Questions

Q1.

A particle P of mass 0.4 kg moves along the x-axis in the positive direction. At time t = 0, P passes through the origin O with speed 10 m s^{-1}. At time t seconds P is x metres from O and the speed of P is v m s^{-1}. The resultant force acting on P has magnitude $\dfrac{8}{(t+4)^2}$ N and is directed towards O.

(a) Show that $v = \dfrac{20}{t+4} + 5$

(5)

When $v = 6$, $x = a + b\ln 5$, where a and b are integers.

(b) Using algebraic integration, find the value of a and the value of b.

(5)

(Total for question = 10 marks)

Q2.

At time $t = 0$, a particle P is at the origin O, moving with speed 8 m s^{-1} in the positive x direction. At time t seconds, $t \geq 0$, the acceleration of P has magnitude $2(t+4)^{-\frac{1}{2}}$ ms^{-2} and is directed towards O.

(a) Show that, at time t seconds, the velocity of P is $16 - 4(t+4)^{\frac{1}{2}}$ ms^{-1}

(5)

(b) Find the distance of P from O when P comes to instantaneous rest.

(7)

(Total for question = 12 marks)

Q3.

A particle P is moving in a straight line. At time t seconds, the distance of P from a fixed point O on the line is x metres and the acceleration of P is $(6 - 2t)$ m s^{-2} in the direction of x increasing. When $t = 0$, P is moving towards O with speed 8 m s^{-1}

(a) Find the velocity of P in terms of t.

(3)

(b) Find the total distance travelled by P in the first 4 seconds.

(5)

(Total for question = 8 marks)

Q4.

A particle P of mass m is fired vertically upwards from a point on the surface of the Earth and initially moves in a straight line directly away from the centre of the Earth. When P is at a distance x from the centre of the Earth, the gravitational force exerted by the Earth on P is directed towards the centre of the Earth and has a magnitude which is inversely proportional to x^2. At the surface of the Earth the acceleration due to gravity is g. The Earth is modelled as a fixed sphere of radius R.

(a) Show that the magnitude of the gravitational force acting on P is $\dfrac{mgR^2}{x^2}$

(2)

The particle was fired with initial speed U and the greatest height above the surface of the Earth reached by P is $\dfrac{R}{20}$

Given that air resistance can be ignored,

(b) find U in terms of g and R.

(7)

(Total for question = 9 marks)

Q5.

A particle *P* of mass 0.6 kg is moving along the positive *x*-axis in the positive direction.

The only force acting on *P* acts in the direction of *x* increasing and has magnitude $\left(3t + \frac{1}{2}\right)$ N, where *t* seconds is the time after *P* leaves the origin *O*.

When $t = 0$, *P* is at rest at *O*.

(a) Find an expression, in terms of *t*, for the velocity of *P* at time *t* seconds.

(2)

The particle passes through the point *A* with speed $\frac{10}{3}$ m s^{-1}.

(b) Find the distance *OA*.

(5)

(Total for question = 7 marks)

Q6.

A particle *P* of mass 0.4 kg moves on the positive *x*-axis under the action of a single force. The force is directed towards the origin *O* and has magnitude $\frac{k}{x^2}$ newtons, where $OP = x$ metres and *k* is a constant. Initially *P* is moving away from *O*. At $x = 2$ the speed of *P* is 5 m s^{-1} and at $x = 5$ the speed of *P* is 2 m s^{-1}.

(a) Find the value of *k*.

(8)

The particle first comes to instantaneous rest at the point *A*.

(b) Find the value of *x* at *A*.

(4)

(Total for question = 12 marks)

Q7.

A particle P of mass 0.5 kg is moving along the positive x-axis under the action of a single force, directed towards the origin O. When $OP = x$ metres, the force has magnitude $\frac{k}{x^2}$ newtons, where k is a constant, and the speed of P is v m s^{-1}.

When P is 2 m from O, the speed of P is 5 m s^{-1} and P is moving in the positive x direction.

When P is 5 m from O, the speed of P is 4 m s^{-1} and P is moving in the positive x direction.

Find v^2 in terms of x.

(Total for question = 8 marks)

Q8.

A particle, P, of mass 0.5 kg is moving along the positive x-axis. At time t seconds, $t \geq 0$, P is x metres from the origin O and is moving away from O with velocity v m s^{-1}, where $v = \frac{1}{(4x+3)}$

When $t = 0$, P is at O.

(a) Find the distance of P from O when $t = 2$

(5)

(b) Find the magnitude of the resultant force acting on P when $t = 2$

(5)

(Total for question = 10 marks)

Q9.

