



Edexcel International A Level (IAL) Maths: Pure 1



Your notes

Simultaneous Equations

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Linear simultaneous equations using elimination

What are linear simultaneous equations?

- When you have more than one equation in more than one unknown, then you are dealing with **simultaneous equations**
- An equation is **linear** if none of the unknowns in it is raised to a power other than one
- Solving a **pair of simultaneous equations** means finding **pairs of values** that make **both equations true** at the same time
- A linear equation in two unknowns will produce a straight line if you graph it... linear = line
- A pair of simultaneous equations will produce lines that will cross each other (if there is a solution!)

How do I use elimination to solve linear simultaneous equations?

Step 1: Multiply one (or both) of the equations by a constant (or constants) to get the numbers in front of **one** of the unknowns to match

Step 2: If the matching numbers have the **same** sign, then **subtract** one equation from the other. If the matching numbers have **different** signs then **add** the equations together

Step 3: Solve the new equation from Step 2 to find the value of one of the unknowns

Step 4: Substitute the value from Step 3 into one of the original equations, and solve to find the value of the other unknown

Step 5: **Check** your solution by substituting the values for the two unknowns into the original equation you **didn't** use in Step 4



Examiner Tips and Tricks

- Don't skip the checking step (it only takes a few seconds) – there are many places to go wrong when solving simultaneous equations!
- Mishandling minus signs is probably the single biggest cause of student error in simultaneous equations questions



Worked Example



Your notes



Solve the simultaneous equations:

$$-2x + 4y = 5$$

$$4x - 5y = -7$$

LABEL THE
EQUATIONS

$$-2x + 4y = 5 \quad \textcircled{1}$$

$$4x - 5y = -7 \quad \textcircled{2}$$

STEP 1: $[2 \times \textcircled{1}] \quad -4x + 8y = 10$
 $[\textcircled{2}] \quad 4x - 5y = -7$

WE ONLY HAVE TO
MULTIPLY ONE OF THE
EQUATIONS HERE

STEP 2: $-4x + 8y = 10$
 $+ (4x - 5y = -7)$
 $\hline 3y = 3$

THE SIGNS IN FRONT OF
THE 4's ARE DIFFERENT,
SO ADD

STEP 3: $y = 3 \div 3 = 1$

STEP 4: $4x - 5(1) = -7$
 $4x = -2$
 $x = -0.5$

USING EQUATION $\textcircled{2}$

STEP 5: $-2(-0.5) + 4(1) = 5$
 $1 + 4 = 5$
 $5 = 5 \checkmark$

USING EQUATION $\textcircled{1}$

$$x = -0.5 \quad y = 1$$

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Linear simultaneous equations using substitution

What are linear simultaneous equations?

- When you have more than one equation in more than one unknown, then you are dealing with **simultaneous equations**
- An equation is **linear** if none of the unknowns in it is raised to a power other than one
- Solving a **pair of simultaneous equations** means finding **pairs of values** that make **both equations true** at the same time
- A linear equation in two unknowns will produce a straight line if you graph it... linear = line
- A pair of simultaneous equations will produce lines that will cross each other (if there is a solution!)

How do I use substitution to solve linear simultaneous equations?

Step 1: Rearrange one of the equations to make one of the unknowns the subject (if one of the equations is already in this form you can skip to Step 2)

Step 2: Substitute the expression found in Step 1 into the equation not used in Step 1

Step 3: Solve the new equation from Step 2 to find the value of one of the unknowns

Step 4: Substitute the value from Step 3 into the rearranged equation from Step 1 to find the value of the other unknown

Step 5: **Check** your solution by substituting the values for the two unknowns into the original equation you **didn't** rearrange in Step 1



Examiner Tips and Tricks

- Although elimination will always work to solve simultaneous linear equations, sometimes substitution can be easier and quicker.
- Knowing both methods can help you a lot in the exam (plus you will need substitution to solve quadratic simultaneous equations).



