Edexcel International A Level (IAL) Maths: Pure 1

Your notes

Trigonometric Functions

Contents

Your notes

Graphs of trigonometric functions

What are the graphs of trigonometric functions?

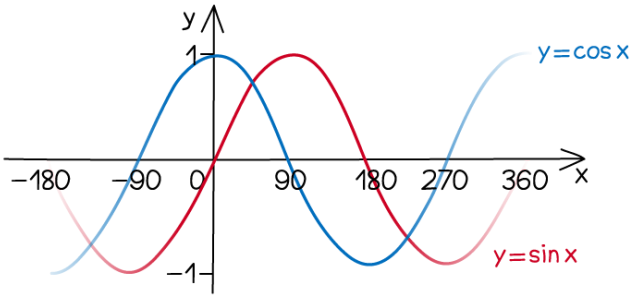
- The Trigonometric Functions Sin, Cos and Tan all have special periodic graphs that you need to be able to sketch and remember
- You'll need to know their properties and how to sketch them to solve equations and for transforming trig functions

Your notes

$y = \sin x$ AND $y = \cos x$

Sin x AND Cos x ARE ALWAYS IN THE RANGE −1 TO 1

Sin x PASSES THROUGH THE ORIGIN
Cos x PASSES THROUGH 1

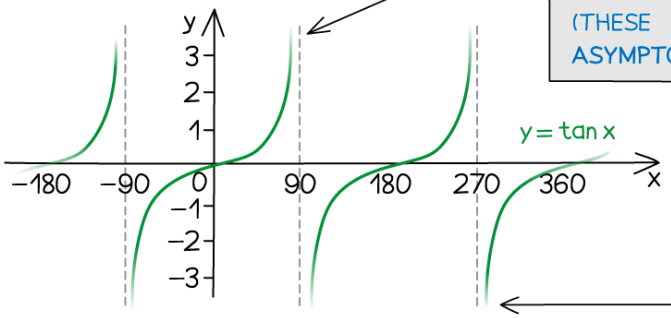


Sin x AND Cos x ARE PERIODIC REPEATING EVERY 360°

Sin x HAS ROTATIONAL SYMMETRY ABOUT THE ORIGIN SO $\sin(-x) = -\sin(x)$
Cos x IS SYMMETRICAL THROUGH THE y−AXIS SO $\cos(-x) = \cos(x)$

