Edexcel International A Level (IAL) Maths: Pure 1

Your notes

Radian Measure

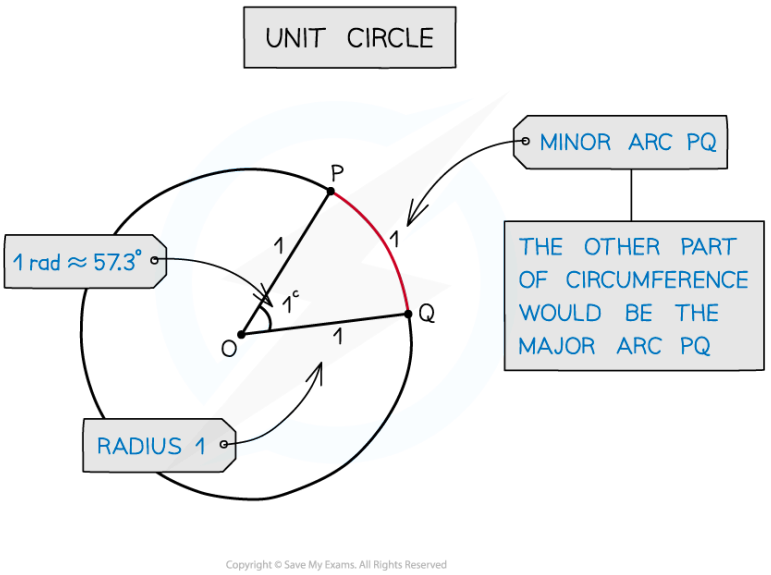
Contents

Your notes

Radian measure

What are radians?

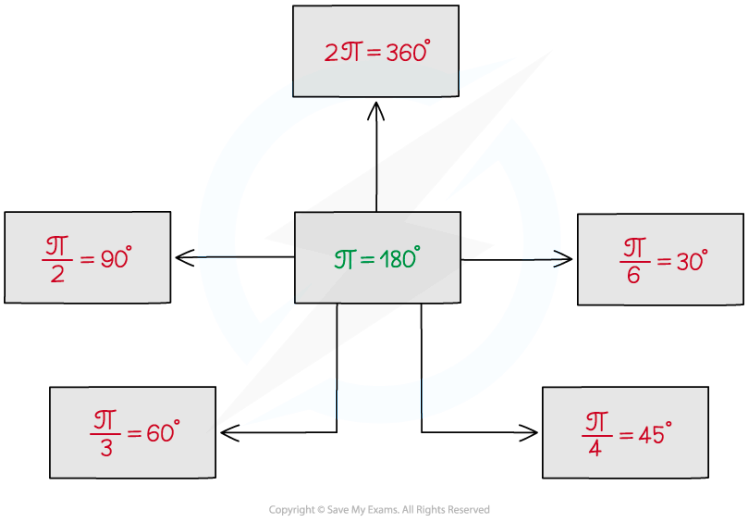
- **Radians** are an alternative to degrees for measuring angles



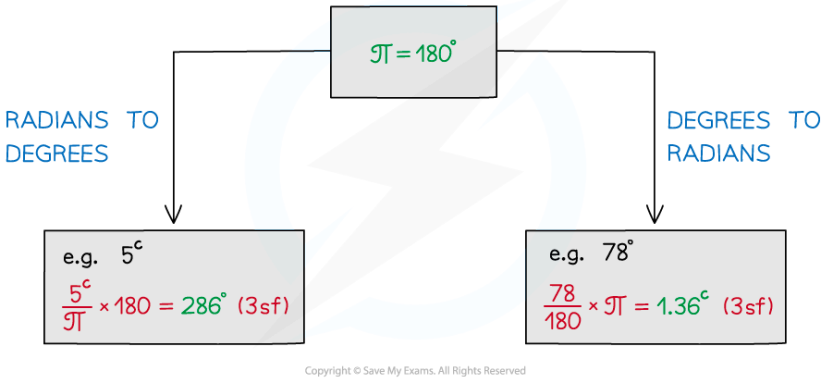
- 1 radian is the angle in a sector of radius 1 and arc length 1
- Radians are normally quoted in terms of π
- This leads to
  - **2π = 360°**
  - **π = 180°**
- The symbol for radians is ^{c} but it is more usual to see **rad**
  - Often, when π is involved, no symbol is given as it is obviously in radians
- Radians are used in **trigonometry** (see Exact Values) and **calculus** (First Principles Differentiation – Trigonometry)

How do I convert between radians and degrees?