Worked Example



Your notes



Solve the simultaneous equations:

$$3x + 5y = -9$$

$$y = 1 - 9x$$

LABEL THE
EQUATIONS

$$3x + 5y = -9 \quad \textcircled{1}$$

$$y = 1 - 9x \quad \textcircled{2}$$

y IS ALREADY THE SUBJECT
OF EQUATION ②, SO WE
CAN SKIP STEP 1

STEP 2: $3x + 5(1 - 9x) = -9$
 $3x + 5 - 45x = -9$
 $-42x = -14$

SUBSTITUTE EQUATION ②
INTO EQUATION ①

STEP 3: $x = -14 \div (-42) = \frac{1}{3}$

STEP 4: $y = 1 - 9(\frac{1}{3}) = 1 - 3 = -2$

USING EQUATION ②

STEP 5: $3(\frac{1}{3}) + 5(-2) = -9$
 $1 - 10 = -9$
 $-9 = -9 \checkmark$

USING EQUATION ①

$$x = \frac{1}{3} \quad y = -2$$

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Quadratic simultaneous equations

What are quadratic simultaneous equations?

- When you have more than one equation in more than one unknown, then you are dealing with **simultaneous equations**
- An equation is quadratic if it contains terms of degree two, but no terms of any higher degrees (and also no unknowns raised to negative or fractional powers)

THESE EQUATIONS ARE QUADRATIC:

$$y = 5x^2 - 2x + 3$$

$$y^2 + 3y - 7x = 4$$

$$y^2 + 4xy - x^2 = -7$$

THESE EQUATIONS ARE NOT QUADRATIC:

$$y = \sqrt{x} - 5$$

$\sqrt{x} = x^{\frac{1}{2}}$ IS A FRACTIONAL POWER OF x

$$xy^2 + 2y - 7x^2 = 19$$

xy^2 IS DEGREE 3

$$y^2 + 2y - x^2 = \frac{1}{x^2}$$

$\frac{1}{x^2} = x^{-2}$ IS A NEGATIVE POWER OF x

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- Solving two simultaneous equations in two unknowns means finding pairs of values that make both of the equations true at the same time
 - At A level usually only one equation will be quadratic and the other will be linear
 - For one quadratic and one linear equation there will usually be **two** solution pairs (although there can be one, or none)

How do I solve quadratic simultaneous equations?

Step 1: Rearrange the **linear** equation so that one of the unknowns becomes the subject (if the linear equation is already in this form, you can skip to Step 2)

Step 2: Substitute the expression found in Step 1 into the quadratic equation

Step 3: Solve the new quadratic equation from Step 2 to find the values of the unknown (there will usually be two of these)

Step 4: Substitute the values from Step 3 into the rearranged equation from Step 1 to find the values of the other unknown

Step 5: Check your solutions by substituting the values for the two unknowns (one pair at a time!) into the original quadratic equation



Your notes



Examiner Tips and Tricks

- You have to use substitution to solve quadratic simultaneous equations – the elimination method won't work.



Worked Example



Your notes



Solve the simultaneous equations:

$$2x - 3y = 23$$

$$3y^2 = 4x^2 + 11$$

LINEAR

$$2x - 3y = 23$$

$$3y^2 = 4x^2 + 11$$

QUADRATIC

STEP 1: $2x - 3y = 23$

$$2x = 3y + 23$$

REARRANGE THE
LINEAR EQUATION

BECAUSE THE QUADRATIC HAS $4x^2 = (2x)^2$
IT'S EASIEST TO STOP AT $2x$ HERE

STEP 2: $3y^2 = 4x^2 + 11$

$$3y^2 = (2x)^2 + 11$$

$$3y^2 = (3y + 23)^2 + 11$$

$$3y^2 = 9y^2 + 138y + 529 + 11$$

$$6y^2 + 138y + 540 = 0$$

$$[\div 6] \quad y^2 + 23y + 90 = 0$$

SUBSTITUTE INTO
THE QUADRATIC

STEP 3: $(y+5)(y+18) = 0$

$$y = -5$$

$$\text{OR } y = -18$$

SOLVE BY FACTORISING
(YOU COULD ALSO USE
THE QUADRATIC FORMULA)

STEP 4: IF $y = -5$

$$2x = 3(-5) + 23 = 8$$

$$x = 4$$

IF $y = -18$

$$2x = 3(-18) + 23 = -31$$

$$x = -15.5$$

SUBSTITUTE INTO REARRANGED
LINEAR EQUATION

STEP 5: $3(-5)^2 = 4(4)^2 + 11$

$$75 = 75 \quad \checkmark$$

$$3(-18)^2 = 4(-15.5)^2 + 11$$

$$972 = 972 \quad \checkmark$$

CHECK BY SUBSTITUTING
INTO QUADRATIC!

THE SOLUTIONS ARE:

$$x = 4, \quad y = -5$$

AND

$$x = -15.5, \quad y = -18$$

INDICATE WHICH x
GOES WITH WHICH y

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