Questions

Q1.

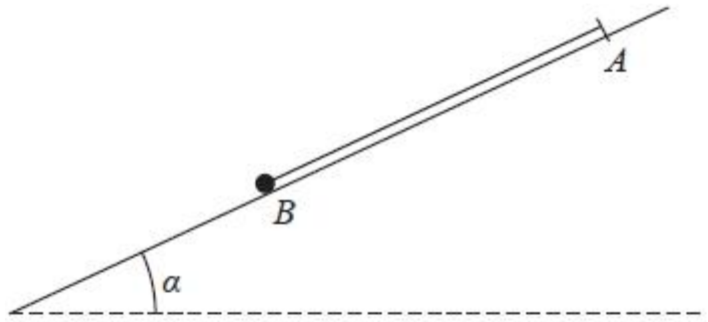

Figure 2

A particle *P* of mass *m* is attached to one end of a light elastic string, of natural length *l* and modulus of elasticity λ. The other end of the string is attached to a fixed point *A* on a smooth plane inclined at angle α to the horizontal, where $\sin \alpha = \frac{3}{5}$. The particle rests in equilibrium on the plane at the point *B* with the string lying along a line of greatest slope of the plane, as shown in Figure 2.

Given that $AB = \frac{6}{5}l$

(a) show that $\lambda = 3mg$

(3)

The particle is pulled down the line of greatest slope to the point *C*, where $BC = \frac{1}{2}l$, and released from rest.

(b) Show that, while the string remains taut, *P* moves with simple harmonic motion about centre *B*.

(4)

(c) Find the greatest magnitude of the acceleration of *P* while the string remains taut.

(2)

The point *D* is the midpoint of *BC*. The time taken by *P* to move directly from *D* to the point where the string becomes slack for the first time is $k\sqrt{\frac{l}{g}}$, where *k* is a constant.

(d) Find, to 2 significant figures, the value of *k*.

(4)

(Total for question = 13 marks)

Q2.

A light elastic spring, of natural length 5*a* and modulus of elasticity 10*mg*, has one end attached to a fixed point *A* on a ceiling. A particle *P* of mass 2*m* is attached to the other end of the spring and *P* hangs freely in equilibrium at the point *O*.

(a) Find the distance *AO*.

(3)

The particle is now pulled vertically downwards a distance $\frac{1}{2}a$ from *O* and released from rest.

(b) Show that *P* moves with simple harmonic motion.

(4)

(c) Find the period of the motion.

(2)

(Total for question = 9 marks)

Q3.

A vertical ladder is fixed to a wall in a harbour. On a particular day the minimum depth of water in the harbour occurs at 0900 hours. The next time the water is at its minimum depth is 2115 hours on the same day. The bottom step of the ladder is 1 m above the lowest level of the water and 9 m below the highest level of the water. The rise and fall of the water level can be modelled as simple harmonic motion and the thickness of the step can be assumed to be negligible.

Find

(a) the speed, in metres per hour, at which the water level is moving when it reaches the bottom step of the ladder,

(7)

(b) the length of time, on this day, between the water reaching the bottom step of the ladder and the ladder being totally out of the water once more.

(4)

(Total for question = 11 marks)

Q4.

One end of a light elastic string, of natural length $5l$ and modulus of elasticity $20mg$, is attached to a fixed point A. A particle P of mass $2m$ is attached to the free end of the string and P hangs freely in equilibrium at the point B.

(a) Find the distance AB.

(3)

The particle is now pulled vertically downwards from B to the point C and released from rest. In the subsequent motion the string does not become slack

(b) Show that P moves with simple harmonic motion with centre B.

(5)

(c) Find the period of this motion.

(2)

The greatest speed of P during this motion is $\frac{1}{5}\sqrt{gl}$

(d) Find the amplitude of this motion.

(3)

The point D is the midpoint of BC and the point E is the highest point reached by P.

(e) Find the time taken by P to move directly from D to E.

(4)

(Total for question = 17 marks)

Q5.

A particle, P, of mass 0.4 kg is attached to the midpoint of a light elastic spring of natural length 0.8 m and modulus of elasticity 20 N. The ends of the spring are attached to the fixed points A and B on a smooth horizontal table, where AB = 1.2 m. Initially P is at rest at the midpoint O of AB where AOB is a straight line. The particle P now receives an impulse of magnitude 2 Ns so that P starts to move directly towards B.

(a) Prove that P moves with simple harmonic motion.

(4)

(b) Write down, in terms of π, the period of the motion.

(1)

(c) Find the amplitude of the motion.

(3)

(d) Find the length of time in each complete oscillation for which AP is greater than 0.5 m.

(5)

(Total for question = 13 marks)

Q6.

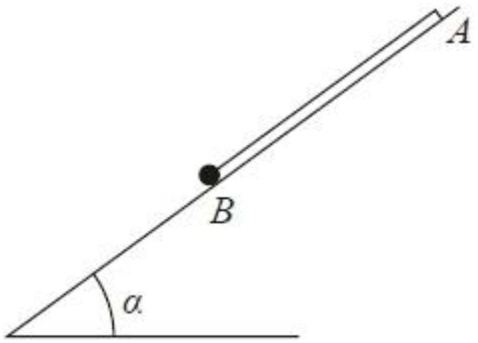

Figure 4

A particle P of mass m is attached to one end of a light elastic string, of natural length l and modulus of elasticity λ. The other end of the string is attached to a fixed point A on a smooth plane which is inclined to the horizontal at angle α, where $\sin \alpha = \frac{3}{5}$. The particle rests in equilibrium on the plane at the point B with the string lying along a line of greatest slope of the plane, as shown in Figure 4.

Given that $AB = \frac{7}{5}l$

(a) show that $\lambda = \frac{3}{2}mg$

(3)

The particle is now pulled down the line of greatest slope to the point C, where $BC = \frac{4}{5}l$, and released from rest.

(b) (i) Show that, while the string remains taut, P moves with simple harmonic motion with centre B.

(ii) Explain briefly why the centre of the motion is at B.

(5)

(c) Find the time taken by P to travel directly from C to B.

(2)

The particle comes to instantaneous rest for the first time at the point D.

(d) Find, in terms of l and g, the time taken by P to travel directly from C to D.

