Edexcel International A Level (IAL) Maths: Pure 1

Your notes

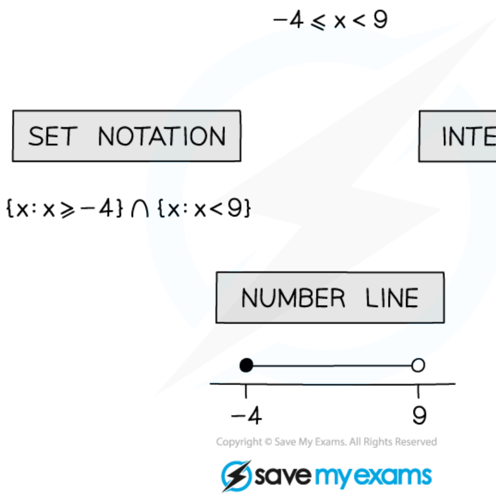
Inequalities

Contents

Linear inequalities

What are linear inequalities?

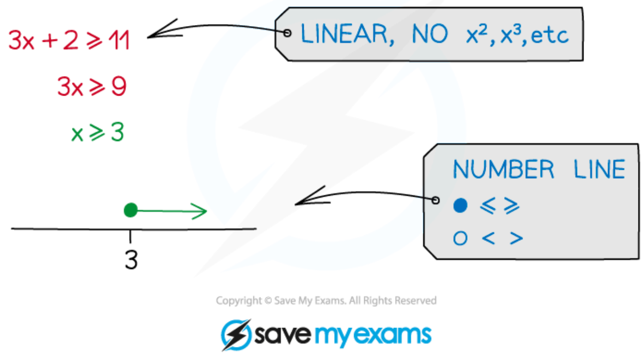
- Linear inequalities are similar to equations but answers take a range of values
- Linear means there will be no terms other than degree 1
  - no squared terms or higher powers, no fractional or negative powers
- Inequalities use the following symbols
  - $>$ Greater than e.g. $5 > 3$
  - $<$ Less than e.g. $-8 < 7$
  - $\geq$ Greater than or equal to
  - $\leq$ Less than or equal to
- Inequalities can be represented in many ways using number lines, set notation and interval notation



How do I use number lines?

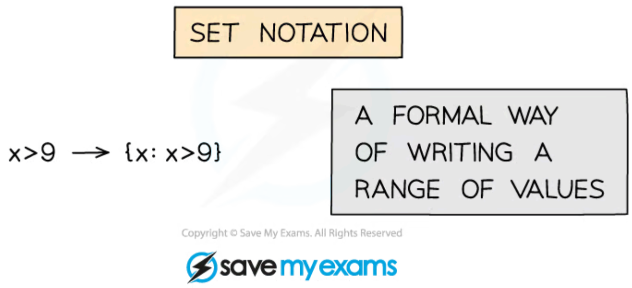
- Number line diagrams are made up from circles and lines set above a number line
  - A filled-in circle or empty circle above a number denotes whether the number is included or not
    - filled in for the greater/less than or equal to symbols
    - empty for the greater/less than symbols

- Arrows show the range of values that are allowed



How do I use set notation for inequalities?

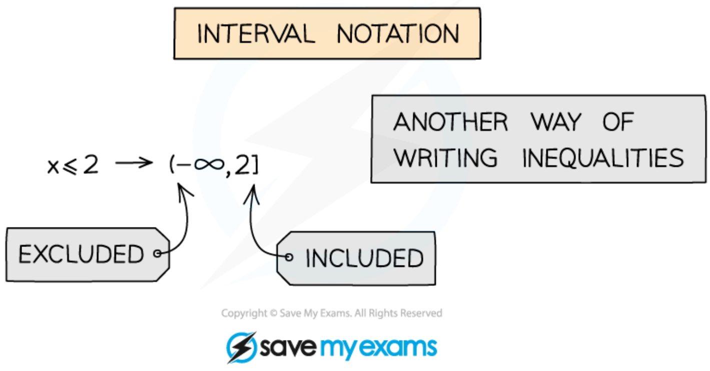
- Set notation is a formal way of writing a range of values
- Use of curly brackets { }
- Intersection ∩ and union ∪ may be used
- Not to be confused with interval notation



How do I use interval notation for inequalities?

