Questions

Q1.

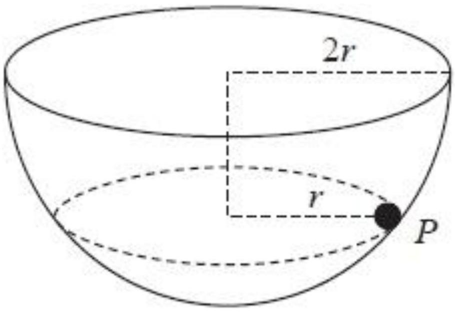


Figure 1

A hemispherical bowl of internal radius $2r$ is fixed with its circular rim horizontal. A particle P is moving in a horizontal circle of radius r on the smooth inner surface of the bowl, as shown in Figure 1. Particle P is moving with constant angular speed ω.

Show that $\omega = \sqrt{\dfrac{g\sqrt{3}}{3r}}$

(6)

(Total for question = 6 marks)

Q2.

A car of mass 800 kg is driven at constant speed v m s^{-1} round a bend in a race track. Around the bend, the track is banked at 20° to the horizontal and the path followed by the car can be modelled as a horizontal circle of radius 20 m. The car is modelled as a particle. The coefficient of friction between the car tyres and the track is 0.5

Given that the tyres do not slip sideways on the track, find the maximum value of v.

(9)

(Total for question = 9 marks)

Q3.

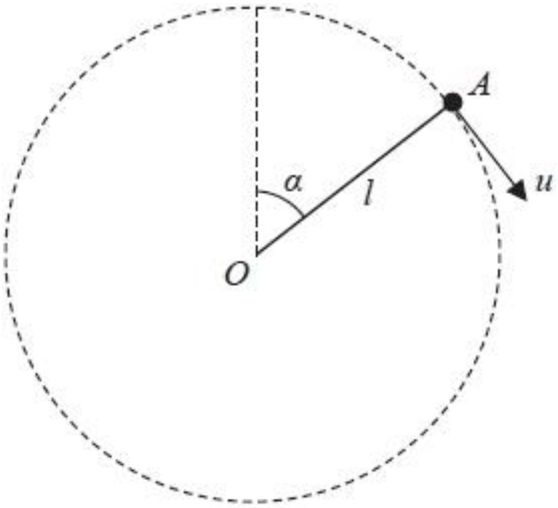


Figure 4

A particle of mass m is attached to one end of a light rod of length l. The other end of the rod is attached to a fixed point O. The rod can turn freely in a vertical plane about a horizontal axis through O. The particle is projected with speed u from a point A, where OA makes an angle α with the upward vertical through O, as shown in Figure 4. The particle moves in complete vertical circles.

Given that $\cos\alpha = \frac{4}{5}$

(a) show that $u > \sqrt{\frac{2gl}{5}}$

(4)

As the rod rotates, the least tension in the rod is T and the greatest tension is $4T$.

(b) Show that $u = \sqrt{\frac{17}{5}gl}$

(11)

(Total for question = 15 marks)

Q4.

A particle is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point A. The particle moves with constant angular speed ω in a horizontal circle. The centre of the circle is vertically below A and the radius of the circle is r.

Show that

$$\omega^2 = \frac{g}{\sqrt{l^2 - r^2}}$$

(8)

(Total for question = 8 marks)

Q5.

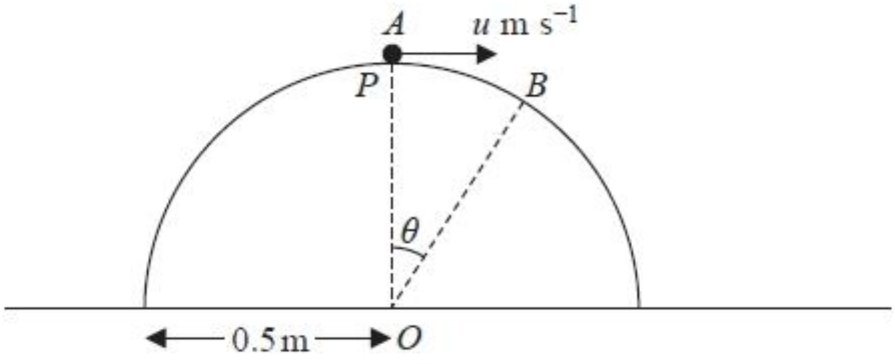


Figure 1

A smooth solid hemisphere of radius 0.5m is fixed with its plane face on a horizontal floor. The plane face has centre O and the highest point of the surface of the hemisphere is A. A particle P has mass 0.2kg. The particle is projected horizontally with speed u m s^{-1} from A and leaves the hemisphere at the point B, where OB makes an angle θ with OA, as shown in Figure 1. The point B is at a vertical distance of 0.1m below the level of A.
The speed of P at B is v m s^{-1}

(a) Show that $v^2 = u^2 + 1.96$

(3)

(b) Find the value of u.

(4)

The particle first strikes the floor at the point C.

(c) Find the length of OC.

(7)

(Total for question = 14 marks)

Q6.

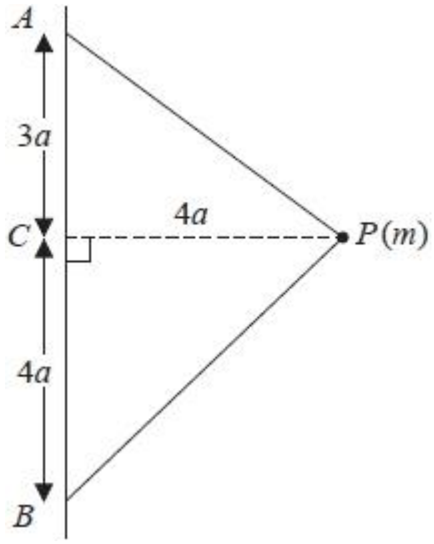


Figure 3

A light inextensible string has its ends attached to two fixed points A and B. The point A is vertically above B and $AB = 7a$. A particle P of mass m is fixed to the string and moves with constant angular speed ω in a horizontal circle of radius $4a$. The centre of the circle is C, where C lies on AB and $AC = 3a$, as shown in Figure 3. Both parts of the string are taut.

(a) Show that the tension in AP is $\frac{5}{7}m(4a\omega^2 + g)$

(8)

(b) Find the tension in BP.

(2)

(c) Deduce that $\omega \geq \sqrt{\frac{g}{ka}}$, stating the value of k.

(2)

(Total for question = 12 marks)

Q7.

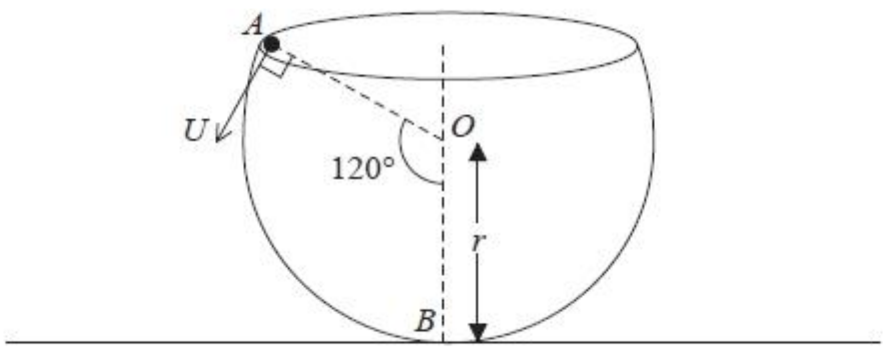


Figure 4

A hollow sphere has internal radius r and centre O. A bowl with a plane circular rim is formed by removing part of the sphere. The bowl is fixed to a horizontal floor with the rim uppermost and horizontal. The point B is the lowest point of the inner surface of the bowl. The point A, where angle AOB = 120°, lies on the rim of the bowl, as shown in Figure 4. A particle P of mass m is projected from A, with speed U at 90° to OA, and moves on the smooth inner surface of the bowl. The motion of P takes place in the vertical plane OAB.

(a) Find, in terms of m, g, U and r, the magnitude of the force exerted on P by the bowl at the instant when P passes through B.

(8)

(b) Find, in terms of g, U and r, the greatest height above the floor reached by P.

(4)

Given that $U > \sqrt{2gr}$

(c) show that, after leaving the surface of the bowl, P does not fall back into the bowl.

(5)

(Total for question = 17 marks)

Q8.

A rough disc is rotating at a constant angular speed of 5 revolutions per minute about a vertical axis. The axis is perpendicular to the plane of the disc and passes through the centre of the disc. A particle, P, of mass m kg is placed on the disc at distance 0.2 m from the axis. The particle does not move relative to the disc. The coefficient of friction between P and the disc is μ.

Find the smallest possible value of μ.

(6)

(Total for question = 6 marks)

Q9.