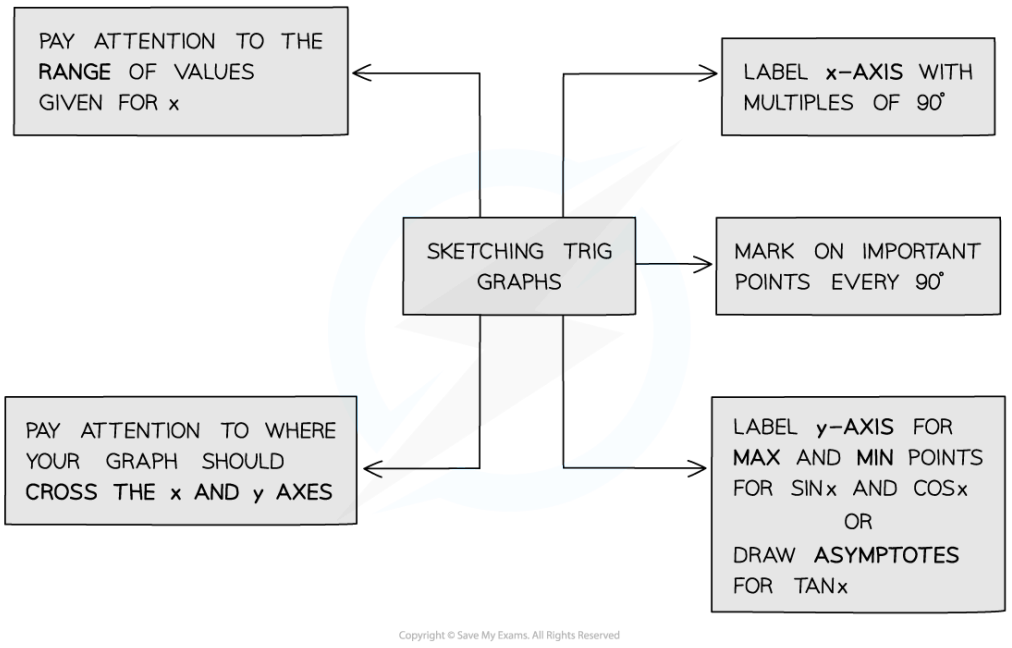
$y = \tan x$

Tan x IS UNDEFINED AT ±90°, ±270°, ±450°... MEANING IT RANGES FROM −∞ TO +∞ (THESE POINTS ARE CALLED ASYMPTOTES)



Tan x IS PERIODIC REPEATING EVERY 180°

How do I sketch trigonometric graphs?



Your notes

How do I use trigonometric graphs?

- By sketching the graph you can read off all the solutions in a given range (or interval)
- Your calculator will only give you the principal value
- You should recognise any values/angles associated with **exact values**
- You should be able to spot the pattern of solutions using the **symmetry** and **periodicity** of the graph

Your notes

STEP 1: SKETCH THE GRAPH FOR THE GIVEN FUNCTION AND RANGE

STEP 2: DRAW A HORIZONTAL LINE TO SEE WHERE THE SOLUTIONS SHOULD BE, YOU MAY BE ABLE TO READ DIRECTLY OFF THE GRAPH TO FIND ALL OF THE SOLUTIONS

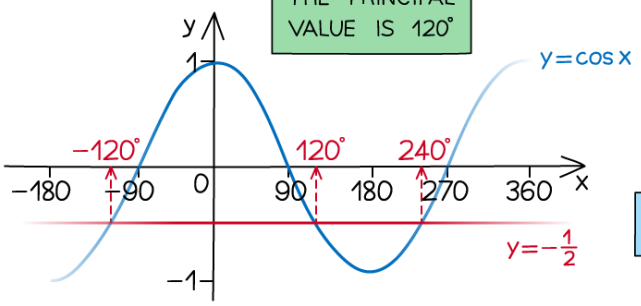
STEP 3: IF NOT, USE YOUR CALCULATOR TO FIND THE PRINCIPAL VALUE (YOU MAY ALREADY KNOW IT IF IT IS A TRIG EXACT VALUE) AND MARK THIS ON THE GRAPH

STEP 4: USE THE SYMMETRY OF THE GRAPH TO FIND ALL OTHER SOLUTIONS BY ADDING / SUBTRACTING FROM POINTS OF REPETITION WITHIN THE SPECIFIED RANGE

FOR THE SOLUTIONS TO THE EQUATION $\cos x = -\frac{1}{2}$ IN THE RANGE $-180° \leqslant x \leqslant 360°$

HERE THE FUNCTION IS $y = \cos x$ AND THE RANGE IS $-180° \leqslant x \leqslant 360°$