- Interval notation uses different brackets to indicate whether a number is included or not
- Use of square [] and round () brackets
- [or] mean included
- (or) mean excluded
  - (4,8] means $4 < x \le 8$
- Note ∞ always uses (or)
- Not to be confused with set notation

Your notes

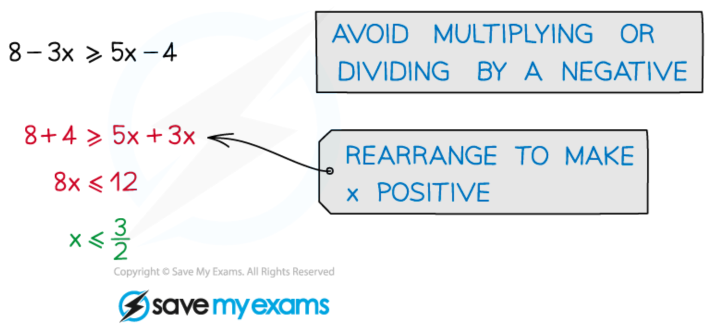


Skills for solving linear inequalities

- representing and interpreting inequalities displayed on a number line
- **writing** and **interpreting** set notation
  - eg **{x : x > 1} ∩ {x : x ≤ 7}** is the same as **1 < x ≤ 7**
- **writing** and **interpreting** interval notation
  - eg **[-4, 6)** is the same as **-4 ≤ x < 6**

How do I solve linear inequalities?

- Treat the inequality as an equation and solve
  - avoid multiplying or dividing by a negative
  - if unavoidable, "flip" the inequality sign so $< \rightarrow >, \geq \rightarrow \leq$, etc
  - try to rearrange to make the x term positive



Worked Example

Your notes

Your notes

Given that $\{x : x \geq 0\}$, solve the simultaneous inequalities $19 - 2x \geq 5$ and $9 < x + 5$, giving your answer in set notation.

$19 - 2x \geqslant 5$

$19 - 5 \geqslant 2x$

$2x \leqslant 14$

$x \leqslant 7$

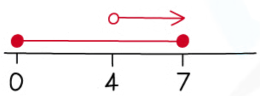
$0 \leqslant x \leqslant 7$

$9 < x + 5$

$x > 4$

SOLVE EACH INEQUALITY SEPARATELY, DEALING WITH NEGATIVES CAREFULLY

THE QUESTION SAID "$x \geqslant 0$"



DRAW A QUICK DIAGRAM TO SEE WHERE ANSWERS COINCIDE (OVERLAP)

$4 < x \leqslant 7$

$\{x : x > 4\} \cap \{x : x \leqslant 7\}$

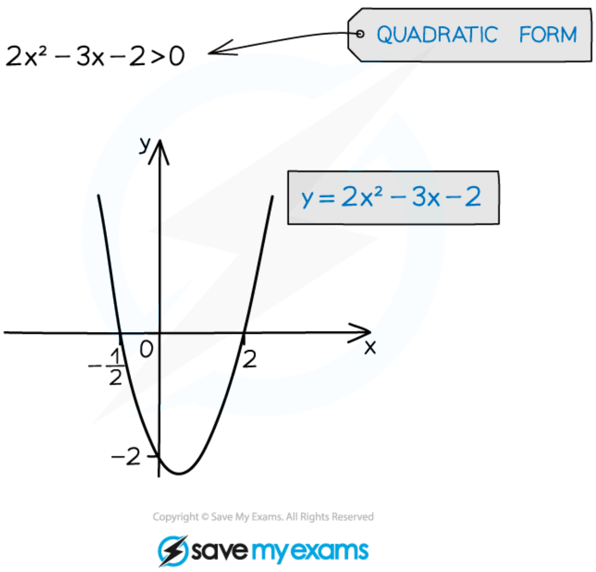
FINAL ANSWER IN SET NOTATION

Your notes

Quadratic inequalities

What are quadratic inequalities?

- Similar to quadratic equations, quadratic inequalities just mean there is a range of values that satisfy the solution
- Sketching a quadratic graph is essential
- Can involve the **discriminant** or **applications** in **mechanics** and **statistics**



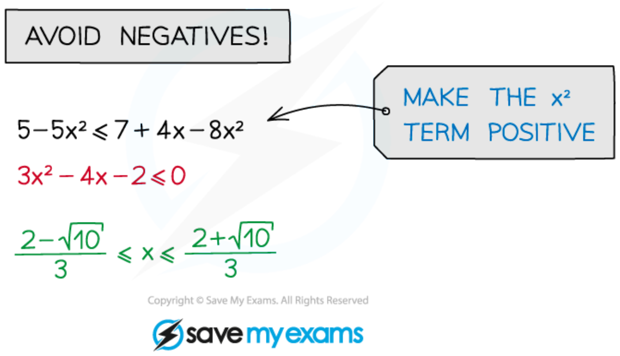
How do I solve quadratic inequalities?