(6)

(Total for question = 16 marks)

Q7.

A particle P of mass 0.5 kg moves in a straight line with simple harmonic motion, completing 4 oscillations per second. The particle comes to instantaneous rest at the fixed points A and B, where $AB = 0.5$ m.

(a) Find the maximum magnitude of the acceleration of P.

(4)

When P is moving at its maximum speed it receives an impulse. The direction of this impulse is opposite to the direction in which P is moving when it receives the impulse. The impulse causes P to reverse its direction of motion but P continues to move with simple harmonic motion. The centre and period of this new simple harmonic motion are the same as the centre and period of the original simple harmonic motion. The amplitude is now half the original amplitude.

(b) Find the magnitude of the impulse.

(5)

(Total for question = 9 marks)

Q8.

A particle P is moving in a straight line with simple harmonic motion about the fixed point O as centre. When P is a distance 0.02m from O, the speed of P is 0.3m s^{-1} and the magnitude of the acceleration of P is 0.5m s^{-2}

(a) Find the period of the motion.

(4)

The amplitude of the motion is a metres.

Find

(b) the value of a,

(3)

(c) the total length of time during each complete oscillation for which P is within $\frac{1}{2}a$ metres of O.

(4)

(Total for question = 11 marks)

Q9.

A particle P is moving in a straight line with simple harmonic motion between two points A and B, where AB is $2a$ metres. The point C lies on the line AB and $AC = \frac{1}{2}a$ metres.

The particle passes through C with speed $\frac{3a\sqrt{3}}{2}\,\mathrm{m\,s^{-1}}$.

(a) Find the period of the motion.

(3)

The maximum magnitude of the acceleration of P is 45 m $\mathrm{s^{-2}}$. Find

(b) the value of a,

(2)

(c) the maximum speed of P.

(2)

The point D lies on AB and P takes a quarter of one period to travel directly from C to D.

(d) Find the distance CD.

(5)

(Total for question = 12 marks)

Q10.

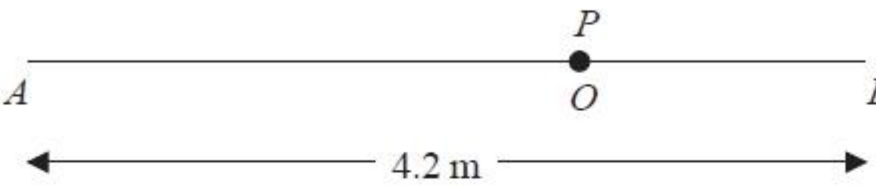

Figure 5

The fixed points *A* and *B* are 4.2 m apart on a smooth horizontal floor. One end of a light elastic spring, of natural length 1.8 m and modulus of elasticity 20 N, is attached to a particle *P* and the other end is attached to *A*. One end of another light elastic spring, of natural length 0.9 m and modulus of elasticity 15 N, is attached to *P* and the other end is attached to *B*. The particle *P* rests in equilibrium at the point *O*, where *AOB* is a straight line, as shown in Figure 5.

(a) Show that *AO* = 2.7 m.

(4)

The particle *P* now receives an impulse acting in the direction *OB* and moves away from *O* towards *B*. In the subsequent motion *P* does not reach *B*.

(b) Show that *P* moves with simple harmonic motion about centre *O*.

(5)

The mass of *P* is 10 kg and the magnitude of the impulse is *J* Ns.

Given that *P* first comes to instantaneous rest at the point *C* where *AC* = 2.9 m,

(c) (i) find the value of *J*,

(ii) find the time taken by *P* to travel a total distance of 0.5 m from when it first leaves *O*.

(8)

(Total for question = 17 marks)

Examiner's Report

Q1.

As always with SHM questions, this discriminated well.
Part (a) was almost always successful, and mostly without any fudging.
The proof in (b) was more often correct than in many past papers but the proofs often felt more memorised than understood. It is unconvincing, although correct, to see a − sign added to one side of the equation as an afterthought.
The memorised standard solution involving a completely undefined and previously unmentioned e was seen a number of times. They need to know that this is worthless. Similarly, those who choose to make the working simpler by calling their extra extension *x*/ need to know that this leads almost inevitably to a dimensionally inconsistent equation. Correct use of $\ddot{x}$ instead of *a* is now much more common than it once was but many still forget to state that they have proved SHM.
Part (c) was very well done, mostly using $\omega^2 a$ but occasionally by N2L. Some wrong formulae were also seen (ωa and ωa^2) and the amplitude was sometimes thought to be either $\frac{l}{5}$ or $\frac{6l}{5}$.
Quite a few candidates didn't attempt (d) at all and, although most who tried had a reasonable idea how to go about it, fully correct solutions were in the minority. It was often assumed that the time from the centre to the natural length was *T*/4 and there was the usual confusion between sin/cos, positive/negative lengths and whether to add or subtract their two times. However, failure to use radians was rare. Finding the value of *k* needed to express the answer in the required form proved a final challenge, even in solutions which had been correct up to that point.

Q2.

Part (a) was generally answered very well. Almost all candidates used Hooke's law correctly and hardly any forgot to add on 5*a* at the end. The proof of SHM was less successful, mainly because candidates did not use a mass of 2*m* with the acceleration term. In general they knew to use Newton's second law and most correctly formed Hooke's law with a variable extension, mostly with the correct centre. Some candidates used *a* rather than $\ddot{x}$ for the acceleration and consequently lost marks as *a* is non-directional. The method mark in (c) was usually gained, although many lost the final mark having used *m* in (b).

Q3.

This question required a careful analysis of the context in which the problem was set with a good diagram not seen nearly as often as it should have been. In part (a), the initial approach required was almost universally realised with an attempt to find ω via the period of the oscillation. However it was surprising to see many mixing up minutes with the decimal of an hour using $T = 12.15$ rather than $T = 12.25$. There were variations on this theme with $T = 1215$ occasionally seen. The next step required the amplitude and value of the displacement at the bottom of the ladder from the centre of motion and it was here that those with a good diagram were least likely to make errors and use the correct values ($a = 5$ and $x = 4$). Common errors were to take the amplitude as 10 or 4 and use *x* as 9 or 1 and combinations of these. A few candidates successfully completed part (a) by using $x = a\sin\omega t$ and then differentiating to find $v = a\omega\cos\omega t$. Those few who worked in seconds rather than hours did need to convert their speed into m/hr at the end; some did and others did not.