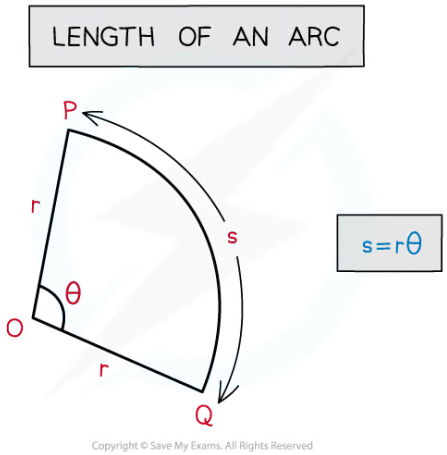
- The most common are below, these are helpful to know



- Multiples of these are quite common too, eg ¾π = 135°
- For less familiar angles use **π = 180°** to convert

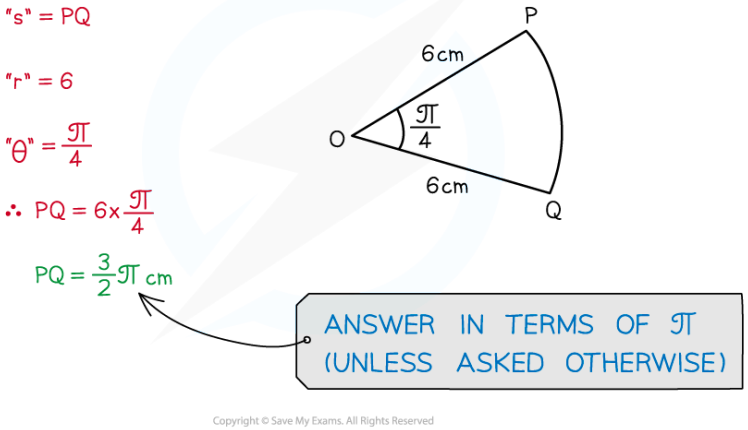


How do I find the arc length of a sector with radians?

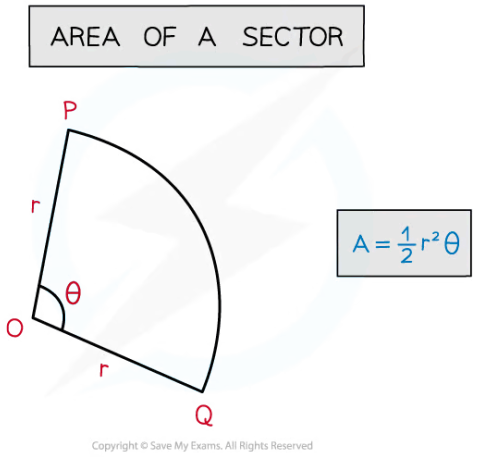


- The length of an arc is **s = rθ**

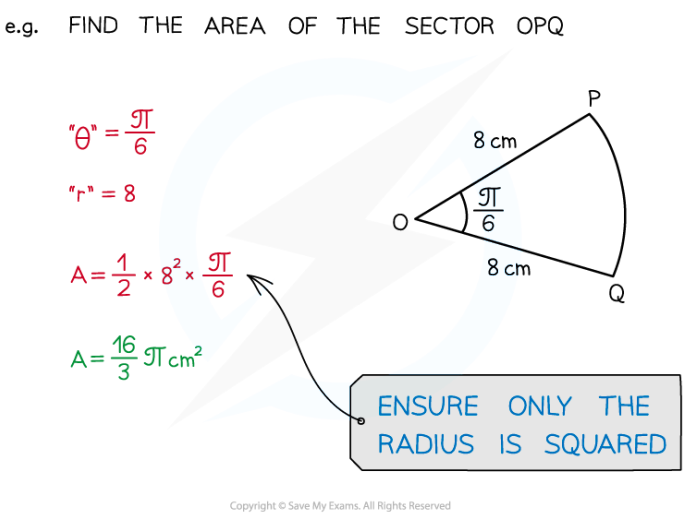
e.g. FIND THE LENGTH OF THE ARC PQ



How do I find the area of a sector with radians?



- The area of a sector is **$A = \frac{1}{2}r^2\theta$**

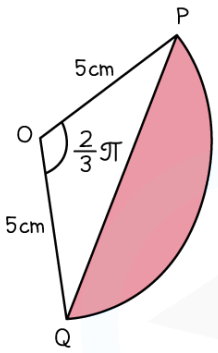


How do I solve sector problems with radians?