THE PRINCIPAL VALUE IS $120°$



DRAW THE LINE $y = -\frac{1}{2}$

ADD AND SUBTRACT $120°$ FROM THE POINTS OF REPETITION: $0°$ AND $360°$ GIVING SOLUTIONS OF $-120°$, $120°$, AND $240°$

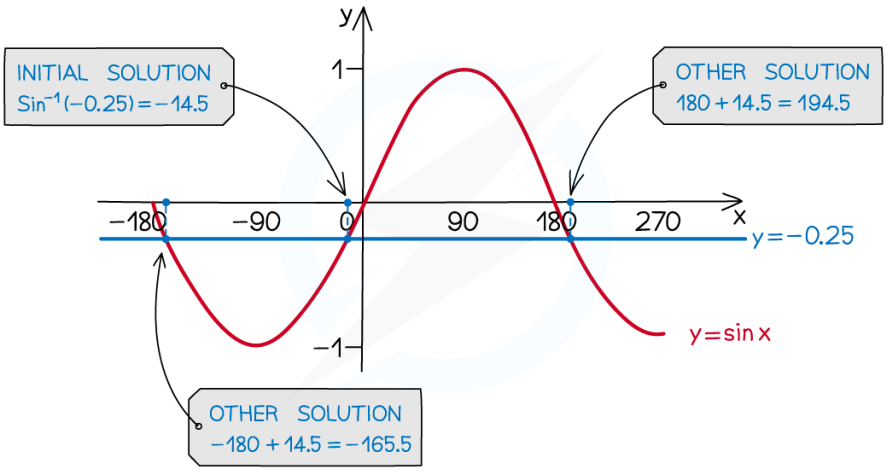
Examiner Tips and Tricks

- Always sketch with a pencil and draw a smooth curve and pay attention to the key features of each graph:
- Where it crosses the x and y axes
    - How often it repeats
    - Whether it is symmetrical
- Remember, when answering exam questions that ask for solutions, a sketch will help ensure you give **all** the appropriate solutions for a given interval.

Your notes

Worked Example

By sketching a graph, find all the solutions to $\sin x = -0.25$ for $-180° \leq x \leq 270°$.



ALL SOLUTIONS ARE:
$x = -165.5$, $x = -14.5$ AND $x = 194.5$

Your notes

Transformations of trigonometric functions

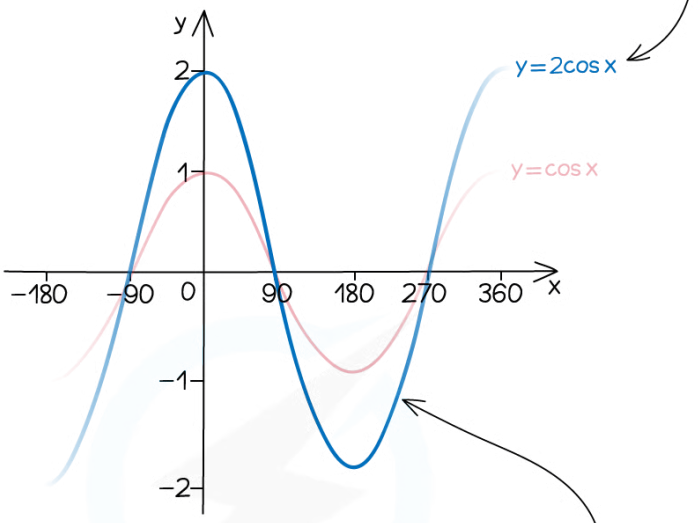
What are transformations of trigonometric functions?

- You should already have a good idea about **translating**, **stretching** and **reflecting** basic functions (see **Transformations of Functions**)
- The basic principles are exactly the same for transforming any trigonometric graph
  - $y = n \cos x$ gives a **vertical stretch or squash**
  - $y = \sin nx$ gives a **horizontal stretch or squash**
  - $y = \tan (x + c)$ gives a **horizontal translation**

Your notes

$y=n\cos x$ GIVES A VERTICAL STRETCH OR SQUASH

THE GRAPH OF $y=\cos x$ STRETCHED VERTICALLY BY A FACTOR OF 2 MAKING IT TWICE AS TALL



STEP 1: SKETCH THE ORIGINAL sin, cos OR tan FUNCTION
STEP 2: PICK KEY CO-ORDINATE POINTS TO PLOT THE NEW GRAPH
STEP 3: SKETCH IN THE NEW TRANSFORMED GRAPH