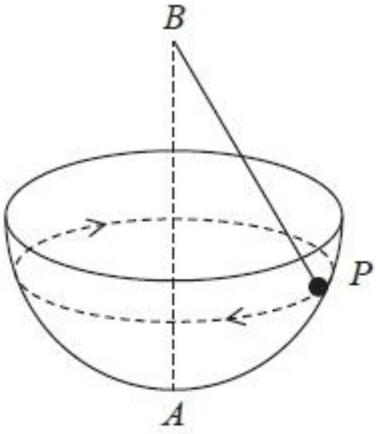


Figure 1

Figure 1 shows a hemispherical bowl of internal radius a fixed with its open plane face uppermost and horizontal. The lowest point of the bowl is A. A light inextensible string of length $a\sqrt{3}$ has one end fixed to the point B, where B is vertically above A and $AB = 2a$. A particle, P, of mass m is attached to the other end of the string.

The particle moves in a horizontal circle on the smooth inner surface of the bowl with constant angular speed ω. The string remains taut and the particle remains in contact with the bowl throughout the motion.

(a) Find, in terms of m, a, ω and g, the tension in the string.

(7)

(b) Show that $\omega \geqslant \sqrt{\dfrac{2g}{3a}}$

(4)

(Total for question = 11 marks)

Q10.

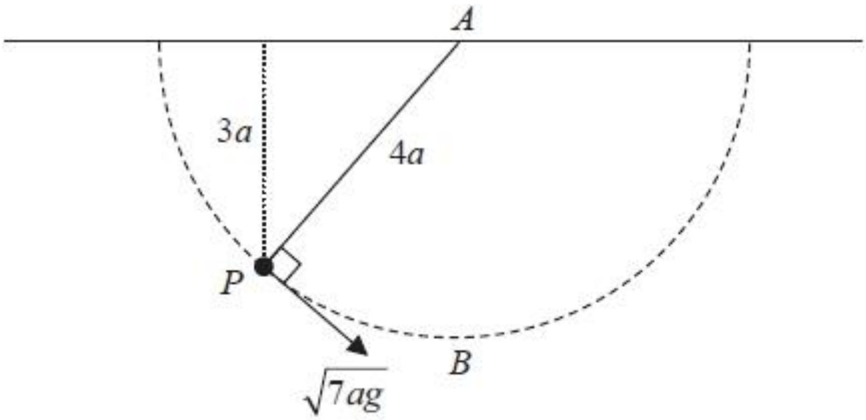


Figure 2

One end of a light inextensible string of length 4*a* is attached to a fixed point *A* on a horizontal ceiling. A particle, *P*, of mass *m* is attached to the other end of the string. The particle is held in equilibrium at a vertical distance 3*a* below the ceiling, with the string taut. The particle is then projected with speed $\sqrt{7ag}$, in the direction perpendicular to the string, in the vertical plane containing *A* and the string, as shown in Figure 2. In the subsequent motion the string remains taut.

(a) Find the speed of *P* at the instant before it hits the ceiling.

(4)

The point *B* is the lowest point of the path of *P*. The first time *P* passes through *B* the tension in the string is T_1 and the second time *P* passes through *B* the tension in the string is T_2

Given that the coefficient of restitution between *P* and the ceiling is $\frac{1}{2}$

(b) find the ratio T_1: T_2 in its simplest form.

(7)

(Total for question = 11 marks)

Q11.

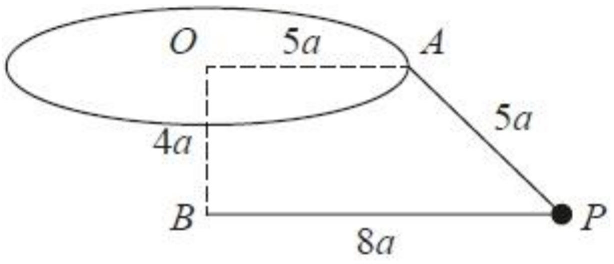


Figure 2

One end of a light inextensible string of length 5*a* is attached to a point *A* on the circumference of a circular disc of radius 5*a* and centre *O*. The other end of the string is attached to a particle *P* of mass *m*. A second light inextensible string has length 8*a*. This string has one end attached to the point *B* vertically below *O*, where *OB* = 4*a*, and the other end attached to *P*, as shown in Figure 2. The disc rotates in a horizontal plane with constant angular speed. The particle *P* moves in a horizontal circle centre *B* with the same constant angular speed as the disc. Both strings are taut and are in a vertical plane through *O* throughout the motion.

Given that string *PB* will break if the tension in it exceeds $\frac{5}{4}mg$, find the greatest possible angular speed of *P*.

(Total for question = 10 marks)

Q12.

A particle *P* moves in a vertical circle, with centre *O* and radius *r*, on the smooth inner surface of a fixed hollow spherical shell. The point *A* lies on the inner surface of the shell and the line *OA* is at 30° below the horizontal. Initially *P* is projected downwards from *A* with speed *u* in a direction perpendicular to *OA*. The particle first loses contact with the shell at the point *B* where the line *OB* is at 30° above the horizontal.

(a) Show that the speed of *P* at *B* is $\sqrt{\frac{rg}{2}}$

(4)

(b) Hence find *u* in terms of *r* and *g*.

(4)

After *P* has lost contact with the shell, *P* moves freely under gravity.

The lowest point of the inner surface of the shell is *C*.

(c) Find the greatest height reached by *P* above the level of *C*.

(5)

(Total for question = 13 marks)

Q13.

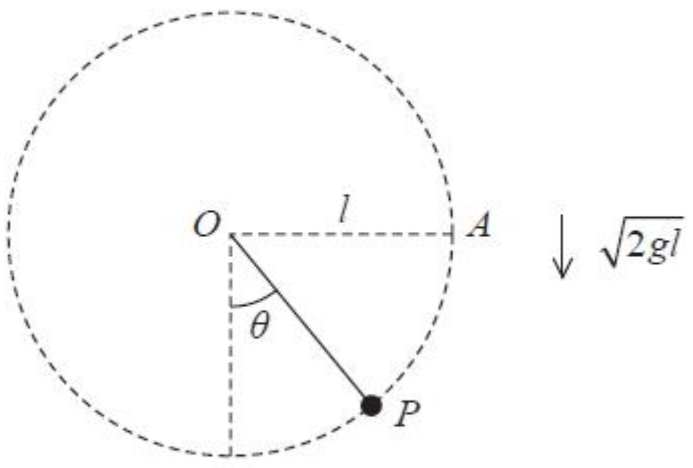


Figure 4

A particle P of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point O. The particle is held at the point A, where $OA = l$ and OA is horizontal.

The particle is then projected vertically downwards from A with speed $\sqrt{2gl}$, as shown in Figure 4. When the string makes an angle θ with the downward vertical through O and the string is still taut, the tension in the string is T.

(a) Show that $T = mg(3\cos\theta + 2)$

(7)

At the instant when the particle reaches the point B, the string becomes slack.

(b) Find the speed of P at B.

(3)

(c) Find the greatest height above O reached by P in the subsequent motion.

(5)

(Total for question = 15 marks)

Q14.

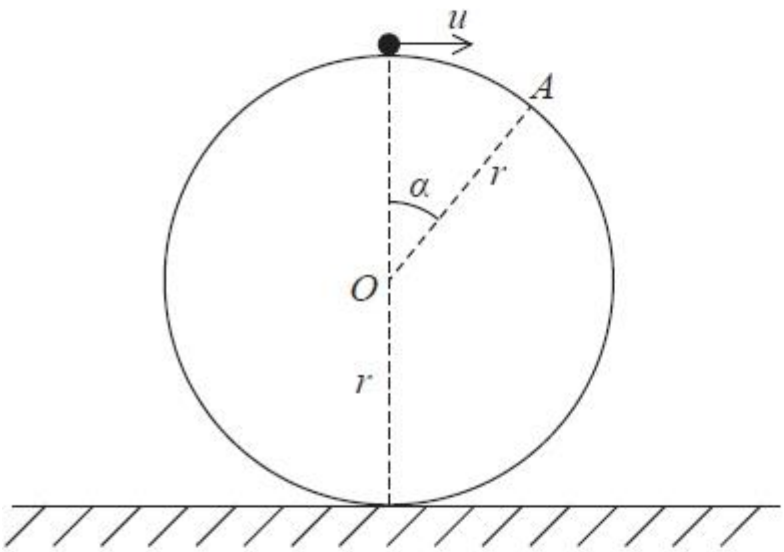


Figure 4

A smooth solid sphere, with centre *O* and radius *r*, is fixed with its lowest point on a horizontal plane. A particle is placed on the surface of the sphere at the highest point of the sphere. The particle is then projected horizontally with speed *u* and starts to move on the surface of the sphere. The particle leaves the surface of the sphere at the point *A* where *OA* makes an angle α, $\alpha > 0$, with the upward vertical, as shown in Figure 4.

(a) Show that $\cos\alpha = \frac{1}{3gr}\left(u^2 + 2gr\right)$

(7)

(b) Show that $u < \sqrt{gr}$

(2)

After leaving the surface of the sphere, the particle strikes the plane with speed $3\sqrt{\frac{gr}{2}}$

(c) Find the value of $\cos\alpha$.