In part (b) there were many correct approaches in finding the time when $x = \pm 4$ but many then found it difficult to form a complete solution. Other than the two approaches in the mark scheme, another succinct method seen was to find the two appropriate solutions to $a \sin \omega t = \pm 4$ or $a \cos \omega t = \pm 4$ and subtract.

Q4.

A few gave themselves difficulties throughout by quoting Hooke's law as $T = kx$ (as used in physics?) and subsequently forgetting that their $k = \lambda/l$. They would do better to accept that there are different versions for the two subjects.

Part (a) was almost always correct although a few forgot to add the extension to 5 l or added it only to l.

Unfortunately, most of the potentially perfect proofs in (b) had $m\ddot{x}$ instead of $2m\ddot{x}$ in the Newton's second law equation and so lost some accuracy marks both here and in later parts of the question. As always, there were many failed attempts to prove SHM. Correct use of $\ddot{x}$ for the acceleration has become much more common but significant numbers still lost marks by using a or forgetting to state that they have proved SHM. Some very brief proofs seemed to have been memorised rather than understood and these often did not make sense; either the $2mg$ term did not genuinely cancel out of the equation or they used undefined terms. No credit was given for generalised proofs using e as the equilibrium extension unless $e = l/2$ had been clearly stated either here or in (a).

Parts (c) and (d) presented no problems and many benefited from the follow through marks.
Part (e) was done very well by stronger students but was left out completely by a significant minority. It was well known that a solution could be found using $x = a\cos\omega t$ or $a\sin\omega t$ but there was much confusion over whether this should then be combined with $T/4$ or $T/2$ using addition or subtraction. Some attempts assumed that the time taken from the mid-amplitude point must be $T/8$ and others tried to involve uniform acceleration equations.

Q5.

In part (a), proving SHM caused more difficulties than in recent papers, largely due to confusion over the lengths involved, with many candidates using a natural length of 0.8, rather than 0.4, although most did manage to find correct extensions. Almost all candidates used the required $\ddot{x}$ notation and it was rare for candidates to forget to give a concluding statement.

Parts (b) and (c) both proved straightforward, with all marks available as follow through, providing that an SHM equation had been reached in (a). Part (d) caused significantly more difficulty. Many candidates did not realise that they needed to use a displacement of ±0.1 and this cost them all of the marks. Sin and Cos were used equally, but candidates often struggled to work out the valid approach to complete the problem, with uncertainty over whether they should be using T or T/2, or whether they should be doubling their time.

Q6.

Parts (a), (b) and (c) were mostly answered well, but fully correct answers to part (d) were uncommon indicating that most candidates had trouble with the final part.

Part (a) caused very few problems, with most candidates reaching the given answer with sufficient steps to score all of the marks.

Part (b) caused more difficulties, with many candidates dropping marks. It was still common for "a" to be used for the acceleration rather than $\ddot{x}$ which scored 2 out of the 4 marks. Most did attempt an equation of motion with both weight and tension, although the tension was not always variable. There was the occasional sign error and others did not clearly state "therefore SHM" in conclusion which was required for the full marks.

Almost all candidates gained the mark for a correct answer to (b) part (ii).

Part (c) was often answered correctly by finding a quarter of the period, although some successfully used $x = a\cos\omega t$ or $x = a\sin\omega t$ with $x = \frac{2l}{5}$ or $x = \frac{2l}{5}$ respectively. A small number of candidates did not realise that they needed only a quarter period.

Part (d) was poorly answered, on the whole, and very few fully correct answers were seen. In many cases a clear diagram would have helped, as candidates seemed to be unclear as to where the string went slack and often assumed SHM or SUVAT when it was not appropriate. An attempt at an energy equation and consideration of E.P.E. was common, but was often used incorrectly or there was often a term missing. A few candidates did use it successfully to find the speed when the string went slack. Candidates who were successful usually found the time under SHM by first finding the time from the centre of the oscillation and then adding a quarter of the period. A significant number of candidates were able to find the speed when the string goes slack by using $v^2 = w^2(a^2 - x^2)$ even if they were unable to find the times correctly. It was not uncommon for candidates to incorrectly assume the acceleration as "g" rather than "3/5g" in use of suvat to find the time up the slope after the string had become slack.

Q7.

Part (a) was not answered as well as should have been expected. Many were not able to translate the four oscillations per second into the correct period and the failure to find led to all marks being lost in (a). A disappointing number also used an incorrect amplitude.

Part (b) was generally better answered (at least for the first few marks), with most students correctly finding the new velocity, following through their amplitude and ω and most realising what was required to find an impulse. However, even students who got (a) correct often made a sign error in the impulse-momentum equation, leading to an answer of $\frac{\pi}{2}$

In both parts, nearly all gave positive answers in order to obtain the magnitude.

Q8.

Part (a) A few students lost a mark by not dealing with the negative sign, when the modulus was required. Some did not use the correct equation to find ω. Use of $v = r\omega$ was seen on a number of occasions to obtain ω = 15. Almost all students could find the period by using $T = \frac{2\pi}{\omega}$.

Part (b) A correct equation was seen in most responses with very few errors. Most did this correctly even if ω was incorrect from part (a). A handful of students used $v^2 = \omega^2\left(a^2 - x^2\right)$ and used the period from (a) instead of ω.

Part (c) This was the most demanding part of the question and students used a variety of methods or in some cases left it blank. The most straightforward was the use of $\sin\omega t$ as on the main scheme although some multiplied by 2 instead of 4 to obtain the final answer. Those who used $\cos\omega t$ made more errors in completing to the final answer. The reference circle was also used successfully by a small number of students.

Q9.

The majority of candidates were successful with parts (a) (b) and (c).

In part (a) almost all used $v^2 = \omega^2 (a^2 - x^2)$ to calculate ω ; only a handful used $x = a\sin \omega t$ and $v = a\omega \cos\omega t$. It was not uncommon for sloppy working to lead to an incorrect value for ω , although full marks were usually obtained in parts (b) and (c) due to the follow through. Incorrect use of "2a" for the amplitude was occasionally seen which scored zero in (a).

