Edexcel International A Level (IAL) Maths: Pure 1

Your notes

Laws of Indices & Surds

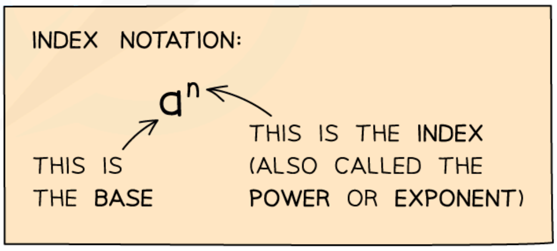
Contents

Laws of indices

What do I need to know about laws of indices?

- Laws of indices (or index laws – 'index' is the singular, 'indices' is the plural) allow you to simplify and manipulate expressions involving powers
- The index laws you need to know and use are summarised here:

  - $a^m \times a^n = a^{m+n}$
  - $a^m \div a^n = a^{m-n}$
  - $(a^m)^n = a^{mn}$
  - $(ab)^n = a^n b^n$
  - $a^{\frac{1}{m}} = \sqrt[m]{a}$
  - $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
  - $a^{-m} = \dfrac{1}{a^m}$
  - $a^1 = a$
  - $a^0 = 1$



How do I change the base of a term?

- Sometimes expressions involve different base values
- You can use index laws to change the base of a term to simplify an expression involving terms with different bases
  - For example $9^4 = (3^2)^4 = 3^{2\times4} = 3^8$
  - Using the above can then help with problems like $9^4 \div 3^7 = 3^8 \div 3^7 = 3^{8-7} = 3^1 = 3$

Examiner Tips and Tricks

Your notes

- Index laws only work with terms that have the same base, so something like $2^3 \times 5^2$ cannot be simplified using index laws

Your notes

Worked Example

Given that $y = \frac{1}{27}x^3$ express each of the following in the form kx^n, where k and n are constants.

a) $y^{\frac{1}{3}}$

b) $\frac{1}{9}y^{-2}$

a) $y^{\frac{1}{3}} = \left(\frac{1}{27}x^3\right)^{\frac{1}{3}}$

$= \left(\frac{1}{27}\right)^{\frac{1}{3}} (x^3)^{\frac{1}{3}}$

$= \sqrt[3]{\frac{1}{27}}\, x^{3 \times \frac{1}{3}}$

$= \frac{1}{3}x^1 \quad = \frac{1}{3}x$

b) $\frac{1}{9}y^{-2} = \frac{1}{9} \left(\frac{1}{27}x^3\right)^{-2}$

$= \frac{1}{9} \left(\frac{1}{27}\right)^{-2} (x^3)^{-2}$

$= \frac{1}{9} \left(\frac{1}{\frac{1}{27}}\right)^2 x^{-6}$

$= \frac{1}{9} (27)^2 x^{-6}$

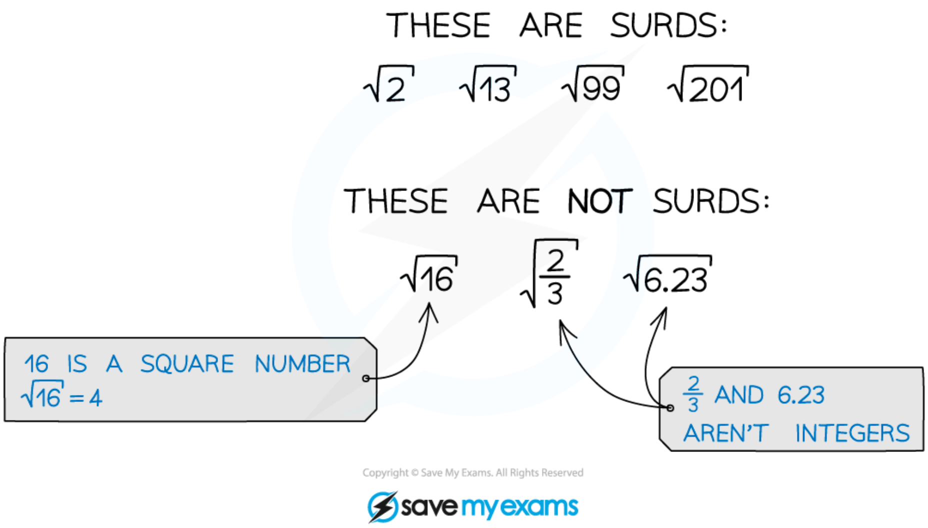
$= 81x^{-6}$

$= \frac{81}{x^6}$

Manipulating surds

What are surds?

- If n is a positive integer and is not a square number, then $\sqrt{n}$ (or any multiple of $\sqrt{n}$) is a **surd**



- Surds are examples of irrational numbers

What do I need to know about manipulating surds?