(5)

(Total for question = 14 marks)

Q15.

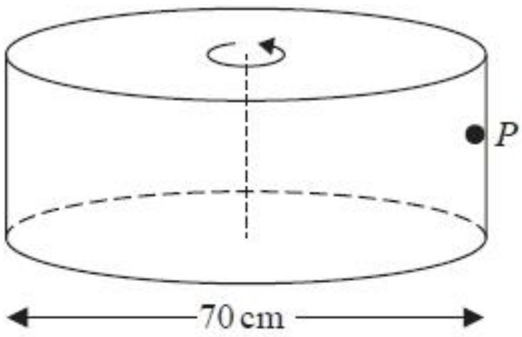


Figure 1

Figure 1 shows a hollow cylinder with diameter 70 cm. The cylinder is rotating at a constant angular speed about its axis, which is vertical. As the cylinder rotates, a particle *P* remains in contact with the same point on the rough inside surface of the cylinder. The particle is moving in a horizontal circle of diameter 70 cm.

The cylinder makes 2 complete revolutions every second.

The coefficient of friction between *P* and the cylinder is μ.

Find the range of possible values of μ.

(Total for question = 6 marks)

Q16.

The path followed by a motorcycle round a circular race track is modelled as a horizontal circle of radius 50 m. The track is banked at an angle θ to the horizontal, where $\sin\theta = \frac{3}{5}$. The motorcycle travels round the track at constant speed. The motorcycle is modelled as a particle and air resistance can be ignored. In an initial model it is assumed that there is no sideways friction between the motorcycle tyres and the track.

(a) Find the speed, in m s^{-1}, of the motorcycle.

(5)

In a refined model it is assumed that there is sideways friction. The coefficient of friction between the motorcycle tyres and the track is $\frac{1}{4}$. It is still assumed that air resistance can be ignored and that the motorcycle is modelled as a particle. The motorcycle's path is unchanged. Using this model,

(b) find the maximum speed, in m s^{-1}, at which the motorcycle can travel without slipping sideways.

(8)

(Total for question = 13 marks)

Q17.

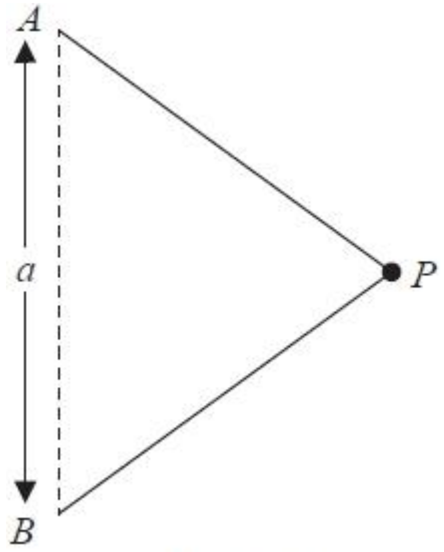


Figure 1

A small ball P of mass m is attached to the midpoint of a light inextensible string of length $2a$. The ends of the string are attached to fixed points A and B, where A is vertically above B and $AB = a$, as shown in Figure 1. The system rotates about the line AB with constant angular speed ω. The ball moves in a horizontal circle with both parts of the string taut. The tension in the string must be less than 3 mg otherwise the string will break.

Given that the time taken by the ball to complete one revolution is S, show that $\pi\sqrt{\frac{a}{g}} < S < \pi\sqrt{\frac{ka}{g}}$ stating the value of the constant k.

(Total for question = 12 marks)

Q18.

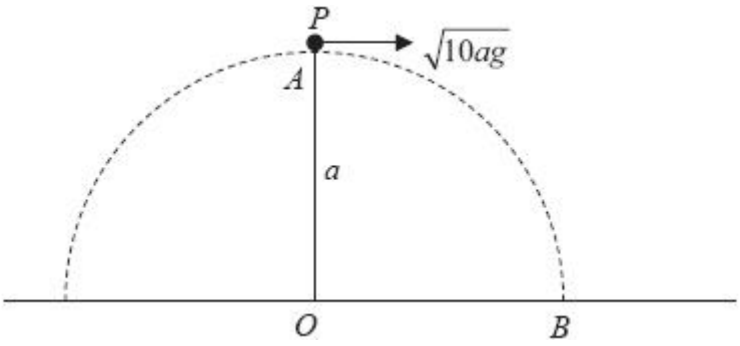


Figure 2

A light inextensible string of length a has one end attached to a fixed point O on a horizontal plane. A particle P is attached to the other end of the string. The particle is held at the point A, where A is vertically above O and $OA = a$. The particle is then projected horizontally with speed $\sqrt{10ag}$, as shown in Figure 2. The particle strikes the plane at the point B. After rebounding from the plane, P passes through A. The coefficient of restitution between the plane and P is e.

(a) Show that $e \geqslant \frac{1}{2}$

(9)

The point C is above the horizontal plane such that $OC = a$ and angle $COB = 120°$

As the particle reaches C, the string breaks. The particle now moves freely under gravity and strikes the plane at the point D.

Given that $e = \frac{\sqrt{3}}{2}$

(b) find the size of the angle between the horizontal and the direction of motion of P at D.

(6)

(Total for question = 15 marks)

Q19.

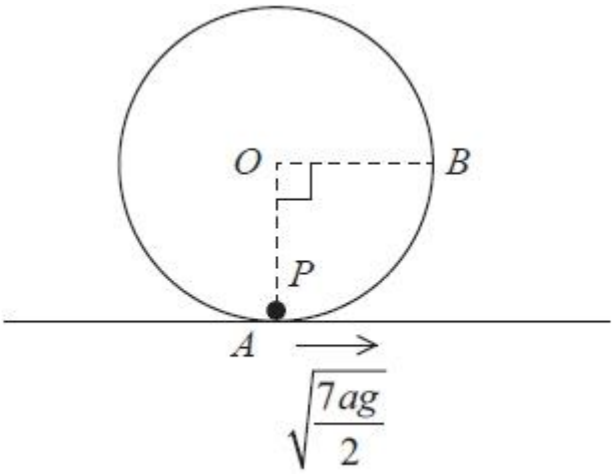


Figure 5

Figure 5 shows a hollow sphere, with centre O and internal radius a, which is fixed to a horizontal surface.

A particle P of mass m is projected horizontally with speed $\sqrt{\frac{7ag}{2}}$ from the lowest point A of the inner surface of the sphere. The particle moves in a vertical circle with centre O on the smooth inner surface of the sphere. The particle passes through the point B, on the inner surface of the sphere, where OB is horizontal.

(a) Find, in terms of m and g, the normal reaction exerted on P by the surface of the sphere when P is at B.

(5)

The particle leaves the inner surface of the sphere at the point C, where OC makes an angle θ, $\theta > 0$, with the upward vertical.

(b) Show that, after leaving the surface of the sphere at C, the particle is next in contact with the surface at A.

(11)

(Total for question = 16 marks)

Q20.

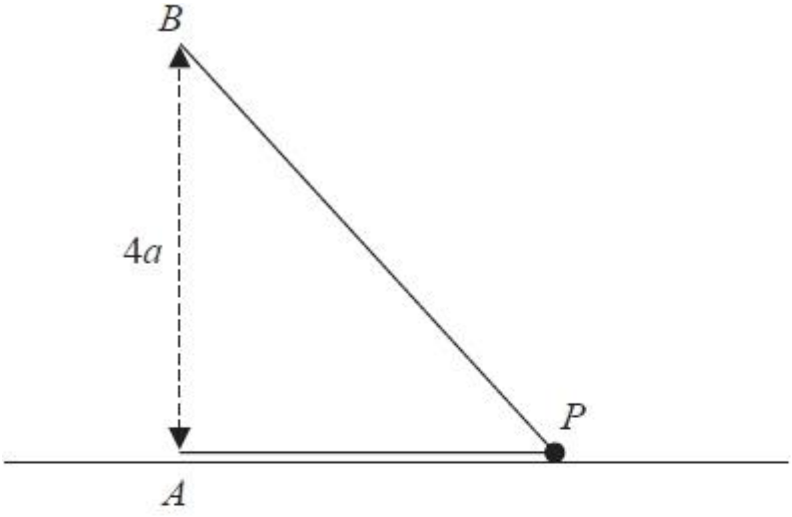


Figure 2

A small smooth bead P is threaded on a light inextensible string of length $8a$. One end of the string is attached to a fixed point A on a smooth horizontal table. The other end of the string is attached to the fixed point B, where B is vertically above A and $AB = 4a$, as shown in Figure 2. The bead moves with constant angular speed, in a horizontal circle, centre A, with AP horizontal. The bead remains in contact with the table and both parts of the string, AP and BP, are taut. The time for P to complete one revolution is S.

Show that $S \geqslant \pi\sqrt{\frac{6a}{g}}$

(Total for question = 12 marks)

Examiner's Report

Q1.

