



Edexcel International A Level (IAL) Maths: Pure 1



Your notes

Integration

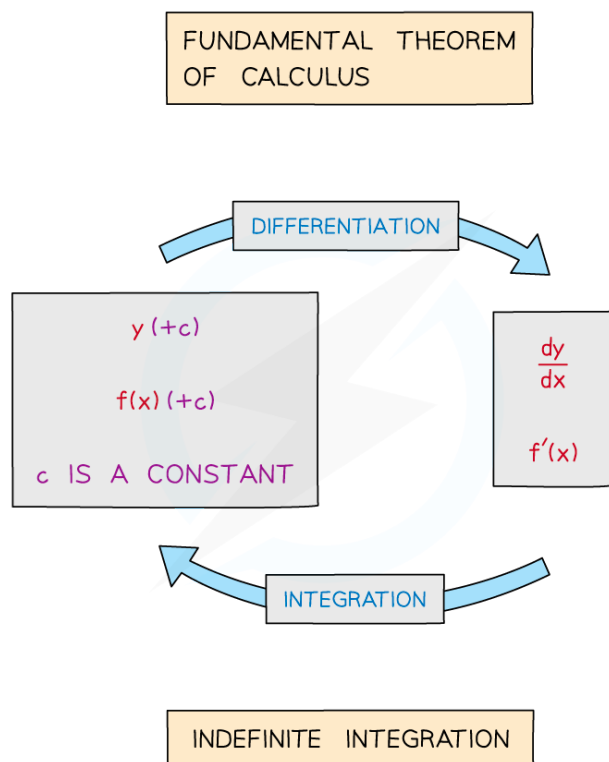
Contents

- * Fundamental Theorem of Calculus
- * Integrating Powers of x



Fundamental theorem of calculus

What is the fundamental theorem of calculus?



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- The Fundamental Theorem of Calculus states that integration is the inverse process of differentiation
 - This form of the Theorem relates to **Indefinite Integration**
 - An alternative version of the Fundamental Theorem of Calculus involves **Definite Integration**

What is a constant of integration?

- When differentiating y , constant terms 'disappear'
 - for constants $y = c$, $\frac{dy}{dx} = 0$



Your notes

- graphs of constants are horizontal lines and so have gradient $\left(\frac{dy}{dx}\right)$ of 0

- Integrating $\frac{dy}{dx}$, to get y , cannot determine the constant

- To acknowledge this constant, “+ c ” is used
- c is called the **constant of integration**

$$y = 5x \qquad \frac{dy}{dx} = 5$$

$$y = 5x + 2 \qquad \frac{dy}{dx} = 5$$

$$y = 5x - 3 \qquad \frac{dy}{dx} = 5$$

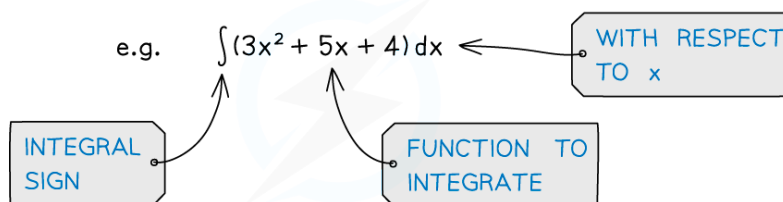
IN REVERSE, $\frac{dy}{dx} = 5$ LEADS TO
 $y = 5x$ BUT THERE COULD BE
A CONSTANT TERM TOO.

$$\text{SO } y = 5x + c$$

WHERE c IS THE CONSTANT OF INTEGRATION

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What notation is used in integration?



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- $\int$ is the sign for integration
- If it has more than one term the function to be integrated (called the **integrand**) should be in brackets
 - “Integrate” -- “all of (...)” -- “with respect to x ”

- **dx** means integrate with respect to **x**, any other letter is treated like a number (ie like a constant)