Part (d) caused the most problems in this question. Although a significant number did arrive at the correct answer most scored a maximum of the first 2 marks because they could calculate a relevant time but did not know what to do with this, with some getting confused over the direction of travel. Those who did provide a correct solution chose a variety of routes, with the majority deciding to use the sin ωt form for the displacement, despite the fact that the motion started at the end of the oscillation. Given that the symmetries of SHM allowed for various valid approaches to the correct solution (as well as some invalid methods that would lead to the correct answer), it would have been helpful for candidates to explain their working more clearly. Frustratingly, a significant number of candidates gave fully correct solutions in terms of a, and so the final mark was lost for not substituting a = 5.

A handful of candidates successfully used the reference circle in a neat solution to the problem.

Q10.

Part (a) was answered fairly well with most students using one of the two main methods in the scheme and showing sufficient working to convincingly arrive at the given result. Some chose to measure their distances from a different point and the calculations to arrive at 2.7 were not always labelled as clearly as is desirable in a "show that" question.

In part (b) it was clear that most students now know what is expected when proving SHM. The majority used the required notation and measured their distances from the equilibrium position. Those that did not, either just dropped the unwanted terms, or left them in, but still claimed SHM. It was rare for students to arrive at the correct form and then fail to state SHM. One fairly common mistake was to not include m, or to drop it part way through, but the most common source of dropped marks was to make a mistake with the fractions and signs, thus obtaining an incorrect expression for ω.

Many students answered part (c) very well. Most had arrived at an SHM equation and from that found their ω. The amplitude was usually correct and the correct method was used to arrive at the impulse. A few students answered (i) without reference to SHM, by considering the energy. This was usually correct, although some students missed some EPE terms, or found differences rather than totals. Almost all students formed a trigonometric equation to start (ii), and, if they used sine, generally went on to a method to find the total time. However those who used cosine rarely went onto a valid method for the total time, nearly all simply adding half the period.

Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$T = mg\sin\alpha = \frac{3}{5}mg$	B1
	$T = \frac{\lambda \times \frac{1}{5}l}{l} = \frac{3}{5}mg$	M1
	$\lambda = 3mg$ *	A1 cso (3)
(b)	$mg\sin\alpha - T = m\ddot{x}$	
	$\frac{3}{5}mg - \frac{3mg\left(\frac{1}{5}l + x\right)}{l} = m\ddot{x}$	M1A1
	$\ddot{x} = -\frac{3g}{l}x$	dM1
	$\therefore$ SHM	A1 (4)
(c)	$\lvert\ddot{x}\rvert_{max} = a\omega^2 = \frac{1}{2}l \times \frac{3g}{l} = \frac{3g}{2}$	M1A1ft (2)
(d)	Time from D to B: $\frac{1}{4}l = \frac{1}{2}l\sin\sqrt{\frac{3g}{l}}t_1$	M1
	$t_1 = \frac{\pi}{6}\sqrt{\frac{l}{3g}}$	
	Time from B to nat. length: $\frac{1}{5}l = \frac{1}{2}l\sin\sqrt{\frac{3g}{l}}t_2$	M1
	$t_2 = \sqrt{\frac{l}{3g}}\sin^{-1}\frac{2}{5}$	
	Total time $= \left(\frac{\pi}{6} + \sin^{-1}\frac{2}{5}\right)\sqrt{\frac{l}{3g}},\ = 0.54\sqrt{\frac{l}{g}}$ $k = 0.54$	ddM1,A1cao (4) [13]

(a)B1	$T = \frac{3}{5}mg$ shown explicitly or used
M1	Attempt Hooke's law
A1cso	Obtain the given result
(b)M1	Equation of motion along the plane inc T in terms of l and x. Allow with acceleration as a
A1	Correct equation, acceleration can still be a. Can still have λ
dM1	Re-arrange to the required form. Acceleration must be $\ddot{x}$ oe now
A1	Correct result as shown or $\ddot{x} = -\frac{\lambda}{ml}x$ and SHM stated. If sub made for g, answer must be 2 or 3 sf. But may be possible to award earlier.
(c)M1	Use $\lvert\ddot{x}\rvert_{max} = a\omega^2$
A1ft	Correct answer follow through their ω Must be positive 14.7 or 15 allowed
(d)M1	Find time from D to B or from C to D
M1	Find time from B to natural length or from C to natural length
ddM1	Add or subtract (as appropriate) the two times obtained
A1	Correct result for k Must be 2 sf but need not be shown explicitly.

Q2.

Question Number	Scheme	Marks
(a)	$T = 2mg = \dfrac{10mgx}{5a}$	M1
	$x = a$	A1
	$AO = 5a + a = 6a$	A1ft (3)
(b)	$2mg - T = 2m\ddot{x}$ or $2ma$	M1
	$2mg - \dfrac{10mg(a+x)}{5a} = 2m\ddot{x}$	M1A1
	$\ddot{x} = -\dfrac{gx}{a}$ $\therefore$ SHM	A1cso (4)
(c)	Period $= \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{a}{g}}$ oe	M1A1 (2)
		[9]

(a)M1 Resolve vertically using HL for the tension
A1 Obtaining $x = a$
A1ft Add their extension to $5a$

(b)M1 Forming an equation of motion, acceleration $\ddot{x}$ or a
M1 Use HL to express the tension in terms of x, (centre not nec at the correct point), acceleration $\ddot{x}$ or a.
A1 Fully correct equation acceleration to be $\ddot{x}$ now
A1cso Simplify to $\ddot{x} = -\dfrac{gx}{a}$ **and** state SHM
All marks available for an algebraic solution (ie no sub made for any/all of mass, λ, l.)

(c)M1 Using period $= \dfrac{2\pi}{\omega}$ with their ω from an "SHM" equation. $\left(\text{ie } \ddot{x} \text{ or } a = \pm\omega^2 x\right)$
A1cso Correct period, any equivalent form but not fractions within fractions. ω must have come from a correct equation and substitutions for mass, λ, l made.

Q3.