- Other **angle**, **circle**, **area**, etc skills may be needed
- For example, area of a triangle **$A = \frac{1}{2}ab\sin C$**

Your notes

e.g. FIND THE AREA OF THE SHADED SEGMENT



"SEGMENT" = "SECTOR" − "TRIANGLE"

AREA OF SECTOR OPQ $= \frac{1}{2} \times 5^2 \times \frac{2}{3}\pi = \frac{25}{3}\pi$ ← "$\frac{1}{2}r^2\theta$"

AREA OF TRIANGLE OPQ $= \frac{1}{2} \times 5 \times 5 \times \sin\frac{2}{3}\pi = \frac{25\sqrt{3}}{4}$ ← "$\frac{1}{2}ab\sin C$"

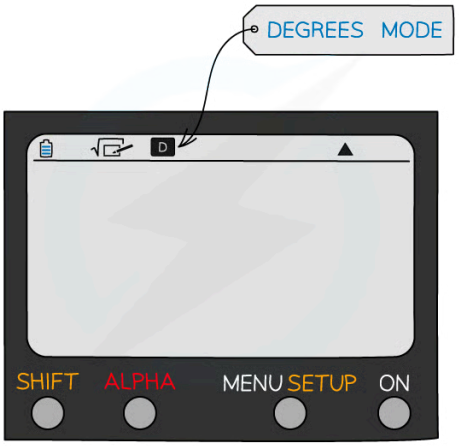
$\therefore$ AREA OF SEGMENT $= \frac{25}{3}\pi - \frac{25\sqrt{3}}{4}$

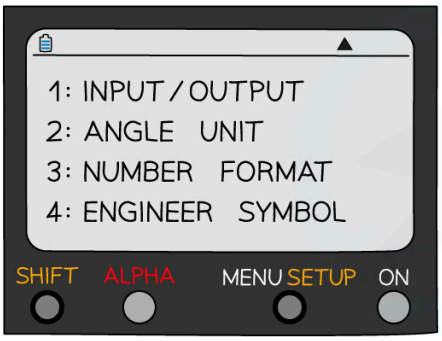
$= \frac{100\pi - 75\sqrt{3}}{12}$ cm^2

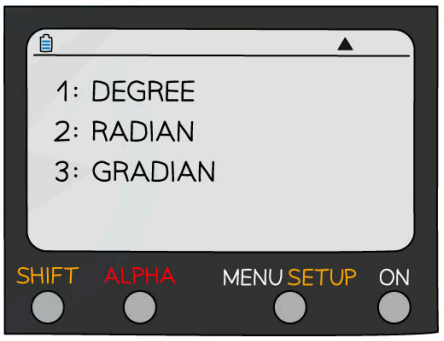
Examiner Tips and Tricks

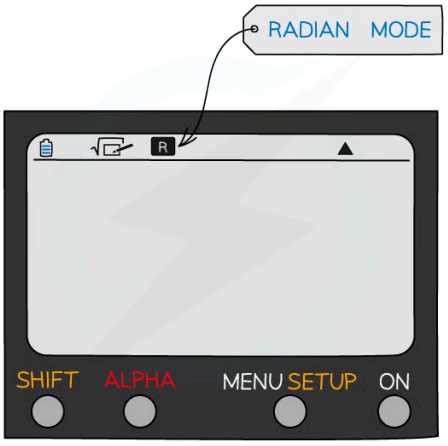
- Exam papers will have a mixture of questions in degrees and radians
- Check the mode (degrees/radians) of your calculator before using it
- Ensure you know how to change the mode on your calculator
- For the Casio fx-991EX Classwiz
  - **SHIFT** then **MENU SETUP**
  - Choose option **2: ANGLE UNIT**
  - Choose either **1: DEGREE** or **2: RADIAN**