- **STEP 1**: **Rearrange** the inequality into quadratic form with a **positive squared term**
  - $\boldsymbol{ax^2 + bx + c > 0}$ (>, <, ≤ or ≥)
- **STEP 2**: Find the **roots** of the quadratic equation
  - **Solve** $ax^2 + bx + c = 0$ to get $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ where $\boldsymbol{x_1 < x_2}$
- **STEP 3**: **Sketch** a graph of the quadratic and **label the roots**
  - As the squared term is positive it will be **"U" shaped**
- **STEP 4**: **Identify** the **region** that satisfies the inequality
  - For $\boldsymbol{ax^2 + bx + c > 0}$ you want the region **above** the x-axis

- The solution is $x < x_1$ or $x > x_2$
- For $ax^2 + bx + c < 0$ you want the region below the x-axis
  - The solution is $x > x_1$ and $x < x_2$
  - This is more commonly written as $x_1 < x < x_2$
- **avoid** multiplying or dividing by a negative number

  if unavoidable, "**flip**" the inequality sign so $< \rightarrow >$, $\geq \rightarrow \leq$, etc
  - **avoid** multiplying or dividing by a **variable** (**x**) that **could** be negative

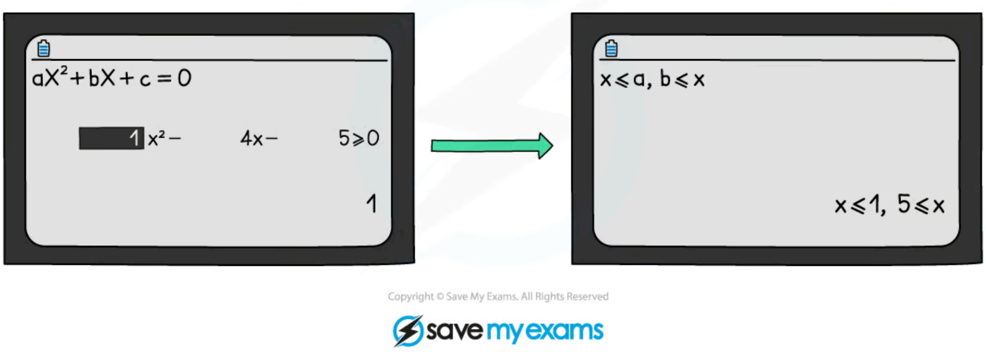
    (multiplying or dividing by x^2 guarantees positivity (unless **x** could be 0) but this can create extra, invalid solutions)
  - **do** rearrange to make the x^2 term positiveBe careful:



How do I solve quadratic inequalities on a calculator?

- Be aware of unconventional ways calculators can display an answer

  eg **8 > x > 2** rather than **2 < x < 8**



Examiner Tips and Tricks

- A calculator can be super-efficient but some marks are for method.
- Use your judgement:
- is it a "show that" or "prove" question?
    - how many marks?
    - how long is the question?

Worked Example

Find the set of values for which $3x^2 + 2x - 6 > x^2 + 4x - 2$ giving your answer in set notation.

