF THE VOLUME OF THE CONTAINER IS 750 m^3

d) GIVEN THAT THE COMPANY MUST KEEP THE PRESSURE BELOW 120 000 Pa FIND, TO THE NEAREST CUBIC METRE, THE SMALLEST VOLUME OF CONTAINER THEY COULD USE

e) STATE ONE ASSUMPTION AND ONE PROBLEM IN USING THIS MODEL

Copyright © Save My Exams. All Rights Reserved
save my exams

Your notes

a) $P \propto \frac{1}{V}$ ← INVERSE PROPORTION

$\therefore P = \frac{k}{V}$ WHERE k IS A CONSTANT

b) SUBSTITUTE P AND V INTO MODEL EQUATION

$350\,000 = \frac{k}{5}$

$k = 1\,750\,000$ ← CAREFUL WITH ALL THOSE ZEROS

STORE IN CALCULATORS' MEMORY

c) $P = \frac{1\,750\,000}{V}$

$P = \frac{1\,750\,000}{750}$

$P = 2330$ Pa (3sf)

d) $P < 120\,000$

$\frac{1\,750\,000}{V} < 120\,000$

$V > \frac{1\,750\,000}{120\,000}$

$V > 14.583...$

$\therefore$ SMALLEST CONTAINER IS $15\,m^3$

e) ASSUMPTION:
THE AMOUNT OF GAS THE COMPANY NEEDS TO STORE DOES NOT CHANGE

PROBLEM:
KEEPING THE TEMPERATURE OF THE GAS/CONTAINER CONSTANT

Copyright © Save My Exams. All Rights Reserved
save my exams

Examiner Tips and Tricks

- Reciprocal graphs generally have two parts/curves

- Only one – usually the positive – may be relevant to the model
- Think about why **x/t/θ** can only take positive values?

Your notes

Worked Example

Your notes

The function

$$f(d) = \frac{L}{d+1}$$

where $d \geq 0$ and L is a constant

is used to model the number of leaves on a tree during the autumn season.

d is the number of days since the start of the autumn season.

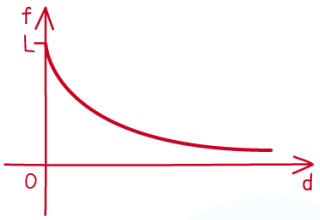
(a) What does the constant L represent?

(b) How long does it take for the number of leaves on the tree to halve?

(c) If the autumn season is 90 days and a tree started with 13500 leaves, how many leaves will be left on the tree at the end of the season

(d) Give a possible problem with the application of this model

A SKETCH OF THE GRAPH MAY HELP



RECIPROCAL GRAPH
$d \geqslant 0$

a) L REPRESENTS THE NUMBER OF LEAVES ON THE TREE AT THE START OF AUTUMN (WHEN d=0)

b) WHEN HALVED $f = \frac{L}{2}$

$\therefore \frac{L}{2} = \frac{L}{d+1}$

$d+1=2$

$d=1$

IT TAKES ONE DAY FOR THE NUMBER OF LEAVES TO HALVE

c) $d=89$ $L=13500$

$\therefore$ 13500

© 2026 Save My Exams, Ltd. | Get more and ace your exams at savemyexams.com

$r = \sqrt{89+1}$

$f = 150$

150 LEAVES ARE LEFT ON THE TREE AT THE END OF THE SEASON

d) IT IS HARD TO PINPOINT EXACTLY WHEN THE AUTUMN SEASON STARTS

THERE ARE MANY ANSWERS TO THESE TYPES OF QUESTIONS

LOOK FOR EXTREMES:
- ANYTHING STRANGE AT/NEAR $x=0$
- DO NEGATIVES GET INVOLVED EVEN THOUGH ALL VALUES SHOULD BE POSITIVE?
- WHAT HAPPENS FOR LARGE VALUES OF x

Copyright © Save My Exams. All Rights Reserved
save my exams

Your notes