Question Number	Scheme	Marks
(a)	Period $=12.25$ hours	B1
	$\frac{2\pi}{\omega}=12.25$	
	$\omega=\frac{2\pi}{12.25}$ oe eg $\omega=\frac{8\pi}{49}$	M1A1ft
	Amplitude $=5$ m	B1
	$v^2=\omega^2\left(a^2-x^2\right)$	
	$x=4 \Rightarrow v^2=\left(\frac{2\pi}{12.25}\right)^2\left(5^2-4^2\right)$	M1A1
	$v=\left(\frac{2\pi}{12.25}\right)\times 3=1.53873...=1.5\text{ m h}^{-1}$ (1.5 or better or $\frac{24\pi}{49}$)	A1cao (7)
(b)	$x=4 \quad 4=5\cos\omega t$	
	$t=\frac{12.25}{2\pi}\cos^{-1}0.8$ $(=1.2546...)$	M1A1
	Time ladder in the water $=12.25-2\times\frac{12.25}{2\pi}\cos^{-1}0.8$	DM1
	$=9.7407...=9.7$ hours (9.7 or better) (9hr 44 min or 35 066 875 seconds)	A1 cao (4) [11]

(a)B1 Correct period for the SHM (Allow 44100 s)

M1 Using period $=\frac{2\pi}{\omega}$ to obtain an expression for ω or any other complete method

A1ft A correct expression for ω follow through their period, inc with units changed.

B1 Correct amplitude seen explicitly or used

M1 Using $v^2=\omega^2\left(a^2-x^2\right)$ with their ω, amplitude and $x=(\text{their amp }-1)$

A1 Correct expression for v^2

A1cao Correct value of v, 2 sf or better Units must be m h^{-1}.

(b)M1 Using $x=a\cos\omega t$ or $a\sin\omega t$ with $x=\pm(\text{their amp }-1)$, their ω and amplitude

A1 Finding a correct time, either from the end point to reaching the bottom step or from the bottom step to the centre of the oscillation. (in hours or seconds)

DM1 Using their time to obtain an expression for the required time

A1cao Time = 9.7 hrs or better

Sine used: Time = $2\times\frac{\text{period}}{4}+2\times\text{time found}$

Q4.

Question Number	Scheme	Marks	
(a)	$2mg = \frac{20mge}{5l}$	M1	
	$e = \frac{l}{2}$	A1	
	$AB = 5.5l$ oe	A1ft	(3)
(b)	$2mg - T = 2m\ddot{x}$	M1	
	$2mg - \frac{20mg\left(0.5l + x\right)}{5l} = 2m\ddot{x}$	dM1A1A1	
	$\ddot{x} = -\frac{2g}{l}x \therefore$ SHM	A1 cso	(5)
(c)	$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{2g}} = \pi\sqrt{\frac{2l}{g}}$ oe	M1A1ft	(2)
(d)	$a\omega = \frac{1}{5}\sqrt{gl} = a\sqrt{\frac{2g}{l}}$	M1A1ft	
	$a = \frac{l}{5\sqrt{2}}$ oe	A1	(3)

(e)	$BD = \frac{1}{2}a$		
	$-\frac{1}{2}a = a\cos\omega t$	M1	
	$\cos t\sqrt{\frac{2g}{l}} = -\frac{1}{2}$	A1	
	$t = \sqrt{\frac{l}{2g}}\cos^{-1}\left(-\frac{1}{2}\right)$	dM1	
	$t = \sqrt{\frac{l}{2g}} \times \frac{2\pi}{3} = \frac{2\pi}{3}\sqrt{\frac{l}{2g}}$	A1cso	(4)
			[17]

Question Number	Scheme	Marks
(a)		
M1	Resolve vertically including using Hooke' law to find the tension. Formula for HL to be correct.	
A1	Solve their equation to find the equilibrium extension	
A1ft	Add $5l$ to their equilibrium extension to obtain AB	
ALT	Use of extension $(AB-l)$	
M1	Resolve vertically inc HL with extension $(AB-l)$ Formula for HL to be correct.	
A1	Fully correct equation	
A1	$AB=5.5l$	
(b)		
M1	Resolve vertically (at a general point). HL not needed for this mark; acceleration can be $\ddot{x}$ or a. Mass $2m$ or m	
dM1	Use HL with extension $(0.5l+x)$ or $(e+x)$ (provided e is clearly defined to be the equilibrium extension by their work in (a)). Dependent on the previous M mark.	
A1	Correct difference of forces either way round; acceleration can be $\ddot{x}$ or a.	
A1	Fully correct equation with acceleration $\ddot{x}$.	
A1cso	Correct equation starting $\ddot{x}=\dots$ and conclusion.	
(c)		
M1	Use Period $=\frac{2\pi}{\omega}$ with their ω from an "SHM" equation (ie $\ddot{x}=\pm\omega^2x$ or $a=\pm\omega^2x$)	
A1ft	Correct period follow through their ω	
(d)		
M1	Use $v=a\omega$ or $v^2=\omega^2(a^2-x^2)$ with $x=0$ later, their ω	
A1ft	Correct equation, follow through their ω	
A1cao	Correct amplitude	
(e)		
M1	Use $x=a\cos\omega t$ or $x=a\sin\omega t$ with their ω from an "SHM" equation (ie $\ddot{x}=\pm\omega^2x$ or $a=\pm\omega^2x$) and $x=\pm\frac{1}{2}a$ Equivalents in terms of l acceptable.	
A1	Correct equation (limited ft) Allow their amplitude in terms of l (if used) but ω to be correct.	
dM1	Solve to $t=\dots$, where t is the required time. Must be in radians. This must be a complete (correct) method, so if eg $x=a\sin\omega t$ and/or $x=\frac{1}{2}a$ used there must be work included to obtain the required answer from their equation.	
A1cso	Correct time, as shown or equivalent. (cso applies to part (e) work only.)	

Q5.