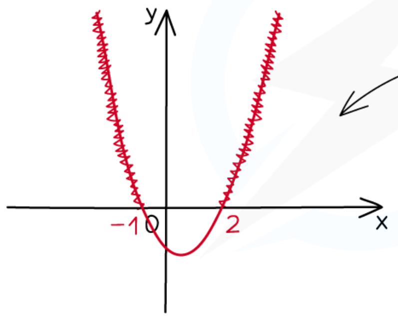
$3x^2 + 2x - 6 - x^2 - 4x + 2 > 0$

$2x^2 - 2x - 4 > 0$ REARRANGE TO QUADRATIC FORM

$2(x^2 - x - 2) > 0$

$x^2 - x - 2 > 0$

$(x-2)(x+1) > 0$



CRUCIAL: SKETCH THE GRAPH TO SEE WHERE THE SOLUTIONS ARE

$x < -1$ OR $x > 2$

$\{x : x < -1\} \cup \{x : x > 2\}$

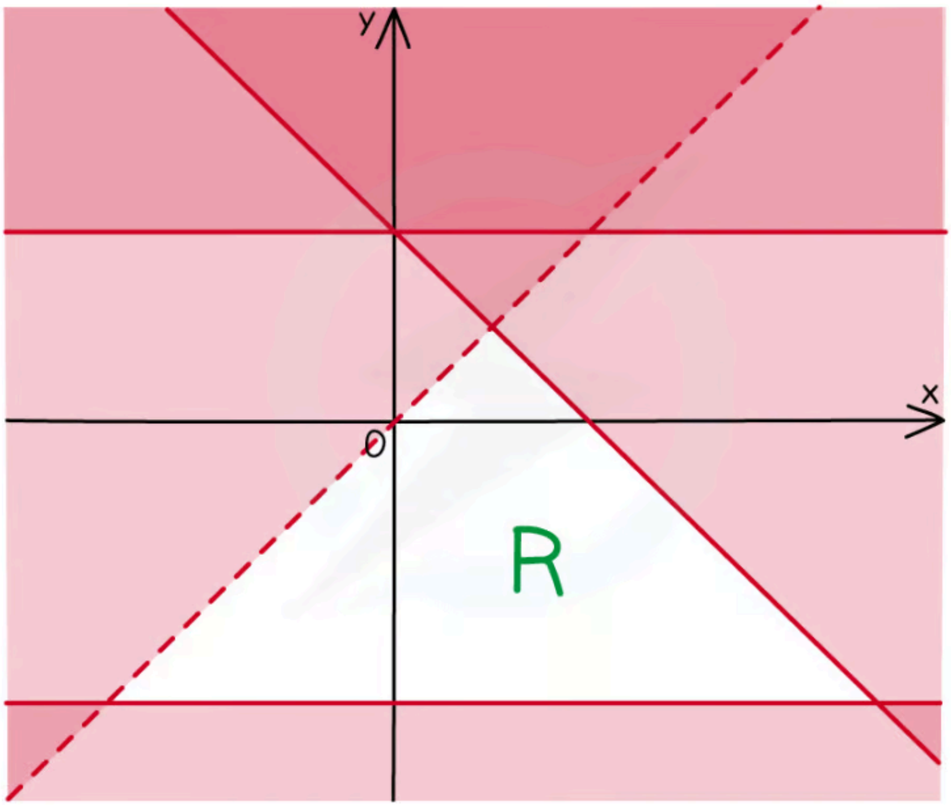
FINAL ANSWER IN SET NOTATION

Your notes

Inequalities on graphs

What are inequalities on graphs?

- Inequalities can be represented on graphs by shaded regions and dotted or solid lines

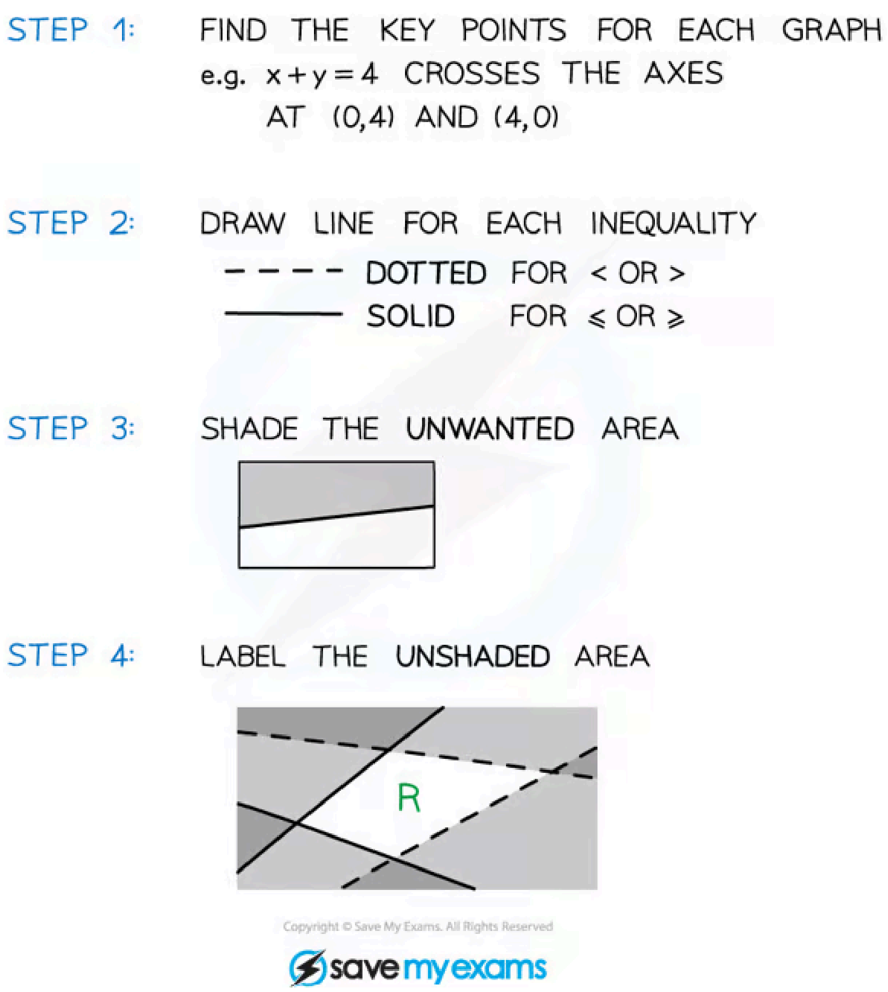


- These inequalities have two variables, **x** and **y**
- Several inequalities are used at once
- The solution is an **area** on a graph (often called a **region**)
- The inequalities can be **linear** or **quadratic**

How do I draw inequalities on a graph?