There were a great many excellent solutions using the method on the mark scheme but a substantial minority reached the required answer by other means. Solutions which may or may not have been based on a triangle proved impossible to mark consistently and it was sometimes hard to judge whether solutions deserved 0 or 6. The statement $mg \tan \theta = m\omega^2 r$ was completely convincing if a relevant triangle was actually shown on the original diagram but without a visible triangle it was not clear whether the equation had been legitimately obtained or just written down because changing $\tan\theta$ to $\sqrt{3}$ led to the given result. Early substitution of 60° for θ was a good way for a candidate to convince the examiner that they knew how to deal correctly with the problem. The most frequent error among the weakest was to use $R - mg \cos \theta = m\omega^2 r$ as if this were a vertical circle. Even these false starts often apparently reached the required answer!

Q2.

A few good (or maybe well-prepared) candidates answered this very well but the majority gave the impression that they hadn't studied banked tracks at all. The modal mark, even for good candidates, was 0. The most surprising fact was that relatively few candidates even attempted to draw a diagram and yet they went ahead and resolved forces they couldn't see using angles they hadn't properly identified. Most started with the wrong statement that $R = mg \cos \theta$ and continued with either an equation assuming that the acceleration was along the slope (in spite of the question clearly stating that the motion was in a horizontal circle) or, less frequently, a correct horizontal equation. Other errors were then irrelevant but it was noticeable that wrong signs occurred frequently in these wrong equations due either to friction being in the wrong direction or $\frac{mv^2}{r}$ being treated as an extra force rather than a mass × acceleration.

A few candidates produced the correct answer directly from the two original equations without showing any intermediate working. They must either have made extensive use of their calculator memories or done working in pencil which they then rubbed out. This is unwise. If no intermediate working is shown, no credit can be given if the answer is wrong.

Some otherwise excellent solutions lost the last mark by leaving the answer as 14.39.

Q3.

In part (a) candidates needed to make it clear that $v > 0$ at the top for complete circles before they could score any marks. Most used an energy equation successfully but those who put $v = 0$ with no explanation lost all the marks. Some thought that the conditions was $T > 0$ at the top so also scored zero.

There were some very good clear solutions for part (b) but there were others where candidates did not know where the tension was greatest or least and who used u indiscriminately to represent all their velocities. The candidates who used Newton's second law at the top and at the bottom and also found the corresponding velocities using energy measured from A were generally successful. Others used energy to find the velocity at the top and then another energy equation from top to bottom and were also successful. A common error was to assume that the v in Newton's second law at the top was the same as the v in the equation at the bottom. These candidates lost many marks as they had omitted essential equations from their work.

Q4.

This was a straightforward and familiar question which produced a very good response. Nearly all candidates resolved vertically and obtained an equation of motion horizontally with just one or two using a 'triangle of forces' type of approach. A small number of candidates arrived at the correct solution without resolving and these always seemed to be working back from the given answer. It is possible that they arrived at their opening line by considering a triangle of forces, but without actually showing the triangle this counted as being "without sufficient working" and no marks were given. A very small number misread the question and had the particle moving in a vertical circle.

Q5.

Generally this question was answered well. Part (a) proved very straightforward and was answered well by almost all candidates. Occasionally $v^2 = u^2 + 2as$ was used rather than energy and scored no marks. In part (b) most candidates were familiar with motion in a vertical circle and wrote down the equation of motion in the radial direction, equated the reaction to zero and, after substituting for v at some stage, obtained a value for u. It was disappointing that some did not realise that at B the reaction was zero and so could make no further progress.
The solution for the projectile motion in part (c) relies on finding the components of the velocity at B and if this was not done marks could not be scored. However some of the marks for this part of the question were available independently of part (b) and a few of those who made little progress in part (b) did manage to score well here. Having found the vertical component of the speed at B there were a number of ways to find the time of flight, some more involved than others. The method in the mark scheme using $s = ut + \frac{1}{2}at^2$ was the most common and successful with candidates using their calculator functions to write down the roots of their quadratic. One alternative seen involved finding the vertical component of velocity at C using $v^2 = u^2 + 2as$ and then $v = u + at$. Most then attempted to find the horizontal distance moved using the horizontal component of velocity but then quite a few stopped at this stage or added 0.5 instead of 0.3 to find OC.
A significant number failed to give their final answer correct to 2 or 3 sf or rounded incorrectly, with 0.6m and 0.599m often seen.

Q6.

Again, the correct method was very well known, especially in (a) and (b). Most errors arose from wrongly identified values for the trigonometric ratios involved. Some decided that the angles were the same for both strings, others that an angle of 60° must be involved somewhere. In (c), although the majority knew the correct starting point for the inequality, those who did not wasted considerable time exploring a variety of alternatives. It should be noted that a question which asks for a result involving ... does not award marks for one using >.

Q7.

For those who knew the relevant theory – a substantial majority - part (a) proved to be very straightforward but some students seemed to have little idea how to get started. Omission of either the energy equation or of the *mg* term in $R - mg = \frac{mv^2}{r}$ both lost five of the eight marks and were not uncommon.

The most common mark for (b) was 0. The preferred method was to conserve energy from the bottom to the top but overlooking the fact that a KE associated with the horizontal velocity needed to be included at the highest point. An easier, and correct, method used the vertical component of *u* and *suvat* equations to find the height above the rim, finally adding $3r/2$.

There were some excellent, clearly reasoned solutions for (c) but these were very much in the minority. Most attempts were muddled, poorly explained and often quite wrong. It was not always realised that energy considerations meant that the velocity of the particle when it reached the rim must be *U*, the same as initially, and quite a few students tried to use the completely irrelevant velocity at the lowest point, found in (a), to investigate the further flight. Others attempted solutions using their greatest height from (b) but, unfortunately, this was usually wrong. There were so many possible methods available here that it was extremely difficult to award method marks to the wrong working unless there was some indication of what had been intended. Horizontal and vertical velocity components and expressions representing times and distances needed to be clearly identified before they could be judged worthy of marks. Quite a few students confused matters further by using

$U = \sqrt{2gr}$ as a given initial velocity. Inequalities should always be treated with caution. It was required to show that a velocity **greater** than this resulted in the particle landing outside the bowl. It is very difficult to write a convincing justification of the further distance travelled unless the inequality has been incorporated into the working.

A final point to make was the number of students who made no errors but then claimed to have shown that the particle landed outside the bowl without ever having mentioned the distance across the rim. Finding this distance was an easy trigonometrical calculation and may have been thought obvious but in a question asking for a result to be shown it is essential to state all the necessary information. This omission lost the final 2 marks.

Q8.

Most candidates knew how to approach this question and the majority gained at least 4 marks. However, many candidates did not know how to find ω, and there was confusion between radians per second and revs per minute. It was also not uncommon for there to be confusion between T and ω. Most candidates did know how to use what they thought was ω, to find a minimum value for m, although some had their inequality the wrong way round. Some candidates went straight to an inequality, without considering the frictional force and centripetal force separately.

Q9.

This was the first question to cause candidates significant difficulties, largely due to the problem of correctly finding the angle. Many candidates struggled to get to grips with the geometry of the situation. If the angle was not found, marks were still available for attempted resolution and solving for T. If the angle (or correct trig values) were found, candidates generally managed to successfully find an expression for T.

Part (b) was generally not answered well, with many candidates setting $T \geq 0$, rather than R and losing all marks. If part (a) was correct and $R \geq 0$ used, then the given result was almost always correctly reached.

Q10.

Part (a) proved straight forward for most candidates and the majority scored full marks. In part (b), most candidates realised that they needed to use energy to find the velocities as the particle passed through the lowest point and most attempted to use restitution correctly for the second velocity. There were, however, frequent mistakes in the resolution, with many forgetting to include weight, using a for the radius instead of $4a$, or making a sign error. A very large number of correct solutions gave a final answer of 52/49, rather than the asked for ratio 52 : 49. Whilst this was accepted, it should be noted that the expectation when a ratio is asked for, is for an answer of the form $a : b$.

Q11.

This question was well answered, with most candidates clearly being comfortable with the approach required for horizontal circular motion. Very few had difficulties with the trigonometry and generally 2 correct equations were produced, with very little confusion over the required radius for the motion. The final 3 marks, however, discriminated between the candidates that really understood the mechanics and how to form a clear argument. Many simply said that $T_b = \frac{5}{4}mg$, with no attempt to justify why this should lead to the maximum angular velocity, losing the last 3 marks. Most candidates that correctly worked from $T_b \leq \frac{5}{4}mg$, did go on to give a value for ω, rather than leaving an inequality.

Q12.

This proved to be very straight forward for many candidates and there were plenty of fully correct responses. However there were still a surprising number of errors made, particularly in part (c).

In part (a) most candidates gained the full 4 marks, although this was often achieved after far more work than was necessary. Many candidates automatically considered energy, completing the work for part (b) and producing very complicated equations of motion, although generally the correct result was found with sufficient working. Candidates should be reminded that the requirements of a show that question are greater than others. In this case, we needed to clearly see the use of correct trigonometry. Candidates who went straight to $\frac{mg}{2}$, which led almost immediately to the given result, could not score full marks

Part (b) was well answered and often followed on from correct working in part (a). Incorrectly using SUVAT was rarely seen. A small number of candidates made a sign error or there was confusion of sin/cos in their energy equation.