Question Number	Scheme	Marks
(a)	$\dfrac{20(0.2-x)}{0.4}-\dfrac{20(0.2+x)}{0.4}=0.4\ddot{x}$	M1A1
	$-100x=0.4\ddot{x}$	
	$\ddot{x}=-250x \quad \therefore$ SHM	dM1A1cso (4)
(b)	Period $=\dfrac{2\pi}{\sqrt{250}}$ oe	B1ft (1)
(c)	$v_{\max}=\dfrac{2}{0.4}=5\text{ m s}^{-1}$	B1
	$a\omega=5 \quad a=\dfrac{5}{\sqrt{250}}=\dfrac{1}{\sqrt{10}} \ (=0.3162...)\text{ m}$	M1A1ft (3)
(d)	$x=a\cos\omega t \quad 0.1=\dfrac{1}{\sqrt{10}}\cos\sqrt{250}t$ or $x=a\sin\omega t \quad 0.1=\dfrac{1}{\sqrt{10}}\sin\sqrt{250}t$	M1A1ft
	$t=\dfrac{1}{\sqrt{250}}\cos^{-1}\left(0.1\times\sqrt{10}\right)$ or $t=\dfrac{1}{\sqrt{250}}\sin^{-1}\left(0.1\times\sqrt{10}\right)$	A1
	Time for which $AP>0.5$	
	$=\dfrac{2\pi}{\sqrt{250}}-2\dfrac{1}{\sqrt{250}}\cos^{-1}\left(0.1\times\sqrt{10}\right)$ or $=\dfrac{\pi}{\sqrt{250}}+2\dfrac{1}{\sqrt{250}}\sin^{-1}\left(0.1\times\sqrt{10}\right)$	dM1
	$=0.2393...\text{s}$	A1cso (5) [13]

(a)	
M1	Attempt an equation of motion using a difference of 2 tensions obtained from Hooke's law and having different variable extensions. $\ddot{x}$ or a allowed. Can be in algebraic form.
A1	Correct equation. $\ddot{x}$ or a allowed but if a used the signs must indicate it is in the same direction as $\ddot{x}$ Can be in algebraic form.
dM1	Rearrange their equation to the required form $\ddot{x}=-\omega^2x$. Must be $\ddot{x}$. They cannot just lose terms to get to the required form.
A1ft	Correct equation, can be numerical as shown or algebraic $\left(e.g.\quad \ddot{x}=-\dfrac{4\lambda}{ml}x\right)$, **and** state conclusion. If algebraic this must include stating that their "ω^2" is positive.
(b)	
B1ft	Correct period (numerical) as shown or equivalent. Follow through their ω from $\ddot{x}$ or $a=\pm\omega^2x$ (0.40 or better)
(c)B1	Correct max speed, seen explicitly or used
M1	Using $v_{\max}=a\omega$ to obtain a value for a
A1ft	Correct value, exact or decimal (0.32 or better)
(d)	
M1	Use $x=a\cos\omega t$ or $x=a\sin\omega t$ with $x=\pm0.1$, their ω, a.
A1ft	Correct equation, follow through their ω, a
A1	Correct expression for time from their choice of equation (if only decimal seen, award for 2sf or better 0.078997.. for cos,0.020349… for sin) Must have come from fully correct work in (a)
dM1	Complete correct method to obtain the required time. Dependent on previous M mark.
A1cso	Correct final answer. 2s.f. or better. Must have come from fully correct work in (a) and (d), although they might not have used $\ddot{x}$

Q6.

(a)	$T = \dfrac{\lambda \frac{2}{5} l}{l}$	
	$mg\sin\alpha = \frac{3}{5}mg = \frac{2}{5}\lambda$	M1A1
	$\lambda = \frac{3}{2}mg$ *	A1 (3)
(b)(i)	$\frac{3}{5}mg - T = m\ddot{x}$	M1
	$\frac{3}{5}mg - \frac{1}{l}\left(\frac{2}{5}l + x\right) \times \frac{3}{2}mg = m\ddot{x}$	dM1A1
	$\ddot{x} = -\frac{3g}{2l}x \quad \therefore$ SHM	A1
(ii)	B is the equilibrium position or $x = 0$ at B	B1 (5)
(c)	Time $= \frac{1}{4} \times \frac{2\pi}{\omega} = \frac{\pi}{2}\sqrt{\frac{2l}{3g}}$ oe	M1A1ft (2)
(d)	$-\frac{2}{5}l = \frac{4}{5}l\cos\omega t = \frac{4}{5}l\cos\sqrt{\frac{3g}{2l}}t$	M1
	$\frac{2}{3}\pi = \sqrt{\frac{3g}{2l}}t \Rightarrow t = \frac{2}{3}\pi\sqrt{\frac{2l}{3g}}$ oe	A1ft
	At nat length $v^2 = \omega^2\left(a^2 - x^2\right) = \frac{3g}{2l}\left(\frac{16}{25}l^2 - \frac{4}{25}l^2\right) = \frac{36gl}{50}$	M1A1ft
	Once string is slack accel is $\frac{3}{5}g$ down the plane.	
	$0 = \frac{6}{5}\sqrt{\frac{gl}{2}} - \frac{3}{5}gt'$	M1
	Total time $= \frac{2}{3}\pi\sqrt{\frac{2l}{3g}} + 2\sqrt{\frac{l}{2g}}$ oe	A1 cso (6)
		[16]

(a)	
M1	Resolve down the slope. Weight must be resolved and Hooke's Law attempted.
A1	Correct equation, including correct use of (unsimplified) Hooke's Law. Trig need not be substituted.
A1*	Given result obtained convincingly. We must see $\frac{2l}{5}$ and trig must be substituted before final result given.
(b) (i)	
M1	Equation of motion along slope, in T. Allow with *ma*.
dM1	Hooke's Law applied, with a variable extension. Allow with *ma*. Dependent on previous M mark.
A1	Correct unsimplified equation. Must now be $m\ddot{x}$
A1	$\ddot{x} = -\frac{3g}{2l}x \quad \therefore$ SHM Must have conclusion.
(ii)	
B1	B is the equilibrium position **or** $x = 0$ at B.
(c)	
M1	Complete method to find a quarter period for their ω. Must have come from an equation of the form $\ddot{x}$ or $a = \pm\omega^2 x$, but ignore dimensions of their ω.
A1ft	Correct time (ft their ω)
(d)	
M1	Complete method to find the time to string becoming slack. Must be using $x = \pm\frac{2}{5}l, a = \frac{4}{5}l$
	If using sin $\frac{2}{5}l = \frac{4}{5}l\sin\omega t_1 \rightarrow t_1 = \frac{\pi}{6}\sqrt{\frac{2l}{3g}} \quad \therefore t = \frac{\pi}{2}\sqrt{\frac{2l}{3g}} + \frac{\pi}{6}\sqrt{\frac{2l}{3g}}$
A1ft	Correct expression for time until string becomes slack. Follow through their ω.
M1	Complete method to find speed when string becomes slack
	Energy alternative $\frac{1}{2}mv^2 = \frac{1}{2}\times\frac{3mg}{2}\times\left(\frac{6l}{5}\right)^2 - mg\times\frac{6l}{5}\times\frac{3}{5}$
A1ft	Correct v or v^2 Follow through their ω.
M1	Complete method to find **total** time for P to come to rest.
A1 cso	Correct time, in any form. **Must have come from correct working in whole question.**

Q7.