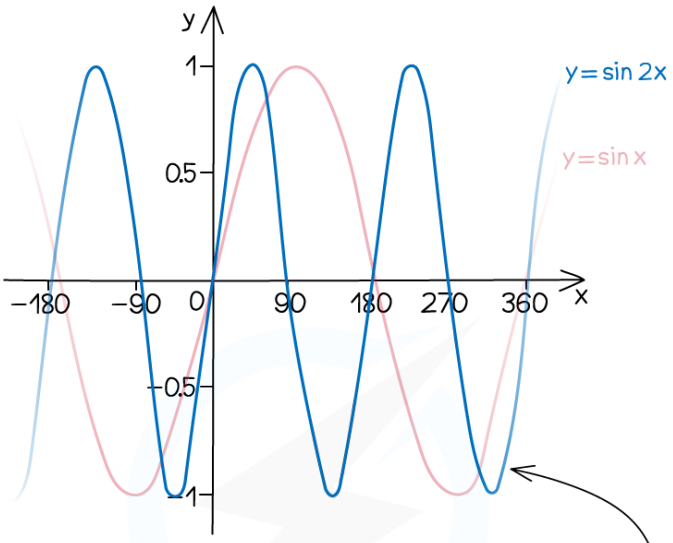
FOR A VERTICAL STRETCH (OR SQUASH)

- IF $n>1$, THE GRAPH IS **STRETCHED VERTICALLY** BY A SCALE FACTOR OF n
- IF $0<n<1$, THE GRAPH IS **SQUASHED** BY THAT SCALE FACTOR
- IF $n<0$, THE GRAPH IS ALSO **REFLECTED** IN THE x-AXIS
- REMEMBER THE x COORDINATES STAY THE SAME; y COORDINATES ARE MULTIPLIED BY n

Your notes

$y = \sin nx$ GIVES A HORIZONTAL STRETCH OR SQUASH

THE GRAPH OF $y = \sin x$ SQUASHED HORIZONTALLY BY A FACTOR OF 2 MEANING IT REPEATS ITSELF TWICE EVERY 360°



STEP 1: SKETCH THE ORIGINAL sin, cos OR tan FUNCTION
STEP 2: PICK KEY CO-ORDINATE POINTS TO PLOT THE NEW GRAPH
STEP 3: SKETCH IN THE NEW TRANSFORMED GRAPH

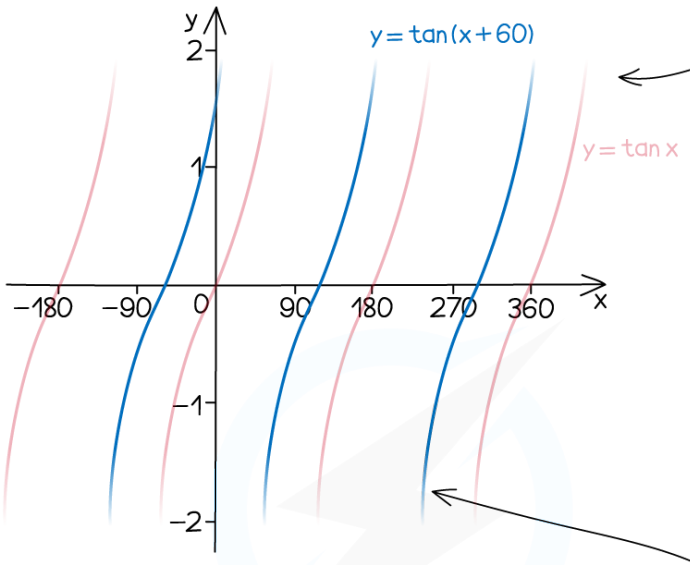
FOR A HORIZONTAL STRETCH (OR SQUASH)

- IF $n>1$, THE GRAPH IS **SQUASHED HORIZONTALLY** BY A SCALE FACTOR OF n
- IF $0<n<1$, THE GRAPH IS **STRETCHED** BY THAT SCALE FACTOR
- IF $n<0$, THE GRAPH IS ALSO **REFLECTED** IN THE y-AXIS
- REMEMBER THE y COORDINATES STAY THE SAME; x COORDINATES ARE MULTIPLIED BY $\frac{1}{n}$