Part (c) was often answered well but there were a significant number of errors seen. Most candidates who attempted to use energy in the projectile motion were unsuccessful. They did not usually consider that there was still kinetic energy at the top of the motion, or they used only the vertical speed in the initial kinetic energy. Of those who used SUVAT sometimes the initial speed was not resolved at all or vsin30 was used instead of vsin60. It was not uncommon to see u, the speed at A, from part (b) being used in some way which scored no marks. A significant number of candidates only added r instead of 1.5r to the height above B essentially finding the height above A rather than C, emphasising again the need to read the question carefully. Likewise some candidates misinterpreted the question and attempted to find the height when the particle was above C rather than the greatest height.

Q13.

In part (a) the energy equation in was usually correct although a few students made a mistake with the PE term. In the equation of motion sometimes the tension was not resolved and there were occasional sign errors.

Part (b) was also done well by most students but some wasted time by not using their energy equation from part (a), and others forgot the demand was to find v and so failed to take the square root of v^2.

Part (c) was much more challenging. Students did not always make it clear which component they were using and some did not resolve at all. Those who did resolve sometimes had sine and cosine the wrong way round − a clear diagram would have helped. There was then confusion over whether to use suvat or energy and when energy was used, some used the original velocity instead of the velocity at *B*. A few lost the last mark, forgetting that they had to add on the height of *B* above *O*.

Q14.

Part (a) This was usually done well. The energy equation from the top to A was almost always correct. The equation of motion at A was often written down correctly without reference to the reaction force, *R*. If *R* was included this was usually correct but sometimes there was a sign error seen. Occasionally the student did not resolve the weight.

Part (b) Many correct solutions were seen usually starting from $\cos \alpha < 1$ although some students did use $R_{top} > 0$. However many students did not know where to start, so either left it blank or tried an invalid approach to produce the given answer. Others used equations and an inequality was never seen although it was given on the paper.

Part (c) The method on the main scheme was by far the most straightforward but the majority did not use it. Energy from A to the plane was the most common but many were unclear about u and v and used u as the speed at *A* instead of using v from part (a). Only a handful of approaches using *suvat* were successful, with many not scoring any marks as they did not consider vertical and horizontal components separately. In many cases, students lost marks through not being able to read their own writing with u and v often being almost identical.

Q15.

This question proved problematic for some. There were many fully correct answers, but others clearly had problems with a situation which they seemed unfamiliar with. Almost all found the correct angular speed. It was a very common and costly error to assume that the friction caused the centripetal force and that R=mg. Candidates often worked with $F = \mu R$ rather than working with an inequality although many did adjust to the final correct inequality at the end. It was common for candidates to incorrectly assume that $\mu < 1$ in their final answer, perhaps thinking that a "range" needed an upper limit as well. A significant minority used $F \geq \mu R$. Some correct responses did not explicitly state that the friction was equal to the weight but wrote mg $\leq \mu R$.

There were a small but surprising number of responses that made the basic errors of using the diameter instead of the radius or using the radius as 35cm rather than in metres.

Q16.

There were many fully correct answers to this question but there were a significant number of poor answers which proved costly.

Part (a) was answered well by many, who obtained two correct equations which almost always led to the correct answer. However there were some poor solutions with, for example, $R = mg\cos\theta$ from an attempt to resolve perpendicular to the plane. One single equation $mg\sin\theta\cos\theta = \frac{mv^2}{50}$ was sometimes seen but the most common error was to assume the force down the slope caused the circular motion: $mg\sin\theta = \frac{mv^2}{50}$. A handful of incorrect solutions resolved the radius using $r = 50\cos\theta$ instead of 50.

Part (b) caused more problems than part (a) although was still answered well by many. Almost all attempted horizontal and vertical equations rather than attempting equations parallel and perpendicular to the slope. The most common error was in the calculation of R. It was often assumed that R was the same as it was in part (a) or that the motorcycle was in equilibrium perpendicular to the plane with $R = mg\cos\theta$ used. This error was often seen accompanied by a full correct horizontal equation of motion, but not always. There was the occasional missing term or an attempt to apply $F = ma$ down the slope without resolving the component of $\frac{mv^2}{r}$ into $\frac{mv^2}{r}\cos\theta$. Another common error was to have the friction force going the wrong way or to have a single sign error in the equations. Sine/cosine confusion was rarely seen. A few very poor answers did not explicitly state what the expression for R was and so it was also unclear that $F = \frac{1}{4}R$ had been used. It is worth advising students to write down all relevant equations before combining them to avoid losing more marks. Again a significant number of students lost one mark through over-specifying their answers following the use of $g = 9.8 \text{ m s}^{-2}$.

Q17.

Not all candidates realised that the tensions in the two strings were different and that the angles were 30° and 60°. Those who did generally resolved vertically and used N2L along the radius scoring the first four marks. A few failed to calculate the radius correctly.

After that, a variety of methods was seen. Some did not find T_A and T_B specifically and others just found one and used it together with the resolution equation to continue. The inequalities were not well understood – often only one was seen, which could be $T_A < 3mg$, $T_B < 3mg$, $T_B > 0$ or $T_B < mg$ since $T_A = T_B + 2mg$. $T_B < 3mg$ led to an answer which was not shown, causing some consternation.

There were very few cases of ≤ or = being used.

The next problem was connecting ω with S. Very few used S early in the solution and when it was used at the end, the direction of the inequalities sometimes caused problems even though the answer was given. Finally, some candidates calculated both inequalities correctly but failed to combine them as required for the final mark.

Q18.

Part (a) was the least well answered part of the paper. The responses were generally very poorly presented and it was often extremely hard to work out what the students were actually doing. Whilst most got as far as finding the velocity at *B* before striking the plane, most did not go on to find the velocity in terms of *e*. Where they did go on to find a second energy equation this was often labelled in an unclear way. The need to find tension at the top was missed by many students. Many did quote a learnt result about the velocity at the top for a complete circle, but many inserted an inequality into their energy equation. Having said that, a significant number did give a clear derivation of the result, either by the mark scheme method, or by finding an inequality for the velocity on leaving *B* and then introducing *e*.

Part (b) was not well presented, but many students managed to gain method marks even if their accuracy went astray. Many failed to use the restitution in the energy equation, but were still able to gain marks for finding components of their velocity at *C* and using these to find an angle. Very few realised that the speed of the particle just before reaching *C* was the same as the speed as it left *B*. In fact, it was surprising how comfortable the students were with the projectile part of the question, even if they could not cope with the preceding restitution/circular motion work. Again, it would have helped if they had written/labelled their working more clearly.

Q19.

Part (a) was successfully completed by the majority of candidates with the most common error seen being the difference of the *KE* terms being the wrong way round and a few cases of finding v rather than R.

Part (b) proved to be very challenging. Some used sine instead of cosine in N2L which led to very complicated trigonometry. Others only used energy and forgot about N2L so continued with v in terms of θ, never finding a value for the size of the angle. The method on the main scheme was not used often. It was more common to see a vertical equation of motion being used first but, because this led to a quadratic, correct solutions were rarely seen and if $\cos\theta$ had not been found the trigonometry was horrendous. Some used $s = ut + \frac{1}{2}at^2s$ but others used $v^2 = u^2 + 2as$ together with $v = u + at$ which tended to be more successful. The components of velocity were not always found correctly, with sine and cosine being interchanged.

There were some completely correct solutions but others gave up when faced with horrendous equations containing powers of sine and cosine. Fortunately it was the last question; some candidates were perhaps short of time as (b) was not attempted.

Q20.

This was a "show that" question and consequently it was essential to find the radius of the circle using Pythagoras or as a minimum justify the existence of a (3,4,5) triangle. Failure to do this cost many students the first two and last mark.

Very common errors were to omit *R*, the reaction, when resolving vertically and to use different tensions when finding the equation of motion along the radius. Omitting R was a very expensive error unless an inequality was introduced at this stage which implied that *R* was greater than zero. Students who did this could go on to obtain full marks but those who worked with equations throughout had the correct expressions but lost a minimum of seven marks. Those who thought the tensions were different sometimes realised that they were the same later and could retrospectively obtain the first A mark in their equation of motion.

Some thought that the inequality was obtained by using an inequality for *T* rather than for *R* even when their equations included *R*.

There were occasional problems with the directions of the inequalities. A few students substituted for ω in terms of *S* early on which led to the correct answer more easily.

Mark Scheme

Q1.

Question Number	Scheme	Marks
	$R\sin 60° = mg$	M1A1
	$R\cos 60° = mr\omega^2$	M1A1
	$\tan 60° = \frac{g}{r\omega^2}$	
	$\omega = \sqrt{\frac{g}{r\sqrt{3}}} = \sqrt{\frac{g\sqrt{3}}{3r}}$ *	ddM1A1cso [6]
M1 A1 M1 A1 ddM1 A1cso	Resolve vertically, allow with $60°$ or θ Correct equation , with $60°$ or θ Equation of motion along radius, acceleration in either form, $60°$ or θ Correct equation , with $60°$ or θ acceleration to be $r\omega^2$ now Eliminate R, substitute a numerical value for θ and solve to $\omega = ...$ or $\omega^2 = ...$ Depends on both previous M marks Complete to the given answer	

Q2.