Question Number	Scheme	Marks
(a)	$\dfrac{2\pi}{\omega} = 0.25, \quad \omega = 8\pi$	M1, A1
	max accel $= a\omega^2 = 0.25 \times 64\pi^2 \ = 16\pi^2 \quad (158)\ \text{m s}^{-2}$	dM1A1 (4)
(b)	$\omega_1 = 8\pi, \ \ a_1 = 0.125$ accept $\dfrac{0.25}{2}$	B1ft
	Max speed for the new motion $= 8\pi \times 0.125\ \text{m s}^{-1}\ (= \pi)$	M1(either)
	Max speed for the original motion $= 8\pi \times 0.25\ \text{m s}^{-1}\ (= 2\pi)$	A1ft(both)
	$\lvert I \rvert = 0.5(8\pi \times 0.25 + 8\pi \times 0.125) = 1.5\pi\ \text{Ns}\ \ (= 4.7123\ldots$ accept 4.7 or better$)$	dM1A1 (5)
		[9]

(a)

M1 Use $\dfrac{2\pi}{\omega} = \dfrac{1}{4}$ or 4 to obtain a value for ω

A1 Correct value of ω exact or decimal

dM1 Use $a\omega^2$ with their ω and $a = 0.25$ to obtain the max magnitude of the acceleration. Depends on the first M mark.

A1 Correct max magnitude, exact or 158 or better

(b)

B1ft New ω and amp, follow through their original ω and amp

M1 One speed needed for this mark (but 2 for the complete problem). Award M1 for either, using their ω and amp. May obtain v or v^2.

A1ft Award A1 if both speeds are correct, follow through their ω and amp.

dM1 Use impulse = change of momentum, with their speeds (neither = 0), to obtain the magnitude of the impulse. Depends on the first M mark of this part. Allow if momenta are subtracted as long as no incorrect formula seen.

A1 Correct magnitude, exact or min 2 sf. Must be positive.

Q8.

Question Number	Scheme	Marks
(a)	$\ddot{x} = -\omega^2 x$	
	$\omega^2 = \frac{0.5}{0.02}, \Rightarrow \omega = 5$	M1,A1
	Period $= \frac{2\pi}{5}$ s Accept 1.3, 1.26 or better	M1A1 (4)
(b)	$v^2 = \omega^2\left(a^2 - x^2\right)$	
	$0.3^2 = \frac{0.5}{0.02}\left(a^2 - 0.02^2\right)$	M1A1ft
	$a = 0.06324...$ Accept 0.063 or better $\left(\text{exact is } \frac{\sqrt{10}}{50} \text{ oe}\right)$	A1 (3)
(c)	$\frac{1}{2}a = a\sin\omega t$	
	$t = \frac{1}{\omega}\sin^{-1}\left(\frac{1}{2}\right), = \frac{\pi}{30} \quad (= 0.104...)$	M1,A1
	Total time $= 4\times\frac{\pi}{30}, = \frac{2\pi}{15}$ s Accept 0.42, 0.419 or better	dM1,A1 (4)
		[11]

(a)	
M1	Use $\ddot{x}$ or $a = -\omega^2 x$ with $\ddot{x} = 0.5$ and $x = 0.02$ to form an equation for ω^2. Allow equation with or without a minus sign.
A1	Obtain correct value for ω or ω^2 from correct working ie no sign errors ($\omega^2 = -25$, $\omega = 5$ scores M1A0)
M1	Use period $\frac{2\pi}{\omega}$ with their ω
A1	Correct period, exact or decimal (can score M1A0M1A1 in (a))
(b)	
M1	Use $v^2 = \omega^2\left(a^2 - x^2\right)$ with $v = 0.3, x = 0.02$ and their ω
A1ft	Correct equation, follow through their ω
A1	Correct value of a, 2 sf or better (inc exact)
(c)	
M1	Use $x = a\sin\omega t$ or $x = a\cos\omega t$ with their ω and $x = \pm\frac{1}{2}a$. May use their value for a or use a
A1	Obtain the correct time from the centre or end of oscillation (depends on the equation used)
dM1	Correct method to complete to obtain the required time. Depends on the first M mark of (c). If $x = a\cos\omega t$ is used this mark needs "period $-4\times$ time found" (or any equivalent to that)
A1	Correct total time, exact or decimal

Q9.

Question Number	Scheme	Marks
(a)	$v^2 = \omega^2\left(a^2 - \frac{a^2}{4}\right) = \frac{3a^2\omega^2}{4}$	
	$\frac{27a^2}{4} = \frac{3a^2\omega^2}{4}$	M1
	$\omega = 3$	A1
	Period $= \frac{2\pi}{3}$	A1ft (3)
(b)	Max mag of accel $= a\omega^2$	
	$45 = 9a, \quad a = 5$	M1,A1ft (2)
(c)	$x = a\sin\omega t \quad \dot{x} = a\omega\cos\omega t$ (or $x = a\cos\omega t \quad \dot{x} = -a\omega\sin\omega t$)	
	$\dot{x}_{max} = 5\times 3 = 15$ (m s^{-1})	M1A1ft (2)
	OR $v_{max} = a\omega = 5\times 3 = 15$ (m s^{-1}) M1A1ft (2)	
(d)	Time A to C $\quad \frac{1}{2}a = a\cos\omega t \Rightarrow \frac{1}{2} = \cos 3t$	
	$t_{AC} = \frac{1}{3}\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{9}$ (0.3490...)	M1A1
	Time A to D $\quad \frac{\pi}{9} + \frac{2\pi}{12} = \frac{5\pi}{18}$	
	$x_D = 5\cos\left(3\times\frac{5\pi}{18}\right) = -\frac{5\sqrt{3}}{2}$ $(= -4.330)$	M1A1
	Distance CD $\frac{5}{2} + \frac{5\sqrt{3}}{2} = \frac{5}{2}\left(1+\sqrt{3}\right)$ or 6.8 (m) (or better)	A1ft (5)
		[12]