- Sketch each graph
  - If the inequality is **strict (< or >)** then use a **dotted line**
  - If the inequality is **weak (≤ or ≥)** then use a **solid line**
- Decide **which side** of the line **satisfies** the inequality
  - Choose a coordinate on each side and **test** it in the inequality

- The origin is an easy point to use
- If it satisfies the inequality then that whole side of the line satisfies the inequality
  - For example: (0,0) satisfies the inequality $y < x^2 + 1$ so you want the side of the curve that contains the origin



Examiner Tips and Tricks

- **Recognise** this type of inequality by the use of **two** variables
- You may have to **deduce** the inequalities from a **given** graph
- Pay careful attention to which **region** you are **asked to shade**
- Sometimes the exam could ask you to shade the region that **satisfies the inequalities**, **this means** you should **shade** the region that is **wanted**
- As long as your final answer is clear you should get the marks!

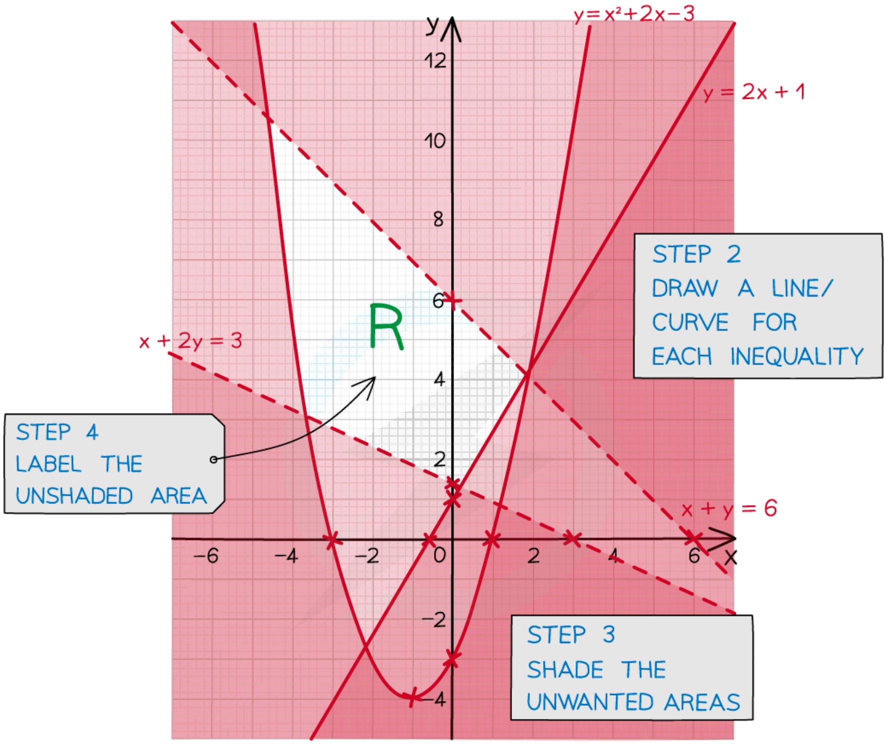
Your notes

Worked Example

On the graph below show the region bounded by the following inequalities:

$$y \geq x^2 + 2x - 3 \qquad x + y < 6$$

$$y \geq 2x + 1 \qquad x + 2y > 3$$



TURNING POINT

$y = x^2 + 2x - 3 = (x+3)(x-1)$

$(0,-3)$ $(-3,0)$ $(1,0)$ $(-1,-4)$ "$\geq$ SOLID"

$x + y = 6$

$(0,6)$ $(6,0)$ "$<$ DOTTED"

$y = 2x + 1$

$(0,1)$ $(\frac{-1}{2},0)$ "$\geq$ SOLID"

$x + 2y = 3$

$(0,\frac{3}{2})$ $(3,0)$ "$>$ DOTTED"

STEP 1
FIND THE KEY POINTS
FOR EACH GRAPH

Your notes