Your notes









Worked Example

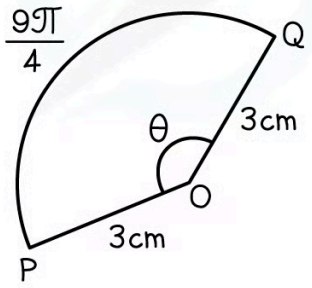
Your notes

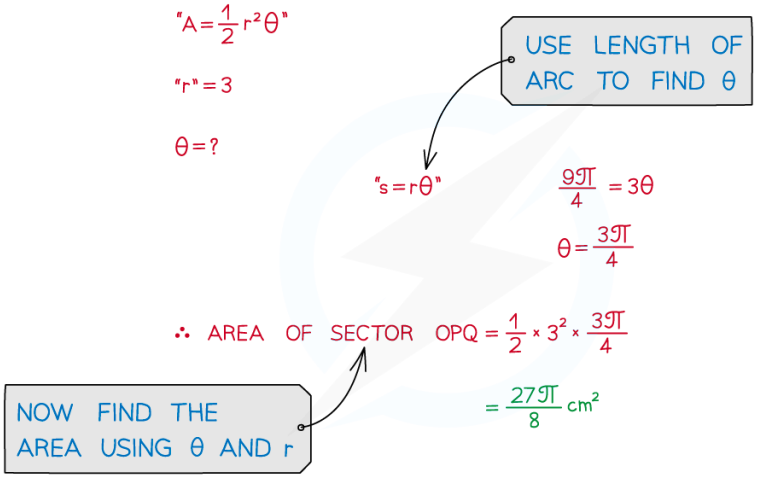
In the diagram below OPQ is the sector of a circle with radius 3 cm.

The length of the arc PQ is $\frac{9\pi}{4}$.

The angle at the centre of the circle is θ, measured in radians.

Find the area of the sector OPQ.



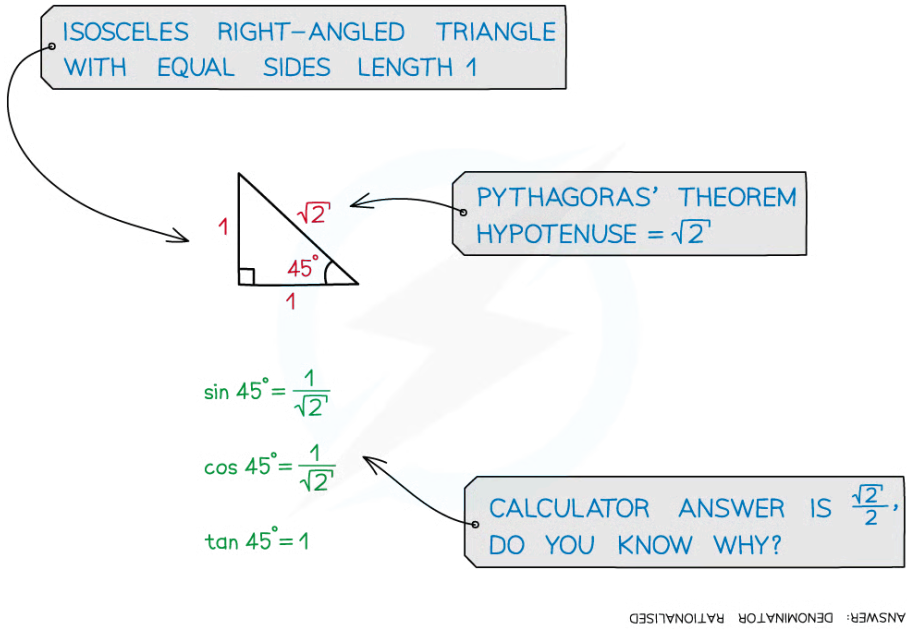


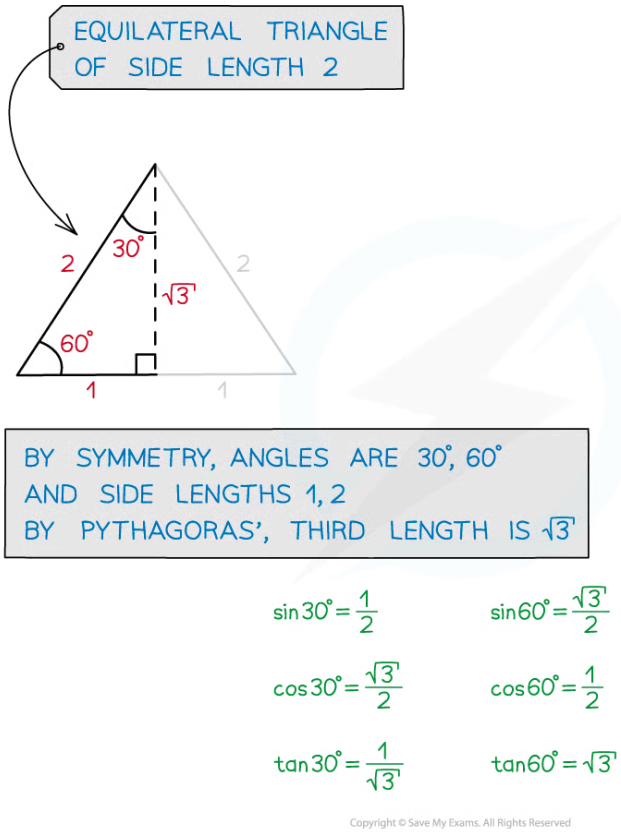
Your notes

Trigonometry exact values

What are exact values in trigonometry?