Your notes



Worked Example

? (a) Find $\frac{dy}{dx}$ for $y = 6x + 3$

(b) Find $\int 6 \, dx$

a) $\frac{dy}{dx} = 6$

b) $y = 6x + c$

CONSTANT OF
INTEGRATION

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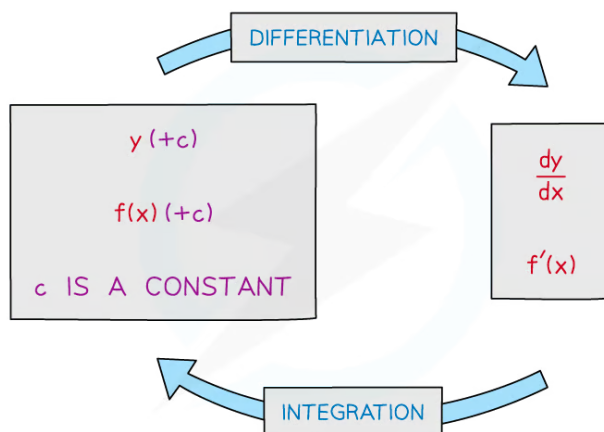




Integrating powers of x

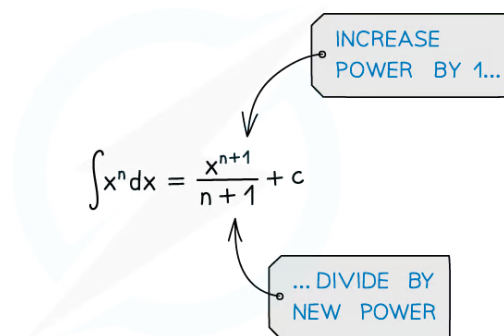
What is integration?

- Integration is the **inverse** ('opposite') of differentiation



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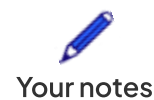
How do I integrate powers of x?



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- For each term ...
 - ... increase the power (of x) by 1
 - ... divide by the new power
- This does not apply when the original power is -1
 - the new power would be 0 and division by 0 is undefined

Finding the constant of integration



How do I find the constant of integration?

- STEP 1 Rewrite the function into a more easily **integrable** form
 - Each term needs to be a power of **x** (or a constant)
- STEP 2 Integrate each term and remember "+c"
 - Increase power by 1 and divide by new power
- STEP 3 Substitute the coordinates of a given point in to form an equation in **c**
 - Solve the equation to find **c**

e.g. FIND THE EQUATION OF THE GRAPH
THAT PASSES THROUGH (2,6) AND
HAS GRADIENT FUNCTION $3x^2(2x-1)$

ANOTHER NAME FOR $\frac{dy}{dx}$

STEP 1 REWRITE IN A MORE EASILY INTEGRABLE FORM

$$\begin{aligned}\frac{dy}{dx} &= 3x^2(2x-1) \\ &= 6x^3 - 3x^2\end{aligned}$$

STEP 2 INTEGRATE...

$$\begin{aligned}y &= \frac{6x^4}{4} - \frac{3x^3}{3} + c \\ y &= \frac{3x^4}{2} - x^3 + c\end{aligned}$$

...REMEMBERING "+c"

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STEP 3 USE THE GIVEN POINT TO FIND **c**

$$\begin{aligned}\text{AT } (2,6) \quad 6 &= \frac{3}{2}(2)^4 - 2^3 + c \\ c &= -10\end{aligned}$$

$$\therefore y = \frac{3}{2}x^4 - x^3 - 10$$

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Worked Example



Your notes

? Given $f'(x) = \frac{(x+3)^2}{\sqrt{x}}$ and $f(1) = 25$,
find $f(x)$.

STEP 1 $f'(x) = \frac{x^2 + 6x + 9}{x^{\frac{1}{2}}}$

REWRITE IN A MORE
EASILY INTEGRABLE FORM

$$f'(x) = \frac{x^2}{x^{\frac{1}{2}}} + \frac{6x}{x^{\frac{1}{2}}} + \frac{9}{x^{\frac{1}{2}}}$$

$$f'(x) = x^{\frac{3}{2}} + 6x^{\frac{1}{2}} + 9x^{-\frac{1}{2}}$$

STEP 2 $f(x) = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{9x^{\frac{1}{2}}}{\frac{1}{2}} + c$

INTEGRATE EACH TERM

REMEMBER
"+c" !

$$f(x) = \frac{2}{5}x^{\frac{5}{2}} + 4x^{\frac{3}{2}} + 18x^{\frac{1}{2}} + c$$

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STEP 3 $f(1) = \frac{2}{5} + 4 + 18 + c = 25$

FIND c

$$c = \frac{13}{5}$$

$$\therefore f(x) = \frac{2}{5}x^{\frac{5}{2}} + 4x^{\frac{3}{2}} + 18x^{\frac{1}{2}} + \frac{13}{5}$$

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