Edexcel International A Level (IAL) Maths: Pure 1

Your notes

Polynomials

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Expanding brackets

How do I expand brackets?

- To expand brackets, the rule is that each term in one set of brackets must be multiplied by each term in the other set of brackets

$$(a+b)(x+y+z) = a(x+y+z) + b(x+y+z)$$
$$= ax + ay + az + bx + by + bz$$

$$(a+b+c)(x+y+z) = a(x+y+z) + b(x+y+z) + c(x+y+z)$$
$$= ax + ay + az + bx + by + bz + cx + cy + cz$$

- 'FOIL' is a special case of this, when each set of brackets contains two terms you can multiply in order
  - e.g. to multiply out or expand these brackets (2x + 3)(x - 5)
    - First (**2x** + 3)(**x** - 5) gives $2x^2$
    - Outside (**2x** + 3)(x **- 5**) gives -10x
    - Inside (2x + **3**)(**x** - 5) gives +3x
    - Last (2x **+ 3**)(x **- 5)** gives -15
  - $(2x + 3)(x - 5) = 2x^2 - 10x + 3x - 15 = 2x^2 - 7x - 15$
- If you spot something like $(a + b)^2$ or $(a + b)^3$ just write the brackets out twice or three times respectively
  - e.g. $(3x + 2)^2 = (3x + 2)(3x + 2) = 9x^2 + 6x + 6x + 4 = 9x^2 + 12x + 4$
- If you are trying to expand something like $\boldsymbol{(a + b)^n}$ for powers of $\boldsymbol{n}$ greater than 2 or 3, use the **binomial expansion**
- If you have to expand more than two sets of brackets, just expand them two at a time

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Worked Example

Expand and simplify $4x(x-3y)(3x+2y-5)$.

EXPAND $4x(x-3y)$ FIRST

$4x(x-3y) = 4x^2 - 12xy$

MULTIPLY EACH TERM IN THE FIRST BRACKET BY EACH TERM IN THE SECOND BRACKET

$(4x^2 - 12xy)(3x + 2y - 5)$

$= 12x^3 + 8x^2y - 20x^2 - 36x^2y - 24xy^2 + 60xy$

$= 12x^3 - 28x^2y - 20x^2 - 24xy^2 + 60xy$

COLLECT LIKE TERMS

Worked Example

Expand $(x-4)(x+3)(x^2+x-1)$

FIRST EXPAND $(x-4)(x+3) = x^2 - x - 12$

THEN CONTINUE: $(x^2 - x - 12)(x^2 + x - 1)$

$= x^2(x^2 + x - 1) - x(x^2 + x - 1) - 12(x^2 + x - 1)$

$= x^4 + x^3 - x^2 - x^3 - x^2 + x - 12x^2 - 12x + 12$

$= x^4 - 14x^2 - 11x + 12$

Factorising expressions

What is meant by factorising expressions?

- Many **expressions** in mathematics are written as a **sum** of terms
  - e.g. $x^2 + 6x - 16$ is the **sum** of **three** the terms x^2, $6x$ and -16
- Many **expressions** are written as a **product** of factors
  - e.g. $(x+8)(x-2)$ is the **product** of the **two** (linear) **factors** $x+8$ and $x-2$
- **Factorising** is the process of rewriting the **sum** of **terms** as a **product** of **factors**
  - The other way round is **expanding**

How do I factorise an expression?

- This will depend on the nature of the expression you are dealing with
- In **all** cases the first thing to consider is if there is a factor (number and/or letter) of all terms in the expression
  - e.g. $2x^3 + 4x^2 - 8x = 2x(x^2 + 2x - 4)$
- A **quadratic** expression may be able to be **factorised** into **two linear factors**
  - Look out for **special** cases
    - **No constant** term: $x^2 + 5x = x(x+5)$
    - **Difference of two squares** (no x term and constant is square): $x^2 - 36 = (x-6)(x+6)$
    - **Perfect squares**: $x^2 + 8x + 16 = (x+4)(x+4) = (x+4)^2$
    - **'Hidden' quadratics:** $3^{2x} - 12 \times 3^x + 27 = (3^x)^2 - 12(3^x) + 27 = (3^x - 3)(3^x - 9)$
    - **More than one variable:** $x^2 - y^2 = (x-y)(x+y)$
- A **cubic** expression (at this level) will **not** contain a **constant** term
  - This means will x be a **factor** (and there might be a number as a factor too)
  - The remaining **expression** will be a **quadratic**
    - this **quadratic** may also be able to be **factorised**
    - e.g. $6x^3 + 3x^2 - 9x = 3x(2x^2 + x - 3) = 3x(2x+3)(x-1)$

How do I factorise harder quadratics?