- There are two basic rules you need to know in order to manipulate and simplify surds:

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

DON'T FORGET THAT THIS SECOND LAW IS THE SAME THING AS

$$\sqrt{a \div b} = \sqrt{a} \div \sqrt{b}$$

ALSO DON'T FORGET HOW THIS WORKS WITH RECIPROCALS:

$$\frac{1}{\sqrt{\frac{a}{b}}} = \frac{1}{\left(\frac{\sqrt{a}}{\sqrt{b}}\right)} = \frac{\sqrt{b}}{\sqrt{a}} = \sqrt{\frac{b}{a}}$$

- When simplifying, look for square factors

LOOK FOR FACTORS THAT ARE SQUARE NUMBERS

THE ANSWERS HERE $\sqrt{8}$ AND $\sqrt{720}$ IN SIMPLEST SURD FORM

$$\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$$

$$\sqrt{720} = \sqrt{144 \times 5} = \sqrt{144} \times \sqrt{5} = 12\sqrt{5}$$

- You can collect like terms with surds like you do with letters in algebra:

$$7\sqrt{3} - 5\sqrt{3} = 2\sqrt{3}$$

$$5\sqrt{2} - 2\sqrt{5} - 8\sqrt{2} + 3\sqrt{5} = -3\sqrt{2} + \sqrt{5}$$

- But be careful not to confuse this with the multiplication and division rules… cannot add or subtract 'under the surd':

$$\sqrt{a} + \sqrt{b} \text{ IS NOT EQUAL TO } \sqrt{a+b}$$

$$\sqrt{a} - \sqrt{b} \text{ IS NOT EQUAL TO } \sqrt{a-b}$$

Your notes

Where can surds be useful?

- Using surds lets you leave answers in exact form
  - e.g. $5\sqrt{2}$ rather than 7.071067812
- Understanding how to simplify surds can help reduce expressions and collect like terms

Examiner Tips and Tricks

- Leaving answers in surd form can be really helpful when you need to carry an exact value through to another part of working
- When simplifying surds, remembering your square numbers is really helpful!

Worked Example

Your notes

Simplify $\sqrt{125} - \sqrt{45}$ giving your answer in the form $a\sqrt{5}$, where a is an integer.

THE FORM OF THE REQUIRED ANSWER LETS YOU KNOW THAT 5 SHOULD BE ONE OF THE FACTORS IN BOTH SURDS HERE!

$$\sqrt{125} - \sqrt{45} = \sqrt{25 \times 5} - \sqrt{9 \times 5}$$

$$= \sqrt{25} \times \sqrt{5} - \sqrt{9} \times \sqrt{5}$$

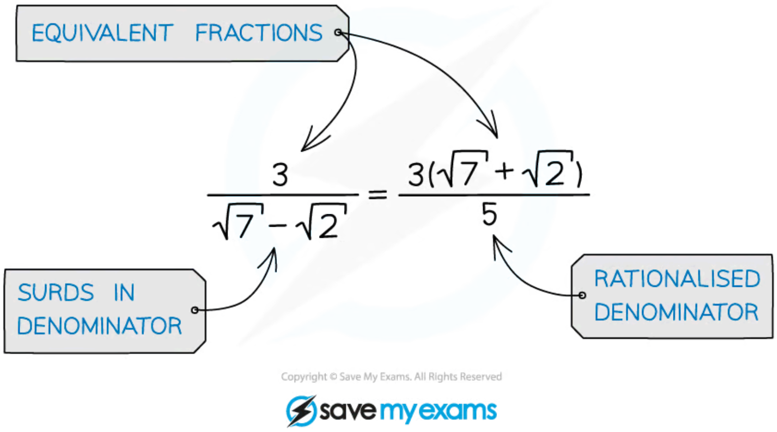
$$= 5\sqrt{5} - 3\sqrt{5}$$

$$= 2\sqrt{5}$$

Rationalising the denominator with surds

How do I rationalise the denominator?

- Rationalising a denominator changes a fraction with surds in its denominator, into an equivalent fraction where the denominator is a rational number (usually an integer) and any surds are in the numerator



- There are three cases you need to know how to deal with when rationalising denominators:

- IF THE FRACTION IS IN THE FORM $\frac{1}{\sqrt{a}}$, MULTIPLY THE NUMERATOR AND DENOMINATOR BY $\sqrt{a}$

- IF THE FRACTION IS IN THE FORM $\frac{1}{\sqrt{a}+\sqrt{b}}$, MULTIPLY THE NUMERATOR AND DENOMINATOR BY $\sqrt{a}-\sqrt{b}$

- IF THE FRACTION IS IN THE FORM $\frac{1}{\sqrt{a}-\sqrt{b}}$, MULTIPLY THE NUMERATOR AND DENOMINATOR BY $\sqrt{a}+\sqrt{b}$

IN THE LAST TWO CASES, EITHER $\sqrt{a}$ OR $\sqrt{b}$ CAN REPLACED BY AN INTEGER.
SIMPLY MODIFY YOUR MULTIPLIER TO MATCH.

FOR EXAMPLE, IF YOUR FRACTION IS IN THE FORM $\frac{1}{a+\sqrt{b}}$ THEN MULTIPLY THE NUMERATOR AND DENOMINATOR BY $a-\sqrt{b}$.

Examiner Tips and Tricks

- If an exam question asks you to give an answer, for example, "in the form $p + q\sqrt{3}$, where p and q are rational numbers", this does NOT mean that p and q have to be integers, or positive!
- Remember: both integers and fractions (both positive and negative) are rational numbers

Your notes

Worked Example

Simplify $\dfrac{4-3\sqrt{3}}{5-\sqrt{3}}$ giving your answer in the form $p+q\sqrt{3}$,

where p and q are rational numbers.

USE FOIL TO MULTIPLY THESE TERMS OUT

$3\sqrt{3} \times \sqrt{3} = 3\sqrt{9} = 9$

$$\frac{4-3\sqrt{3}}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}} = \frac{20+4\sqrt{3}-15\sqrt{3}-9}{25-3}$$

DIFFERENCE OF SQUARES, WITH $(\sqrt{3})^2 = 3$

$$= \frac{11-11\sqrt{3}}{22}$$

$$= \frac{11}{22} - \frac{11\sqrt{3}}{22}$$

$$= \frac{1}{2} - \frac{1}{2}\sqrt{3}$$