A particle P of mass 0.8kg moves along the x-axis in the positive x direction under the action of a resultant force. This force acts in the direction of x increasing. At time t seconds, $t \geq 0$, P is x metres from the origin O, P is moving with speed v m s^{-1} and the force has magnitude $\frac{4}{(x+1)^3}$ N.

When $t = 0$, P is at rest at O.

(a) Show that $v^2 = 5\left(\frac{(x+1)^2 - 1}{(x+1)^2}\right)$

(6)

When $t = 2$, P is at the point A. When $t = 4$, P is at the point B.

(b) Using algebraic integration, find the distance AB.

(7)

(Total for question = 13 marks)

Q10.

A particle P moves on the x-axis. At time t seconds, $t \geq 0$, the displacement of P from the origin O is x metres and the acceleration of P is $\left(\frac{7}{2} - 2x\right)$ m s^{-2}, measured in the positive x direction. At time $t = 0$, P passes through O moving with speed 3 m s^{-1} in the positive x direction. Find the distance of P from O when P first comes to instantaneous rest.

(Total for question = 6 marks)

Examiner's Report

Q1.

Again, very well answered. Most students got full marks in (a), with the only problem being the missing minus sign. Most showed sufficient working to convincingly arrive at a value for c, even if they had not divided through by 0.4 at that stage. In (b) most knew what to do and generally managed the integration correctly. A small number of students failed to find the constant and generally did not seem to register that their final answer, although in roughly the correct form, had the wrong value in the log. Almost everybody correctly identified and used $t = 16$. The definite integration approach was rare, but, when used, was usually successful.

Q2.

This was probably the easiest question on the paper. Most got (a) correct, although a significant number again managed to get the sign wrong. Sadly for them, rather than realising that they needed a minus sign before integrating, they just wrote down the answer. In most cases they must have realised that they were doing something wrong, as they inevitably had no trouble with their signs in (b). Most clearly showed where the 16 came from, although as ever many did not give as much working as would have been desired for a given answer. The majority knew exactly what to do in (b), correctly finding the required time and performing the integration. Whilst indefinite integration was the most popular approach, since many found $t = 2$ first, there were many correct attempts at definite integration.

Q3.

Relatively few completely correct solutions were seen although it was pleasing to see that almost all used the correct form of the acceleration. In part (a) the main problem was misunderstanding the directions of the velocity and/or the acceleration. Many candidates used $v = 8$ to obtain the constant of integration resulting in an incorrect expression for v. The majority of candidates did not realise that there was a change of direction during the time interval in part (b) so integrated their expression for v between the limits 0 and 4. Those who did part (a) correctly and put $v = 0$ obtained a quadratic which factorised and generally went on to complete the solution correctly. Those who had used $v = 8$ had a quadratic which had to be solved using the formula and which only gave one valid solution implying that there was no change of direction. A few put the acceleration equal to zero and integrated between 0 and 3 and then 3 and 4.

Q4.

Part (a) should be a completely standard and learnt proof but very few explicitly gave a general proportion statement, followed by a clear statement about mg and R, leading to an expression for k. Many went straight in at $mg = k/R^2$, whilst others started with the general statement before introducing mg and R on separate lines, losing the connection between them. There were plenty of physicists that insisted on using GMm, which just made their working harder to keep accurate.
In part (b) most knew what was required but many did not set their work out as formally as they ought to, making it hard to be certain whether correct signs had been used. The most common approach was to use definite integration, but this was often only half shown, with the candidates going straight to a difference in v^2, where one of the velocities went on to become 0. The more convincing candidates went to $-u^2$ and a negative integral, so it was clear that signs were being used consistently, but others lost marks because there appeared to be a double sign error. Some candidates used a distance of $R/20$ and others paired up the distances and speeds incorrectly.

Q5.

The majority answered this perfectly but it was noticeable how many students wrote solutions to the quadratic equation in (b) without showing any working. This is acceptable if the answers are correct but no method marks can be given if either the equation or the solutions are wrong. It is therefore advisable to work through the solution fully, showing either factors or formula. Some major method mistakes among the incorrect solutions were:

- attempting to use $v\frac{\mathrm{d}v}{\mathrm{d}x}$ instead of $\frac{\mathrm{d}v}{\mathrm{d}t}$ for the acceleration in (a)
- using the instantaneous velocity $\frac{10}{3}$ as $\frac{\mathrm{d}x}{\mathrm{d}t}$ in (b) instead of the variable answer from (a).

A number of lesser, but significant, errors seen a number of times were:

- numerical slips when dividing by 3/5 in (a); this led to all the required answers being wrong
- using limits of 0 and 10/3 (the velocity) or –4/3 and 1 instead of 0 and 1 when finding $\int v\,\mathrm{d}t$.