Question Number	Scheme	Marks
(a)		
M1	Use of $v^2 = \omega^2\left(a^2 - x^2\right)$ with $v = \frac{3a\sqrt{3}}{2}$, $x = \frac{1}{2}a$, amp $= a$ (or any other complete method)	
A1	Correct value for ω	
A1ft	Correct period, follow through their ω	
(b)		
M1	Use max mag of accel $= a\omega^2$ with their ω	
A1ft	$a = 5$	
(c)		
M1	Use either method shown with their values of a and ω to obtain a value for the max speed	
A1ft	$v_{\max} = 15\ \left(\mathrm{m\,s^{-1}}\right)$	
(d)		
M1	Attempt time A to C with their value of ω and $x = \frac{1}{2}a$ or half their amp. Must reach a value for t using radians	
A1	Correct time, exact or min 3 sf (no penalty for using an incorrect amp) The above 2 marks can be awarded for a time even if no indication of which time the are finding (ie not stated to be time from end to C or centre to C. Following marks can only be awarded if work is consistent with their work for these 2 marks.	
M1	Add ¼ period and use this time to obtain a value for x at D using their value for a or just a	
A1	Correct value of x exact or min 3 sf or a multiple of a	
A1ft	Correct distance CD, follow through their x_D Must be positive Min 2 sf for decimal	
ALT (d)	Time C to centre O $\quad \frac{1}{2}a = a\sin\omega t \Rightarrow \frac{1}{2} = \sin 3t$	
	$t_{CO} = \frac{1}{3}\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{18}\ (0.1745...)$	M1A1
	Time O to D $\quad \frac{\pi}{6} - \frac{\pi}{18} = \frac{\pi}{9}$	
	$OD = a\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}a$	M1A1
	Distance CD $\frac{5}{2} + \frac{5\sqrt{3}}{2} = \frac{5}{2}\left(1+\sqrt{3}\right)$ or 6.83 (m)	A1ft

Q10.

Question Number	Scheme	Marks
(a)	$\dfrac{20e}{1.8}=\dfrac{15(1.5-e)}{0.9}$	M1A1
	$2e=4.5-3e \quad e=\dfrac{4.5}{5}=0.9$	A1
	$AO=2.7\text{ m}$ *	A1cso (4)
ALT	With AO as unknown:	
	$\dfrac{20(AO-1.8)}{1.8}=\dfrac{15(4.2-AO-0.9)}{0.9}$ M1A1A1	
	$AO=2.7\text{ m}$ * A1cso	
(b)	$\dfrac{15(0.6-x)}{0.9}-\dfrac{20(0.9+x)}{1.8}=m\ddot{x}$ or $\dfrac{20(0.9-x)}{1.8}-\dfrac{15(0.6+x)}{0.9}=m\ddot{x}$	M1A1A1
	$-\dfrac{50}{1.8m}x=\ddot{x}$ oe	M1
	$(m>0)$ $\therefore$ SHM	A1cso (5)

(c)	$-\dfrac{50}{1.8\times10}x=\ddot{x}$	
	$\omega=\sqrt{\dfrac{50}{18}}\left(=\dfrac{5}{3}\right)$	B1ft
	$a=0.2\text{ m}$	B1
	$v_{\max}=a\omega=0.2\times\dfrac{5}{3}$	M1
(i)	$J=10\times0.2\times\dfrac{5}{3}=\dfrac{10}{3}$ $(=3.3\ldots\ =3.3$ or better$)$	M1A1
	$x=-0.1$	
(ii)	$-0.1=0.2\sin t\left(\dfrac{5}{3}\right)$	M1
	$t=\dfrac{3}{5}\sin^{-1}(-0.5)=\dfrac{3}{5}\times\dfrac{7\pi}{6}=2.199\ldots\text{ s}$ (accept 2.2 or better inc $\dfrac{7\pi}{10}$)	M1A1 (8)
		[17]

Question Number	Scheme	Marks
(a)		
M1	Attempt an equation equating 2 tensions. Tensions to be obtained using Hooke's Law. The two extensions must add to 1.5	
A1	Correct equation	
A1	Correct extension for either string	
A1cso	Correct completion to length AO	
(b)		
M1	Form an equation of motion with the difference of 2 tensions with different extensions. Tensions must be of the form $k\dfrac{\lambda x}{l}$. Acceleration can be $\ddot{x}$ or a. NB: x can be measured in either direction.	
A1 A1	Deduct one per error (Award A1A0 for one deduction. Allow if a used instead of $\ddot{x}$ **provided** the direction of a is the same as that of $\ddot{x}$	
M1	Attempt to simplify their equation to the standard form for SHM. Acceleration must be $\ddot{x}$ now. Equation must simplify to $\ddot{x} = \pm\,\omega^2 x$	
A1cso	Correct equation with conclusion.	
(c)		
B1ft	Correct value of ω, seen explicitly or used. Follow through from their equation of the form $\ddot{x}$ or $a = \pm\omega^2 x$ If equation has been left in the form $m\ddot{x}$ or $a = ...$ in (b) but RHS divided by m here to obtain ω this mark is available (but no extra marks for (b)).	
B1	Correct amplitude, seen explicitly or used	
M1	Attempt the maximum speed with their a and ω	
(i)M1	Use an impulse-momentum equation with their max speed to obtain a value for J	
A1	Correct value for J	
(ii)M1	Use $x = a\sin\omega t$ or $x = a\cos\omega t$ with $x = \pm 0.1$, their ω and a	
M1	Solve their equation and use a correct method to find the required time. Must use radians	
A1	Correct time 2.2 or better	