Your notes

y = tan(x+c) GIVES A HORIZONTAL TRANSLATION

THE GRAPH OF y = tan x TRANSLATED HORIZONTALLY TO THE LEFT BY 60



STEP 1: SKETCH THE ORIGINAL sin, cos OR tan FUNCTION
STEP 2: PICK KEY CO-ORDINATE POINTS TO PLOT THE NEW GRAPH
STEP 3: SKETCH IN THE NEW TRANSFORMED GRAPH

FOR A HORIZONTAL TRANSLATION

- IF c>0, THE GRAPH IS **TRANSLATED LEFT** BY c
  e.g. sin(x+40) IS sinx MOVED 40 LEFT
- IF c<0, THE GRAPH IS **TRANSLATED RIGHT** BY c
  e.g. cos(x−20) IS cosx MOVED 20 RIGHT
- REMEMBER THE y COORDINATES STAY THE SAME; x COORDINATES WILL BE ±c

Examiner Tips and Tricks

- Always sketch with a pencil and draw a smooth curve.
- When you sketch the transformation of a graph, be sure to indicate the new coordinates of any points that are marked on the original graph.

- Try to indicate the coordinates of points where the new graph intersects the axes.

Your notes

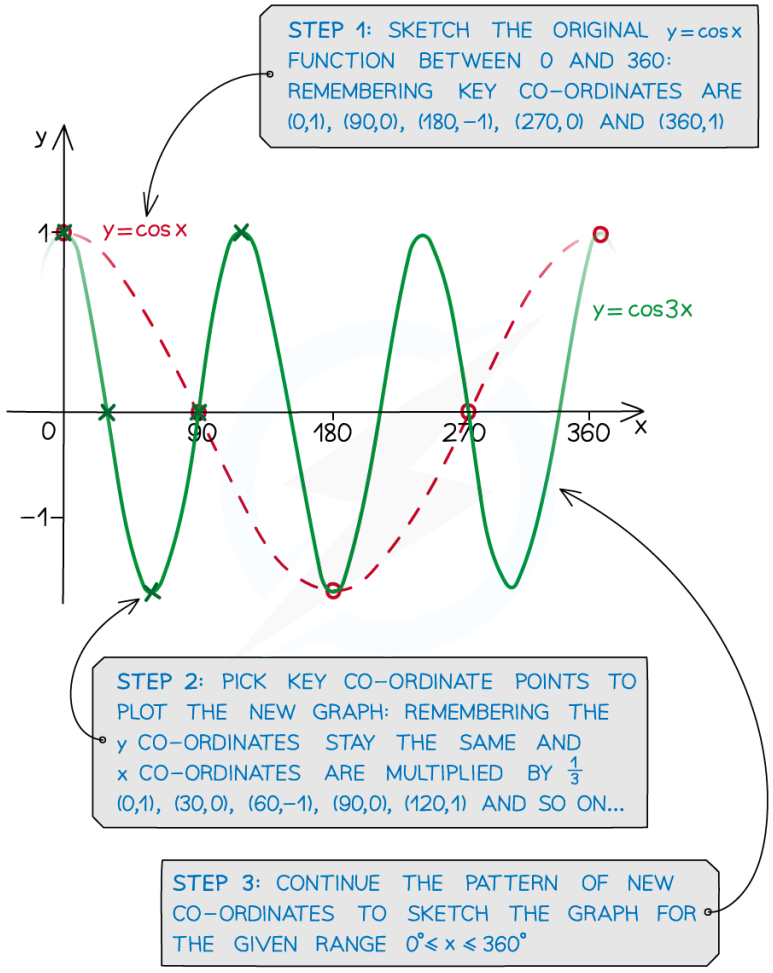
Worked Example

Your notes

a) Sketch the graph of $y = \cos 3x$, for $0° \leq x \leq 360°$.

b) Write down all the values of x where $\cos 3x = 0$, for $0° \leq x \leq 360°$.

a)



b) (30,0), (90,0), (150,0), (210,0), (270,0), (330,0)

LIST OUT THE CO-ORDINATES WHERE y = cos3x CROSSES THE x-AXIS, AGAIN YOU SHOULD BE ABLE TO SEE A PATTERN (+60 EACH TIME)