Q6.

In part (a) almost all could set up a correct equation of motion and write this in the required form prior to integration. A significant number of students would first quote $\frac{1}{2}v^2 = \int a\,\mathrm{d}x$ or $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = a$ with only some using *KE* = work done. Integration was almost always successful, the majority using indefinite integration and a handful using limits. Some students did make careless computational errors in calculating the constants but most had no problems. By far the most common error was to omit the minus sign from the equation of motion at the start resulting in a loss of 3 out of the 9 marks available. A few students missed out the mass in the initial equation of motion.

Part (b) was very well answered with students able to use $v = 0$ in their equation. Marks lost here were usually as a result of errors in part (a). It was common for both constants to be found in part (a) and then transferred to this part.

Q7.

This question proved to be a very accessible start to the paper, with very few candidates failing to score and many fully correct responses. Most candidates could set up the initial equations and integrate correctly. By far the most common error was not having a minus sign in the equation of motion. Candidates need to read the question carefully as many did not interpret the fact that the force was directed towards O and should be negative. This was a costly error as only 4 out of 8 marks were then available. Candidates were careful with their working and very few numerical slips were made in finding the values of the constants. A small number used limits in the integration and were able to obtain the value of k, but a significant number of these candidates could then not also find the value of c.

Q8.

Part (a) was generally answered well, with the majority of candidates able to separate variables and integrate correctly. Ommission of c was very rare and most candidates successfully solved their quadratic and rejected the answer of -2. The most common difficulty arose from simply integrating v with respect to x, rather than separating variables, and this lost all marks in (a).

Part (b) generally proved more of a challenge. Not all candidates knew that they needed to find $v\frac{dv}{dx}$, and when they did, the differentiation often went wrong, with the 4 disappearing. Other common mistakes were to only find acceleration, rather than the force, which lost the final 2 marks.

Q9.

Part (a) A significant number started from use of a formula such as $\int a\,\mathrm{d}x = \frac{1}{2}v^2$ or $\int F\,\mathrm{d}x = \frac{1}{2}mv^2$ or $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = a$ rather than explicitly showing $F = mv\frac{\mathrm{d}v}{\mathrm{d}x}$ and separation of variables. The integration was not a problem and all could find the constant of integration easily. On the whole the quality of working, to a given answer, was good.

Part (b) This was often not attempted and many did not know where to start. A surprising number of students made careless errors and did not start with the correct expression for $\frac{\mathrm{d}x}{\mathrm{d}t}$ or made errors when separating the variables. Many struggled to rearrange a correct integral into a form that could be integrated. Some students did not have the correct integral and solved their integral by other means which gained no marks. A handful of very poor attempts were seen such as use of $s = \int v\mathrm{d}x$ and then using $x = 2$ and $x = 4$ as limits. A significant minority did however achieve full marks and the correct answer for part (b) was seen.

Q10.

This was a straightforward start for most and the majority scored full marks. The most common error was using $a = \frac{\mathrm{d}v}{\mathrm{d}x}$ or $\frac{\mathrm{d}v}{\mathrm{d}t}$ instead of $v\frac{\mathrm{d}v}{\mathrm{d}x}$. Omitting a constant of integration was less common, but costly, as it led to a 2 term, rather than the required 3 term, quadratic. Where a mistake was made in producing a 3 term quadratic, many candidates then lost a mark by failing to show sufficiently detailed working in solving their equation.

Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$0.4\frac{dv}{dt}=-\frac{8}{(t+4)^2}$	M1
	$v=\int -\frac{20}{(t+4)^2}dt$	
	$v=\frac{20}{(t+4)}\ (+c)$	DM1A1
	$t=0,\ v=10\Rightarrow\ 10=5+c,\ c=5$	DM1
	$v=\frac{20}{(t+4)}+5$ *	A1cso (5)
(b)	$x=\int \frac{20}{(t+4)}+5\ dt$	
	$x=20\ln(t+4)+5t+c$	M1A1
	$t=0,\ x=0\Rightarrow 0=20\ln 4+c\quad c=-20\ln 4$	A1
	$v=6\quad \frac{20}{t+4}=1\quad t=16$	B1
	$x=20\ln 20+80-20\ln 4=80+20\ln 5$	
	$a=80,\ b=20$	A1cao (5) [10]

(a)	0.4 or m for the mass for the first 4 marks
M1	Form an equation of motion with the acceleration in form shown. Minus sign may be missing.
DM1	Attempt the integration
A1	Correct integration, including correct sign. Constant not needed
DM1	Use $t=0,\ v=10$ to obtain a value for c
A1cso	Correct answer only, no errors seen
(b)	
M1	Integrate the **given** expression for v to obtain an expression for x. Constant not needed.
A1	Correct integration, constant not needed
A1	Obtain the correct constant
B1	Correct value for t when $v=6$
A1cao	Correct values for a and b Need not be shown explicitly.