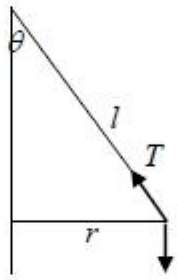
Question Number	Scheme	Marks
	R(↑) $R\cos 20° = F\cos 70° + 800g$	M1A1A1
	NL2(→) $R\cos 70° + F\cos 20° = 800\frac{v^2}{20}$	M1A1A1
	$F = \mu R = 0.5R$	
	$R\cos 20° = 0.5R\cos 70° + 800g$	
	$R = \frac{800g}{(\cos 20° - 0.5\cos 70°)}$	
	$v^2 = \frac{1}{40}\frac{800g(\cos 70° + 0.5\cos 20°)}{(\cos 20° - 0.5\cos 70°)}$ OR $\frac{v^2}{20g} = \frac{(\cos 70° + 0.5\cos 20°)}{(\cos 20° - 0.5\cos 70°)}$	ddM1
	$v = 14.38... = 14.4$ or 14 m s^{-1}	dddM1A1cao [9]
M1 A1A1 M1 A1 A1 ddM1 dddM1 A1	Resolve vertically A1A1 if equation completely correct; A1A0 one error; A0A0 two or more errors Equation of motion along the radius, acceleration in either form LHS correct RHS correct inc acceleration in form $\frac{v^2}{r}$ m or 800 for these 2 marks Use $F = \mu R$ and eliminate R to obtain $v^2 = ...$ Depends on both previous M marks Complete to a numerical value for v^2 or v All previous M marks needed. Correct value of v 2 or 3 sf only	

Q3.

Question Number	Scheme	Marks
(a)	For complete circles there must be a speed at the top. (or equiv statement)	B1
	$\frac{1}{2}mu^2 > \frac{mgl}{5}$	M1A1
	$u > \sqrt{\frac{2gl}{5}}$ *	A1 (4)
(b)	NL2 at bottom $T_{max} - mg = m\frac{v^2}{l}$	M1A1
	Energy to bottom $\frac{1}{2} \times mv^2 - \frac{1}{2} \times mu^2 = mgl(1+\cos\alpha) = \frac{mg \times 9}{5}$	M1A1
	$T_{max} = mg + \frac{m}{l}\left(\frac{18gl}{5} + u^2\right)$	A1
	NL2 at top $T_{min} + mg = m\frac{v^2}{l}$	M1A1
	$T_{min} = \frac{m}{l}\left(u^2 - \frac{2}{5}gl\right) - mg$	M1A1
	$4\left(\frac{m}{l}\left(u^2 - \frac{2}{5}gl\right) - mg\right) = mg + \frac{m}{l}\left(\frac{18gl}{5} + u^2\right)$	ddddM1
	$4\left(u^2 - \frac{2}{5}gl - gl\right) = gl + \frac{18gl}{5} + u^2$	
	$u = \sqrt{\frac{51gl}{15}} = \sqrt{\frac{17gl}{5}}$ *	A1cso (11)
		[15]

(a)	
B1	Statement (oe) shown seen in working.
M1	Use energy (and above statement) to obtain an inequality for u^2
A1	Correct inequality
A1	Deduce the given result
ALT	Energy equation including speed at top
B1,M1	State $v^2 > 0$ and use in energy equation to obtain an inequality
A1A1	As above
(b)	
M1	Attempt NL2 at bottom. Acceleration in either form.
A1	Correct equation inc acceleration in $\frac{v^2}{r}$ form
M1	Energy equation from A to lowest point
A1	Correct energy equation
A1	Eliminate v^2 to obtain the max tension in terms of m, g, l u as shown oe
M1	Attempt NL2 at top. Acceleration in either form.
A1	Correct equation inc acceleration in $\frac{v^2}{r}$ form
M1	Use energy to top to obtain an expression for least tension in terms of m, g, l u
A1	Correct expression as shown oe
ddddM1	Connect the two tensions using 4 on either side
A1cso	Obtain the given result.

Q4.

Question Number	Scheme	Marks
		
	$T\cos\theta = mg$	M1A1
	$T\sin\theta = mr\omega^2$	M1A1
	$\tan\theta = \dfrac{r\omega^2}{g}$	DM1
	$\tan\theta = \dfrac{r}{\sqrt{l^2 - r^2}}$	B1
	$\dfrac{r\omega^2}{g} = \dfrac{r}{\sqrt{l^2 - r^2}}$	DM1
	$\omega^2 = \dfrac{g}{\sqrt{l^2 - r^2}}$ *	A1cso
		[8]

M1	Attempting to resolve vertically
A1	Completely correct equation
M1	Attempting NL2 along the radius, acceleration in either form
A1	Completely correct equation inc acceleration as $r\omega^2$
DM1	Eliminate T Dependent on 1st and 2nd M marks
B1	$\tan\theta$ or $\cos\theta$ (ie vert side of triangle used)
DM1	Eliminate the angle Dependent on 1st and 2nd M marks
A1cso	Obtain the GIVEN result.

$mg\sin\theta = mr\omega^2\cos\theta$ as initial equation scores M2A2

Triangle of forces methods must show the triangle (not just the equations) for the working to be complete.

Q5.

Question Number	Scheme	Marks
(a)	Energy A to B $\quad \frac{1}{2}\times 0.2v^2 - \frac{1}{2}\times 0.2u^2 = 0.2g\times 0.1$	M1A1
	$v^2 - u^2 = 0.2g$	
	$v^2 = u^2 + 1.96$ *	A1 cso (3)
(b)	$mg\cos\theta \ \ (-R) = m\frac{v^2}{r}$	M1
	$R = 0 \Rightarrow g\times\frac{4}{5} = \frac{(u^2+1.96)}{0.5}$	A1
	$0.4g = u^2 + 1.96$	
	$u^2 = 1.96, \ u = 1.4$ or 1.40	DM1,A1 (4)
(c)	Vert speed $= v\sin\theta = \frac{3}{5}\sqrt{0.4g}$ $(=1.1879...)$ or $\frac{21\sqrt{2}}{25}$	M1
	$0.4 = \frac{3}{5}\sqrt{0.4g}\,t + \frac{1}{2}gt^2$	M1A1ft
	$4.9t^2 + 1.1879...t - 0.4 = 0$	
	$t = \frac{-1.1879 \pm \sqrt{1.1879^2 + 4\times 4.9\times 0.4}}{9.8}$	
	$t = 0.1891... \ (t > 0)$	A1
	Horiz speed $= v\cos\theta = \frac{4}{5}\sqrt{0.4g}$	M1
	Horiz distance $= \frac{4}{5}\sqrt{0.4g}\times 0.1891... = 0.2995$	A1
	$OC = 0.3 + 0.2995... = 0.5995...$ $\quad OC = 0.60$ or 0.600 m	A1ft (7) [14]

(a)M1	Energy equation from A to B: A difference of KE terms = loss of GPE mass m, 0.2 or already cancelled. **Must** be clearly an energy equation and not uniform acceleration.
A1	Completely correct equation mass m, 0.2 or already cancelled.
A1cso	Rearrange to the GIVEN form mass m, 0.2 or previously cancelled.
(b)M1	Attempt NL2 at B. Normal contact force is zero here so need not be seen in the equation
A1	Fully correct equation with $R = 0$ and substitution for v^2
DM1	Solve for a numerical value for u^2 or u
A1	Correct value for u, must be 1.4
(c)M1	Attempt the vertical speed, $v\sin\theta$ or $v\cos\theta$, with their value for v (not their value of u) and an attempt at a numerical value for the trig function.
M1	Use $s = ut + \frac{1}{2}at^2$ with their vertical speed. Must have attempted to resolve the speed.
A1ft	Correct equation, follow through their vertical speed
A1	Correct value of t (no ft here)
M1	Attempt the horizontal speed, (trig function used to be different to that used for the vertical) with their value for v and an attempt at a numerical value for the trig function. Allow if value of u is used, provided there is an attempt to resolve.
A1	Correct horizontal distance
A1ft	Add their horizontal distance to 0.3. Answer must be 2 or 3 sf.

Q6.