- In trigonometry some values of sin, cos and tan are "nice"
- For example **sin 30° = ½**, **tan 45° = 1**
- We can find those associated with the angles **30°**, **45°** and **60°** using SOHCAHTOA on isosceles and equilateral triangles





Your notes

- **sin**, **cos** and **tan** values for **30°**, **45°** and **60°** should be recalled easily
- Memorise either the actual values or how to work them out from the triangle
- **sin**, **cos** and **tan** for **0°**, **90°**, **180°** should also be very familiar
- They can be recalled using the relevant **trigonometric graph** (see Graphs of Trigonometric Functions)

What exact values do I need to know in trigonometry?

- All of the above applies to radians as well as degrees
- Here is a table of all exact values including **radians** (see Radian Measure)

Your notes

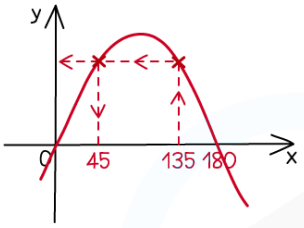
DEGREES	0°	30°	45°	60°	90°	180°	360°
RADIANS	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	UNDEFINED	0	0

How do I find exact values of other angles?

- Exact **sin**, **cos** and **tan** values of **multiples** of **30°**, **45°**, **60°** can also be found
- This is a combination of recalling the basic values and using the graph

e.g. FIND THE EXACT VALUE OF $\sin 135^\circ$

STEP 1



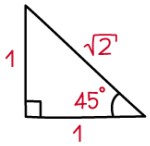
SKETCH THE GRAPH OF $\sin x$

STEP 2

$\sin 135^\circ = \sin 45^\circ$

SYMMETRY OF GRAPH RELATES 135° TO 45°

STEP 3



$\sin 45^\circ = \frac{1}{\sqrt{2}}$

RECALL EXACT VALUE
DRAW DIAGRAM TO HELP

$\therefore \sin 135^\circ = \frac{1}{\sqrt{2}}$

Your notes

Examiner Tips and Tricks

- Draw the triangles for **sin**, **cos** and **tan** of **30°**, **45°** and **60°** on the exam paper so that you can then use them as many times as you need throughout the paper.

Worked Example

Your notes

Write down the exact values of the following:

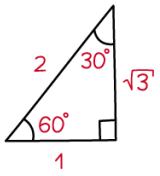
(i) $\sin 30°$

(ii) $\cos\frac{\pi}{3}$

(iii) $\tan 135°$

(iv) $\sin\frac{4\pi}{3}$

(i)



USE TRIANGLE TO RECALL EXACT VALUE

$\sin 30° = \frac{1}{2}$

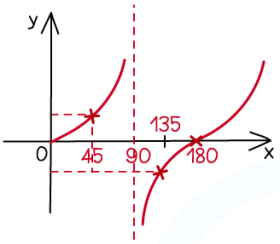
(ii) $\frac{\pi}{3} = 60°$

USE DIAGRAM ABOVE

$\cos\frac{\pi}{3} = \frac{1}{2}$

(iii) STEP 1

SKETCH GRAPH OF tanx

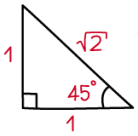


STEP 2

USE SYMMETRY OF GRAPH

$\tan 135° = -\tan 45°$

STEP 3



RECALL EXACT VALUE

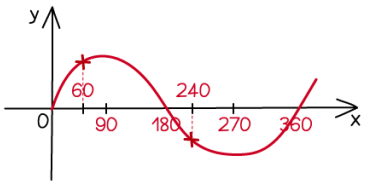
$\tan 135° = -1$

(iv) $\frac{4\pi}{3} = 4\left(\frac{\pi}{3}\right) = 4 \times 60 = 240°$

CONVERSION NOT ESSENTIAL

Your notes

STEP 1



SKETCH THE GRAPH OF sinx

STEP 2

$\sin 240^\circ = -\sin 60^\circ$

SYMMETRY

$\sin \frac{4\pi}{3} = -\sin \frac{\pi}{3}$

STEP 3

$\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

USING TRIANGLE FROM PART (i)