Q2.

Question Number	Scheme	Marks
(a)	$\frac{\mathrm{d}v}{\mathrm{d}t}=-2(t+4)^{-\frac{1}{2}}$	M1
	$v=-\int 2(t+4)^{-\frac{1}{2}}\,\mathrm{d}t$	
	$v=-4(t+4)^{\frac{1}{2}}\quad(+c)$	DM1A1
	$t=0, v=8 \Rightarrow c=16$	M1
	$v=16-4(t+4)^{\frac{1}{2}}\quad(\mathrm{m\,s^{-1}})$ *	A1cso (5)
(b)	$v=0\quad 16=4(t+4)^{\frac{1}{2}}$	M1
	$16=t+4\quad t=12$	A1
	$x=4\int\left(4-(t+4)^{\frac{1}{2}}\right)\mathrm{d}t$	
	$x=4\left(4t-\frac{2}{3}(t+4)^{\frac{3}{2}}\right)\quad(+d)$	M1A1
	$t=0,\ x=0\quad d=4\times\frac{2}{3}\times4^{\frac{3}{2}}=\frac{64}{3}$ oe	A1
	$t=12\quad x=4\left(4\times12-\frac{2}{3}\times16^{\frac{3}{2}}\right)+\frac{64}{3}=42\frac{2}{3}$ (m) oe eg 43 or better	DM1A1 (7)
		[12]

(a) **M1** Attempting an expression for the acceleration in the form $\frac{\mathrm{d}v}{\mathrm{d}t}$; minus may be omitted.

DM1 Attempting the integration

A1 Correct integration, constant of integration may be omitted (no ft)

M1 Using the initial conditions to obtain a value for the constant of integration

A1cso Substitute the value of c and obtain the final GIVEN answer

(b) **M1** Setting the **given** expression for v equal to 0

A1 Solving to get $t=12$

M1 Setting $v=\frac{\mathrm{d}x}{\mathrm{d}t}$ and attempting the integration wrt t At least one term must clearly be Integrated.

A1 Correct integration, constant may be omitted

A1 Substituting $t=0,\ x=0$ and obtaining the correct value of d. Any equivalent number, inc decimals.

DM1 Substituting their value for t and obtaining a value for the required distance. Dependent on The second M mark.

A1 Correct final answer, any equivalent form.

Q3.

Question Number	Scheme	Marks
(a)	$a=\dfrac{dv}{dt}=6-2t$	
	$v=6t-t^2 \quad (+c)$	M1A1
	$t=0, v=-8 \Rightarrow c=-8$	
	$v=6t-t^2-8$	A1 (3)
(b)	$v=6t-t^2-8=0$	
	$(t-2)(t-4)=0$	
	$t=2,\ t=4$	M1
	$s=\int\left(6t-t^2-8\right)dt=3t^2-\dfrac{1}{3}t^3-8t \quad (+c)$	M1
	$\left[3t^2-\dfrac{1}{3}t^3-8t\right]_2^4=48-\dfrac{64}{3}-32-\left(12-\dfrac{8}{3}-16\right)=1\dfrac{1}{3}$	A1
	$\left[3t^2-\dfrac{1}{3}t^3-8t\right]_0^2=12-\dfrac{8}{3}-16\ (-0)=-6\dfrac{2}{3}$	A1
	Total distance = 8 m	A1ft (5) [8]

(a)	
M1	Using $a=\dfrac{dv}{dt}$ and attempting the integration Use of $a=v\dfrac{dv}{dx}$ scores M0
A1	Correct integration w/wo constant
A1	Find constant and correct statement for the velocity
(b)	
M1	Find the times when P is at rest (Usual rules for factorising or formula)
M1	Integrate v to obtain an expression for s (c not needed)
A1	Correct displacement for either time interval
A1	Correct displacement for second time interval
A1ft	Add 2 (positive) distances
	NB: Using $v=8$ Is NOT a misread.

Q4.