Question Number	Scheme	Marks
(a)	$T_a \sin\theta = mg + T_b \sin\phi$	M1
	$\frac{3}{5}T_a = T_b \sin\phi + mg$	A1A1
	$T_a \cos\theta + T_b \cos\phi = m \times 4a\omega^2$	M1
	$\frac{4}{5}T_a + T_b \cos\phi = 4ma\omega^2$	A1A1
	$\frac{3}{5}T_a + \frac{4}{5}T_a = mg + 4ma\omega^2$ (inc use of $\cos\phi = \sin\phi$)	dM1
	$T_a = \frac{5}{7}m\left(4a\omega^2 + g\right)$ *	A1cso (8)
(b)	$\frac{1}{\sqrt{2}}T_b = 4ma\omega^2 - \frac{4}{7}m\left(4a\omega^2 + g\right)$	M1
	$T_b = \sqrt{2}\left(\frac{12}{7}ma\omega^2 - \frac{4}{7}mg\right)$ oe $\left(\text{eg } \frac{4\sqrt{2}}{7}m\left(3a\omega^2 - g\right)\right)$	A1 (2)
(c)	$T_b \geqslant 0 \Rightarrow 3a\omega^2 \geqslant g$	M1
	$\omega \geqslant \sqrt{\frac{g}{3a}}$ $k = 3$	A1 (2)
		[12]

(a)M1	Resolve vertically. Both tensions to be resolved.
A1	Both tension terms correct. θ, ϕ allowed
A1	Completely correct equation. $\sin\theta$ replaced with 3/5 now or later, $\sin\phi, \sin 45°$ or $1/\sqrt{2}$ accepted
M1	NL2 along radius. Both tensions to be resolved. Accel in either form.
A1	Both tension terms correct, θ, ϕ allowed.
A1	Completely correct equation. $\cos\theta$ replaced with 4/5 now or later, $\cos\phi, \cos 45°$ or $1/\sqrt{2}$ accepted. Acceleration to be $4a\omega^2$ now.
dM1	Eliminate T_b between the 2 equations. Dep on both previous M marks and use of $\cos\phi = \sin\phi$ or numerical values for all trig functions. There must be evidence of the solution of the pair of equations.
A1cso	Given result obtained from correct working.
(b)	
M1	Obtain T_b by any valid means. If sub T_a in one equation the given expression (or equivalent) must be used.
A1cao	Correct expression for T_b in any equivalent form.
NB	One or both marks for (b) can be gained for using the given expression for T_a in an equation obtained in (a), but watch for "circular" arguments which obtain both tensions this way. Marks for the equation used can only be awarded for work shown in (a)
(c)	
M1	Setting their $T_b \geqslant 0$ and deducing an inequality connecting ω^2 (or ω) and g
A1	Completing to the required form with a correct value for k (need not be shown explicitly)

Q7.

Question Number	Scheme	Marks
(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mU^2 = mgr(1+\cos 60)$	M1A1A1
	$R - mg = m\frac{v^2}{r}$	M1A1A1
	$R - mg = 2mg \times \frac{3}{2} + m\frac{U^2}{r}$	dM1
	$R = 4mg + m\frac{U^2}{r}$	A1 (8)
(b)	P leaves the bowl with speed U at $60°$ to the horizontal	
	Vert speed $= U\sin 60 \quad \left(= \frac{U\sqrt{3}}{2}\right)$	B1
	$0 = (U\sin 60)^2 - 2gs$	M1
	$s = \frac{3U^2}{8g}$	A1
	Greatest height $= \frac{3}{2}r + \frac{3U^2}{8g}$	A1ft (4)
(c)	$0 = U\sin 60t - \frac{1}{2}gt^2$	M1
	Time to level of rim $= \frac{2U\sin 60}{g} \quad \left(= \frac{U\sqrt{3}}{g}\right)$	A1
	Horiz speed $= U\cos 60 \ = \left(\frac{1}{2}U\right)$	
	Horiz dist $= \frac{2U^2\sin 60}{2g}$	B1
	$U^2 > 2gr \ \therefore$ horiz dist $> \frac{4rg\sin 60}{2g} = 2r\sin 60$	M1
	$\therefore$ P does not fall back into the bowl	A1cso (5) [17]

ALT 1	**Alternatives for (c)** **Work to the highest point:** $0 = U\sin 60 - gt, \quad t = \dfrac{U\sqrt{3}}{2g}$ Horiz dist to highest point $= \dfrac{U^2\sqrt{3}}{4g}$ $U^2 > 2gr \;\therefore$ horiz dist $> r\sin 60, \quad \therefore P$ does not fall back into the bowl	 M1,A1 B1 M1,A1cso

Question Number	Scheme	Marks
ALT 2	**Find the vertical height when "above" A** time to $A = \dfrac{2r\sqrt{3}}{U}$ Vert dist $= U\dfrac{\sqrt{3}}{2}\times\dfrac{2r\sqrt{3}}{U} - \dfrac{1}{2}g\dfrac{4r^2\times 3}{U^2} \left(= 3r - \dfrac{6gr^2}{U^2}\right)$ $U^2 > 2gr \;\therefore$ vert dist $> 0, \quad \therefore P$ does not fall back into the bowl	 B1 M1A1 M1,A1cso

(a)	
M1	Attempt an energy equation from A to B with 2 KE terms and a PE term (which must include a trig function.
A1	Correct KE terms (must be a difference)
A1	Correct PE term and all signs correct
M1	Attempt NL2 along the radius at B, acceleration in either form. If the equation at a general point is given this mark is not earned until the equation at B is reached.
A1	Correct difference of forces
A1	Correct mass x accel term and completely correct equation
dM1	Eliminate v between the two equations. Dependent on both previous M marks
A1	Correct expression for R. Any equivalent form accepted but must be positive.
(b)	
B1	Correct vertical speed on leaving the bowl
M1	Use $v^2 = u^2 + 2as$ with their initial vertical speed (or any other complete method) to find the greatest height of P above the rim of the bowl.
A1	Solve to obtain the correct greatest height above the rim
A1ft	Add $\frac{3r}{2}r$ to their previous answer to obtain the greatest height above the floor.
	Energy methods: (i) Using vertical speed only: marks exactly as above. (ii) If initial speed U is used, account must be taken of the horizontal energy at the top. B1 for the correct horizontal speed (or KE) M1 for the equation with correct number of terms A1A1ft as above. Attempts which do not include the horizontal energy will score 0/4 (apart from exceptional cases where the horizontal or vertical speed has been found correctly and B1 is earned)
(c)	
NB	There is no mark in (c) for the vertical speed. The mark in (b) **cannot** be awarded in (c)
M1	Use $s = ut + \frac{1}{2}at^2$ with their initial vertical speed and $s = 0$ to find the time until P returns to the level of the rim.
A1	Correct time obtained
B1	Correct horizontal distance. Allow with one, two trig functions or none
dM1	Compare their horizontal distance with their diameter of the rim using the inequality. Depends on the previous M mark
A1cso	Correct conclusion from correct work

Question Number	Scheme	Marks
	Alternatives for (c)	
ALT 1	Find the horizontal distance to the highest point	
M1	Use $v = u + at$ with their $v = 0$ to find the time until P reaches it's highest point OR use any other complete method	
A1	Correct time found	
B1	Correct horizontal distance. Allow with one, two trig functions or none	
dM1	Compare their horizontal distance with the radius of the rim using the inequality	
A1cso	Correct conclusion from correct work	
ALT 2	Find the vertical height when above A:	
B1	Use horizontal motion to find the time to A	
M1	Find the vertical height "above" A	
A1	Correct height "above" A	
dM1	Use the inequality determine whether or not P is above the rim	
A1cso	Correct conclusion from correct work	
NB	If the limiting value of U is assumed $\left(\text{ie } U = \sqrt{2gr}\right)$ M1A1B1 can be awarded for correct work. The final 2 marks are awarded only if consideration of the projectile motion is used to explain why P falls outside. eg increasing the initial speed will increase the (horizontal) range (M1)and correct conclusion from correct work (A1)	

Q8.

Question Number	Scheme	Marks
	$\omega = \dfrac{10\pi}{60}$ (rad s^{-1})	B1
	$F = mg\mu$ (N)	B1
	$F = m \times 0.2\left(\dfrac{\pi}{6}\right)^2 = \dfrac{m\pi^2}{180}$	M1A1ft
	$mg\mu \geq \dfrac{m\pi^2}{180}$	dM1
	$\mu_{\text{min}} = \dfrac{\pi^2}{180g}, \quad (0.0056, \quad 0.00560)$	A1
		[6]
B1 B1 M1 A1ft dM1 A1	Correct angular speed in radians per second, seen anywhere Correct inequality or equation for Friction, seen or used anywhere Attempt the equation of motion along the radius. Must only contain friction and resultant force (give BOD unless clearly not friction). Allow with their ω or just ω. Correct equation. Follow through their ω Eliminate F and solve to find μ. Allow with an inequality or equation. Dependent on previous M1. Correct answer, as shown or 2/3 sf decimal (0.00560). Must not be an inequality now.	

Special Case: If $F \geq mg\mu$ or $F < mg\mu$ used, leading to $\mu = \dfrac{\pi^2}{180g}$ award max B1B0 M1A1 M1A0

Q9.