- There are many shortcuts to factorise quadratic expressions, but they often only apply under certain conditions (such as when $a = 1$)
  - the method below works for **any** quadratic expression
  - it is most useful when the **coefficient** of the x^2 term is **greater** than 1 (and **not prime**)
- Follow the steps:
  - **STEP 1** Starting with $ax^2 + bx + c$ find the product ac
    - For example: for $6x^2 + 7x - 3$ $ac = 6 \times -3 = -18$
  - **STEP 2** Find two numbers *m* & *n* whose **product** is ac and **sum** is b
    - For example: $9 \times -2 = -18 = ac$ & $9 + (-2) = 7 = b$
    - So $m = 9$ & $n = -2$
  - **STEP 3** Split the bx term into $mx + nx$
    - For example: $6x^2 - 2x + 9x - 3$
  - **STEP 4** Factorise the first two terms and the last two terms
    - For example: $2x(3x - 1) + 3(3x - 1)$
  - **STEP 5** Factorise once more for the final answer
    - For example: $(3x - 1)(2x + 3)$
- If *a* and/or c are **prime**, factorising can be done "**by inspection**"
  - For example: the only way to split (prime) 3 into factors would be 3 and 1

Why does the 'ac' method work?

- Suppose $ax^2 + bx + c \equiv (px + r)(qx + s)$
  - then expanding and simplifying gives
    - $ax^2 + bx + c \equiv pqx^2 + psx + qrx + rs \equiv pqx^2 + (ps + qr)x + rs$
- By **comparing coefficients**
  - $a = pq$
  - $b = ps + qr$
  - $c = rs$
- Let $m = ps$ and $n = qr$ then:

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- $m + n = ps + qr = b$
- $m \times n = psqr = ac$
- Therefore these are the two numbers whose **product** is *ac* and **sum** is *b*

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Worked Example

Fully factorise the following expressions

(i) $10x^2 + 5x$

(ii) $4x^2 + 19x - 5$

(iii) $9x^2 - 49y^2$

(iv) $4x^3 + 22x^2 + 30x$

(i) $10x^2 + 5x$ THIS IS A QUADRATIC EXPRESSION WITH NO CONSTANT TERM

$\therefore 10x^2 + 5x = 5x(2x + 1)$

(ii) $4x^2 + 19x - 5$ A STANDARD QUADRATIC BUT $a > 1$ (AND NOT PRIME)

STEP 1: FIND ac, $ac = 4 \times -5 = -20$

STEP 2: $\times \longrightarrow ac$, $+ \longrightarrow b$, $b = 19$ $20 \times -1 = -20$, $20 + -1 = 19$

STEP 3: SPLIT THE x TERM, $4x^2 + 20x - x - 5$

STEP 4: FACTORISE FIRST TWO TERMS, LAST TWO TERMS:

$4x(x + 5) - (x + 5)$

STEP 5: FACTORISE ONCE MORE: $(x + 5)(4x - 1)$

$\therefore 4x^2 + 19x - 5 = (x+5)(4x - 1)$

(iii) $9x^2 - 49y^2$ DIFFERENCE OF TWO SQUARES (AND MORE THAN ONE VARIABLE

$\therefore 9x^2 - 49y^2 = (3x - 7y)(3x + 7y)$

(iv) $4x^3 + 22x^2 + 30x$ A CUBIC EXPRESSION

$= 2x(2x^2 + 11x + 15)$

$\therefore 4x^3 + 22x^2 + 30x = 2x(2x + 5)(x + 3)$

Examiner Tips and Tricks

- Do use your tried and tested shortcuts for factorising quadratics
  - We've explained it in full above to help you understand the process rather than to learn 'tricks'
- You **don't need to learn** why the '*ac*' method works - but we thought you might think that the algebra is cool