Question Number	Scheme	Marks
(a)	$F=(-)\dfrac{k}{x^2}$	
	$x=R \quad F=(-)mg$	
	$mg=(-)\dfrac{k}{R^2}$	M1
	Mag of $F=\dfrac{mgR^2}{x^2}$ *	A1cso (2)
(b)	$(m)v\dfrac{\mathrm{d}v}{\mathrm{d}x}=-(m)\dfrac{gR^2}{x^2}$	M1
	$\int v\mathrm{d}v=-\int gR^2x^{-2}\mathrm{d}x$	
	$\dfrac{1}{2}v^2=gR^2x^{-1}+(c)$	DM1A1ft
	$x=R \;\; v=U \quad \dfrac{1}{2}U^2=gR+c$	M1
	$\dfrac{1}{2}v^2=\dfrac{gR^2}{x}+\dfrac{1}{2}U^2-gR$ oe	A1
	$x=\dfrac{21}{20}R \;\; v=0 \quad 0=\dfrac{gR^2}{\frac{21}{20}R}+\dfrac{1}{2}U^2-gR$	M1
	$\dfrac{1}{2}U^2=gR-\dfrac{20}{21}gR$	
	$U=\sqrt{\dfrac{2gR}{21}}$ oe	A1 (7)
		[9]

(a)M1	Using NL2 at the surface of the Earth with the proportionality condition, force to be mg, distance to be R. Can use $F = \dfrac{GMm}{x^2}$
A1cso	Find the constant of proportionality and hence deduce the stated magnitude.
(b)M1	Using NL2 with acceleration in form $v\dfrac{dv}{dx}$. Minus sign may be missing. m may be cancelled.
DM1	Attempt integration of their expression. v^2 or x^{-1} must be seen.
A1ft	Correct integration follow through a missing minus sign. Constant of integration may be missing.
M1	Using $x = R\ \ v = U$ OR substitute $x = \dfrac{21}{20}R\ \ v = 0$ in an equation containing c
A1	Substituting a **correct** constant to obtain a correct expression for $\dfrac{1}{2}v^2$
M1	Substitute $x = \dfrac{21}{20}R\ \ v = 0$ OR substitute $x = R\ \ v = U$
A1	Complete to a correct expression for U.

Definite integration in (b):

$(m)v\dfrac{dv}{dx} = -(m)\dfrac{gR^2}{x^2}$ M1

$\int_U^0 v\,dv = -\int_R^{\frac{21R}{20}} gR^2x^{-2}\,dx$

$\left[\dfrac{1}{2}v^2\right] = \left[gR^2x^{-1}\right]$ DM1A1ft (ft missing sign, as main scheme) Limits not needed

$\left[\dfrac{1}{2}v^2\right]_U^0 = \left[gR^2x^{-1}\right]_R^{\frac{21R}{20}}$ M1 limits seen, correct "values" but may be paired incorrectly

A1 correct limits, 0 and $\dfrac{21R}{20}$ at top or bottom on both integrals, U and R in the other positions

M1 Substitute their limits A1 correct answer.

NB It is possible to appear to omit the minus sign and reverse the limits on one integral. However, without some explanation this will be insufficient working and scores as a missing minus case.

Q5.

Question Number	Scheme	Marks
(a)	$0.6a = \left(3t + \frac{1}{2}\right)$	
	$\frac{dv}{dt} = \frac{5}{3}\left(3t + \frac{1}{2}\right)$	M1
	$v = \frac{5}{2}t^2 + \frac{5}{6}t \quad (+c)$	
	$(t = 0, v = 0 \ \Rightarrow c = 0) \qquad \therefore v = \frac{5}{2}t^2 + \frac{5}{6}t$	A1 (2)
(b)	$\frac{10}{3} = \frac{5}{2}t^2 + \frac{5}{6}t$	M1
	$3t^2 + t - 4 = 0$	
	$(3t + 4)(t - 1) = 0$	
	$t = 1$ at A	dM1A1
	$s = \left[\frac{5t^3}{6} + \frac{5t^2}{12}\right]_0^1, \quad = \frac{5}{4}\text{m} \quad$ oe	M1, A1 (5)
		[7]

(a)	
M1	Form an equation of motion with the acceleration in form dv/dt. Must include the mass.
A1	Integrate and complete. Award for a correct expression even if no reference has been made to the constant of integration.
(b)	
M1	Set their expression equal to 10/3
dM1	Factorise (or use formula or completing the square) to reach t = ... (May include a second value.)
A1	Correct value for t. (Ignore extra answer provided limits on following integral are correct.)
M1	Integrate the expression for v, limits needed, lower limit must be 0 (seen or implied), their t for upper limit.
A1	Substitute correct limits to obtain 5/4 or 1.25 m
	Indefinite integration in (b) - last 2 marks: M1 for the integration with or without the constant, A1 for the correct answer.

Q6.

Question Number	Scheme	Marks
(a)	$0.4\ddot{x} = -\dfrac{k}{x^2}$	M1
	$0.4v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\dfrac{k}{x^2}$	M1
	$0.2v^2 = \int -kx^{-2}\mathrm{d}x$	
	$0.2v^2 = \dfrac{k}{x}\ (+c)$	dM1A1ft
	$x = 2,\ v = 5 \Rightarrow 5 = \dfrac{k}{2} + c$	dM1
	$x = 5,\ v = 2 \Rightarrow 0.8 = \dfrac{k}{5} + c$	A1
	$4.2 = k\left(\dfrac{1}{2} - \dfrac{1}{5}\right)$	dM1
	$k = 14$	A1 cso (8)
(b)	$c = 5 - 7 = -2$	
	$0.2v^2 = \dfrac{14}{x} - 2$	M1A1ft
	$v = 0 \Rightarrow x = 7$	dM1A1 cso(4) [12]

(a)M1 Form an equation of motion, minus sign may be missing.

M1 Writing the acceleration in the form $v\dfrac{\mathrm{d}v}{\mathrm{d}x}$ These two M marks may be awarded together.
Can be implied by $\dfrac{1}{2}mv^2$ after integrating.

dM1 Attempt to integrate both sides of the equation wrt x Depends on both M marks above

A1ft Correct integration with correct signs. Constant may be missing. Follow through a missing minus sign.

NB For the first 4 marks m or 0.4 may be used

dM1 Substitute either $x = 2,\ v = 5$ or $x = 5,\ v = 2$ Depends on all M marks above.

A1 Both substitutions made and 2 correct equations in k and c found

dM1 Solve these simultaneous equations to obtain a value for k. Solving 1 linear equation (as c was omitted) scores M0. Depends on all M marks above.

A1 Correct value of k obtained.

(b)

M1 Obtain a value of c and form an expression for v^2. (Often seen in (a); award marks if (b) is attempted.)

A1ft Correct expression for v^2. Follow through $k = -14$ which gives $c = -5$

dM1 Substitute $v = 0$ in their expression for v^2 and solve for x

A1cso Correct value of x obtained.

Q7.

Note: if they don't have $\frac{1}{4}$ (or equivalent) on second line LHS, they will get the correct result, but should lose all A marks.

Question Number	Scheme	Marks
	$\frac{1}{2}v\frac{dv}{dx}=-\frac{k}{x^2}$	M1
	$\frac{1}{4}v^2=\frac{k}{x}\quad(+c)$	M1A1
	$t=0\;\; x=2\;\; v=5\;\Rightarrow\frac{25}{4}=\frac{k}{2}+c$	dM1
	$x=5\;\; v=4\;\Rightarrow\frac{4^2}{4}=\frac{k}{5}+c$	A1
	$k=\frac{15}{2}\quad c=\frac{5}{2}$	dM1A1
	$v^2=\frac{30}{x}+10$	A1 (8)
		[8]

M1	For equation of motion with acceleration in the form $v\frac{dv}{dx}$ oe. Must have used a mass.
M1	Separating the variables and integrating. They must have had $v\frac{dv}{dx}$ oe
A1	Correct integration, constant not needed. They must have started with a minus sign. Note that k is positive since $\frac{k}{x^2}$ is a magnitude.
dM1	Using one of the boundary conditions to form an equation in k and c. Dependent on the first M1.
A1	Both equations correct
dM1	Solving their equations to find values for k and c. Dependent on the previous M mark.
A1	Correct values for k and c
A1	Correct expression for v^2
ALT	-if they use definite integration to find k and then indefinite to find c
	First 3 marks as per the main scheme, then
dM1	$\int_5^4\frac{1}{2}v\,dv=-\int_2^5\frac{k}{x^2}dx$
A1	$k=\frac{15}{2}$ from correct working.
dM1	Using indefinite integration and one set of limits to find c.
A1	$c=\frac{5}{2}$ o.e. from correct working.
A1	Correct expression for v^2

Q8.

Question Number	Scheme	Marks
(a)	$v = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{(4x+3)}$	
	$\frac{\mathrm{d}t}{\mathrm{d}x} = 4x+3$	
	$t = \int (4x+3)\,\mathrm{d}x, = \frac{1}{2} \times 4x^2 + 3x + c$	
	OR $\int_0^2 \mathrm{d}t = \int_0^X (4x+3)\,\mathrm{d}x,\ = \left[\frac{1}{2} \times 4x^2 + 3x\right]_0^X$	M1,dM1A1
	$c = 0$	
	$t = 2 = \frac{1}{2} \times 4x^2 + 3x \quad 2x^2 + 3x - 2 = 0$	dM1
	$x = \frac{1}{2}\ (x = -2)$	A1cso (5)
(b)	$a = v\frac{\mathrm{d}v}{\mathrm{d}x} \qquad alt:\ a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	M1
	$= \frac{1}{4x+3} \times \frac{-4}{(4x+3)^2}$	dM1A1
	$\lvert F\rvert = \frac{1}{2} \times \frac{4}{(2+3)^3} = \frac{2}{125} = 0.016\mathrm{N}$	M1 A1cso (5)
		[10]

(a)	
M1	Rewrite as $\frac{dx}{dt}$ **and** separate variables to reach a form ready for integration
dM1	Attempt the integration (at least one power going up).
A1	Correct integration. Constant/limits not needed.
dM1	Use $t = 2$ in their expression or substitute correct limits, and solve their 3 term quadratic to find x. If solving an incorrect quadratic, evidence of a correct method must be seen. Depends on the previous M mark.
A1cso	Obtain $x = \frac{1}{2}$ (and reject -2 if seen) from completely correct work. Constant of integration must have been seen, although we do not need to see evidence of evaluation.
(b)	
M1	Use $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$
dM1	Differentiate the given expression for v and obtain an expression for a. We need to see a power of 2 (or a power of 3 if using $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$). Depends on the previous M mark.
A1	Correct expression, any form.
M1	Use their acceleration in an equation of motion to obtain a value for F. Mass must be included and they must use their value of x. Independent, but must have found an expression for acceleration.
A1cso	Correct magnitude of F. Correct solution only. Can be fraction or decimal. Must be positive.

Q9.

Question Number	Scheme	Marks
(a)	$0.8v\dfrac{\mathrm{d}v}{\mathrm{d}x}=\dfrac{4}{(x+1)^3}$	M1A1
	$v\dfrac{\mathrm{d}v}{\mathrm{d}x}=\dfrac{5}{(x+1)^3}$	
	$\dfrac{1}{2}v^2=-\dfrac{5}{2(x+1)^2}\ (+c)$	dM1A1
	$(t=0, v=0, x=0)\ \Rightarrow c=\dfrac{5}{2}$	ddM1
	$v^2=5\left(1-\dfrac{1}{(x+1)^2}\right)=5\left(\dfrac{(x+1)^2-1}{(x+1)^2}\right)$ *	A1cso (6)
(b)	$v=\dfrac{\mathrm{d}x}{\mathrm{d}t}=\sqrt{5\left(\dfrac{(x+1)^2-1}{(x+1)^2}\right)}$	M1
	$\int\dfrac{(x+1)}{\sqrt{(x+1)^2-1}}\mathrm{d}x=\int\sqrt{5}\,\mathrm{d}t$	M1A1
	$\left((x+1)^2-1\right)^{\frac{1}{2}}=\sqrt{5}t\ (+k)$	M1A1
	$t=0, x=0\ \Rightarrow k=0$	
	$(x+1)^2-1=5t^2\quad x=\sqrt{5t^2+1}-1$	
	Dist $=\sqrt{81}-1-\left(\sqrt{21}-1\right)=9-\sqrt{21}$ or 4.417 m (accept 4.4 or better)	M1A1cso(7) [13]

(a)M1	Attempt an equation of motion with acceleration $=v\dfrac{\mathrm{d}v}{\mathrm{d}x}$. Can be implied by subsequent work.
A1	Fully correct equation. Can be implied by subsequent work.
dM1	Attempt the necessary integration. $\dfrac{k}{(x+1)^3}\to\pm\dfrac{k'}{(x+1)^2}$ Depends on the first M mark
A1	Correct integration (not ft). Constant may be omitted.
ddM1	Use the given initial conditions to obtain a value for the constant. Depends on both preceding M marks. Initial conditions need not be shown explicitly.
A1cso	Reach the **given** answer in the form shown in the question. No errors in the work.

(b)M1	Replace v with $\mathrm{d}x/\mathrm{d}t$ in either the answer in (a) (with or without taking square root first) or in an equivalent expression drawn from their working in (a)
M1	Separate the variables ready to integrate. Must reach an integrand which can be integrated.
A1	All correct so far
M1	Attempt the integration by any valid means (eg inspection or substitution). Must include $\dfrac{(x+1)}{\sqrt{(x+1)^2-1}} \to A\left((x+1)^2-1\right)^{\frac{1}{2}}$ or $\dfrac{(x+1)}{\sqrt{x^2+2x}} \to B\left(x^2+2x\right)^{\frac{1}{2}}$ oe
A1	Correct integration (not ft). Constant may be omitted.
M1	Use the given values of t to obtain the distances OA and OB and hence the distance AB. The function must have been integrated but third M mark may have been lost.
A1cso	Correct distance. Exact or decimal accepted. Constant must have been included and shown to be 0.

	By definite integration:	
(a)	$0.8v\dfrac{\mathrm{d}v}{\mathrm{d}x} = \dfrac{4}{(x+1)^3}$	M1A1
	$\int_0^v v\,\mathrm{d}v = \int_0^x \dfrac{5}{(x+1)^3}\,\mathrm{d}x$	
	$\left[\dfrac{1}{2}v^2\right]_0^v = \left[-\dfrac{5}{2(x+1)^2}\right]_0^x$ (Integration condition as main scheme)	dM1A1 Ignore limits
	$\dfrac{1}{2}v^2 = -\dfrac{5}{2(x+1)^2} + \dfrac{5}{2}$	ddM1 Sub correct limits
	$v^2 = 5\left(1 - \dfrac{1}{(x+1)^2}\right) = 5\left(\dfrac{(x+1)^2-1}{(x+1)^2}\right)$	A1cso (6)
(b)	Complete solution following definite integration requires distances O to A and O to B being found followed by finding their difference.	
	$v = \dfrac{\mathrm{d}x}{\mathrm{d}t} = \sqrt{5\left(\dfrac{(x+1)^2-1}{(x+1)^2}\right)}$	M1
	$\int_0^X \dfrac{(x+1)}{\sqrt{(x+1)^2-1}}\,\mathrm{d}x = \int_0^T \sqrt{5}\,\mathrm{d}t$	M1A1 ignore limits
	$\left[\left((x+1)^2-1\right)^{\frac{1}{2}}\right]_0^X = \left[\sqrt{5}t\right]_0^T$	M1A1 Integration only. Ignore limits
	$(X+1)^2 - 1 = 5T^2 \quad X = \sqrt{5T^2+1} - 1$	
	Dist $= \sqrt{81} - 1 - \left(\sqrt{21} - 1\right) = 9 - \sqrt{21}$ or 4.417 m (accept 4.4 or better)	M1A1 Correct completion

Q10.

Question Number	Scheme	Marks
	$v\dfrac{dv}{dx}=\dfrac{7}{2}-2x$	M1
	$\dfrac{1}{2}v^2=\dfrac{7}{2}x-x^2\ (+c)$	A1
	$x=0\ \ v=3 \Rightarrow c=\dfrac{9}{2}$	A1
	$v=0 \quad 0=\dfrac{7}{2}x-x^2+\dfrac{9}{2}$	M1
	$2x^2-7x-9=0$	
	$(2x-9)(x+1)=0$	M1
	$x=4.5$ oe	A1cso
	By definite integration:	
	$v\dfrac{dv}{dx}=\dfrac{7}{2}-2x$	M1
	$\int v\,dv=\int\left(\dfrac{7}{2}-2x\right)dx \Rightarrow \left[\dfrac{1}{2}v^2\right]_3^0=\left[\dfrac{7}{2}x-x^2\right]_0^X$	A1A1
	$(0)-\dfrac{1}{2}\times 3^2=\dfrac{7}{2}X-X^2\ (-0)$	M1
	$2X^2-7X-9=0 \Rightarrow X=4.5$ oe	M1A1cso
		[6]

M1	For an equation of motion with the acceleration in the form $v\dfrac{dv}{dx}$ oe
	May be implied by sight of $(1/2)v^2$ after integration
A1	Correct integration, constant not needed
A1	Use $x=0\ \ v=3$ to obtain $c=9/2$
M1	Substitute $v=0$ in their expression for v^2 or v. Award M0 if this expression includes t
M1	Solve the resulting 3TQ in x only, by any valid means. Must reach $x=$ ...(less than 3 terms scores M0)
A1cso	Correct value for x obtained from correct working. If $x=-1$ is seen it must be eliminated.
	By definite integration:
M1	For an equation of motion as above
A1	Correct integration, ignore limits
A1	Correct limits, as shown or both sets reversed
M1	Substitute their limits, zeros need not be shown
M1	Solve the resulting 3TQ by any valid means. Must reach $X=$ ...(less than 3 terms scores M0)
A1cso	Correct value for x obtained from correct working.
NB	**Solving a 3TQ,**
	Calculator solutions: Correct equation: correct answer implies correct method. (Incorrect answer M0) -1 need not be seen. Incorrect equation: No working, award M0
	By formula: Correct general formula seen and used (even with incorrect sub) scores M1. With no general formula, award M1 if the sub in the formula is correct for their equation.