(a)	$\angle PBA = 30°$	B1

Question Number	Scheme	Marks
(b)	R$(\uparrow)$ $T\cos 30° + R\cos 60° = mg$	M1
	NL2 horizontally: $T\cos 60° + R\cos 30° = mr\omega^2, = ma\omega^2 \cos 30°$	M1A1,A1
	$T = \frac{m\sqrt{3}}{2}\left(2g - a\omega^2\right)$ o.e.	dM1A1 (7)
	$R = 2mg - \frac{3m}{2}\left(2g - a\omega^2\right) = \frac{3ma\omega^2}{2} - mg$	M1A1
	Use $R \geq 0$	M1
	$\omega \geq \sqrt{\frac{2g}{3a}}$ *	A1cso (4)
		[11]
(a)		
B1	Correct angle, seen explicitly, implied by a correct trig ratio, or used.	
M1	Attempt a vertical equation with 3 forces, T and R resolved. Angles can be algebraic. Condone sin/cos confusion and use of the same angle for both forces.	
M1	Equation of motion horizontally, two forces resolved and acceleration in either form. Attempt at radius not needed. Angles can be algebraic. Condone sin/cos confusion and use of the same angle for both forces.	
A1	Correct LHS	
A1	Correct acceleration with correct radius (which might be seen later in part (a)).	
dM1	Eliminate R and solve to find expression for T. Depends on both previous M marks. Allow this mark even if they have not found an angle.	
A1	Correct expression for T (any correct equivalent).	
(b)		
M1	Attempt to obtain an expression in R. Independent of the M marks in (a), but must have come from 2 equations in T and R.	
A1	Correct unsimplified expression in R	
M1	Use of the correct inequality for R	
A1cso*	Obtain given result from fully correct working.	

Q10.

Question Number	Scheme	Marks
(a)	$3amg = \frac{1}{2}m \times 7ag - \frac{1}{2}mv^2$	M1A2
	$v^2 = ag \quad v = \sqrt{ag}$	A1 (4)
(b)	$amg = \frac{1}{2}mw^2 - \frac{1}{2}m \times 7ag$	M1
	$w^2 = 9ag$	
	$T_1 - mg = \frac{mw^2}{4a}$	M1
	$T_1 = \frac{13mg}{4}$	A1
	Speed immediately after impact $= \frac{1}{2}\sqrt{ag}$	
	$4amg = \frac{1}{2}mV^2 - \frac{1}{2}m \times \frac{1}{4}ag$	M1
	$V^2 = \frac{33}{4}ag$	
	$T_2 - mg = \frac{mV^2}{4a}$	M1
	$T_2 = \frac{49}{16}mg$	A1
	$T_1 : T_2 = \frac{13}{4} : \frac{49}{16} = 52 : 49$	A1 (7)
		[11]

(a)	
M1	Energy equation from projection to reaching the ceiling. Must have at least one GPE term and 2 KE terms
A2	Correct equation. -1 for each error.
A1cso	Correct expression for v from fully correct work
(b)	
M1	Energy equation from the point of projection to B. Must have all required terms
M1	Form equation of motion at B and eliminate w^2 to obtain an expression for T_1 Must have attempted a velocity at B. Condone $r = a$.
A1	Correct expression for T_1
M1	Form energy equation from leaving the ceiling to reaching B. Must have attempted to use the coeff of restitution to find the initial speed for this equation. Condone $r = a$.
M1	Attempt an equation of motion at B and eliminate V^2 to obtain an expression for T_2. Must have attempted a velocity at B.
A1	Correct expression for T_2
A1cao	Correct ratio. Question asks for simplest form, so must be 52:49 (Condone $\frac{52}{49}$)

Q11.

	Any correct trig function	B1
	$T_b + T_a \cos\theta = 8ma\omega^2$	M1A1A1
	$T_a \sin\theta = mg$	M1A1
	$T_b = 8ma\omega^2 - \dfrac{mg}{\sin\theta} \times \cos\theta$	dM1
	$8ma\omega^2 - \dfrac{mg}{\sin\theta}\cos\theta \le \dfrac{5}{4}mg$	dM1
	$\omega^2 \le \dfrac{g}{4a}$	A1
	$\omega = \dfrac{1}{2}\sqrt{\dfrac{g}{a}}$ o.e.	A1
		[10]

B1	$\sin\theta = \dfrac{4}{5}$ or $\cos\theta = \dfrac{3}{5}$ seen (or reversed if they have angle of string to vertical)
M1	Resolving horizontally, with two tensions (T_a resolved) and acceleration in either form.
A1	Both tensions correct
A1	Correct form of acceleration
M1	Attempt at vertical equation, with T_a resolved and weight not resolved
A1	Correct equation
dM1	Eliminate T_a. Dependent on both previous M marks.
dM1	Set $T_b \le \dfrac{5}{4}mg$ to form inequality in ω^2. Trig need not be substituted. Dependent on previous M mark. Condone use of <
A1	Correct inequality for ω^2 Condone use of <
A1	$\omega = \frac{1}{2}\sqrt{\frac{g}{a}}$ o.e. Must have used $\le$ for this mark.

ALT – for last 4 marks

	$8ma\omega^2 = T_b + \dfrac{mg}{\sin\theta} \times \cos\theta$	dM1
	$8ma\omega_{Max}{}^2 = \left(\dfrac{5}{4} + \dfrac{3}{4}\right)mg$	dM1
	$\omega_{Max}{}^2 = \dfrac{g}{4a}$	A1
	$\omega = \dfrac{1}{2}\sqrt{\dfrac{g}{a}}$ o.e.	A1

dM1	Eliminate T_a
dM1	Set $T_b = \dfrac{5}{4}mg$ to form and equation in $\omega_{Max}{}^2$ Some justification required that ω_{Max} will be when tension is greatest in PB. Trig need not be substituted.
A1	Correct expression for $\omega_{Max}{}^2$
A1	$\omega = \frac{1}{2}\sqrt{\frac{g}{a}}$ o.e.

Q12.

(a)	At B: $mg\cos 60° = \dfrac{mv^2}{r}$	M1A1A1
	$v = \sqrt{\dfrac{rg}{2}}$ *	A1 (4)
(b)	$\dfrac{1}{2}m\left(u^2 - v^2\right) = 2mgr\sin 30°$	M1A1A1
	$u^2 = 4gr \times \dfrac{1}{2} + \dfrac{rg}{2}$	
	$u = \sqrt{\dfrac{5rg}{2}}$	A1 (4)
(c)	$0^{(2)} = (v\sin 60)^2 - 2gh$	M1A1
	$h = \dfrac{3v^2}{8g} = \dfrac{3r}{16}$	A1
	Height above $C = \dfrac{3r}{16} + r + r\cos 60°$	M1
	$= \dfrac{27}{16}r$	A1 (5)
		[13]

(a)	
M1	Attempt at equation of motion along the radius, with weight resolved and acceleration in either form. If R is included, it must be set to zero before this mark is awarded.
A1A1	Fully correct equation. -1 for each error. Acceleration must be in correct form.
A1*	Given answer reached from fully correct working. There must be at least one line of working between At B: $mg\cos 60° = \dfrac{mv^2}{r}$ and the given result.
(b)	
M1	Energy equation with 2 KE terms and a change in GPE. Use of SUVAT scores M0. The work might be seen in (a), but must be used in (b).
A1	Correct GPE or change in KE.
A1	Fully correct equation.
A1	Correct expression for u.
(c)	
M1	Complete SUVAT method to find height gained after leaving sphere. Initial velocity must be resolved. If energy used, they must take account of the fact that there is still KE at the highest point.
A1	Fully correct equation.
A1	Correct height gained.
M1	Complete method to find height above C, by adding 3 distances. Independent of previous M mark, as long a dimensionally correct height gained has been found.
A1	$\dfrac{27}{16}r\,(=1.6875r)$ cso

Q13.

Question Number	Scheme	Marks
(a)	$\frac{1}{2}\times mv^2-\frac{1}{2}\times m\times 2lg=mgl\cos\theta$	M1A1
	$T-mg\cos\theta=m\frac{v^2}{l}$	M1A1A1
	$T=mg\cos\theta+\frac{1}{l}(2mgl\cos\theta+2mgl)$	DM1
	$T=mg(3\cos\theta+2)$ *	A1cso (7)
(b)	$T=0\Rightarrow\cos\theta=-\frac{2}{3}$	B1
	$v^2=-gl\cos\theta$ or $v^2=2gl\cos\theta+2gl$	
	$v^2=\frac{2gl}{3},\quad v=\sqrt{\frac{2gl}{3}}$	M1,A1 (3)
(c)	Horiz speed at $B=v\lvert\cos\theta\rvert=\sqrt{\frac{2gl}{3}}\times\frac{2}{3}$	M1A1ft on v
	Energy: $\frac{1}{2}m\left(\frac{2gl}{3}\right)-\frac{1}{2}m\times\frac{4}{9}\left(\frac{2gl}{3}\right)=mgh$	M1
	$h=\frac{5l}{27}$	A1
	Height above $O=\frac{5l}{27}+l\cos(180-\theta)=\frac{5l}{27}+\frac{2l}{3}=\frac{23l}{27}$ (0.85l or 0.852l)	A1 cao (5) [15]

ALT(c)	Vert speed at $B=v\cos(\theta-90)=v\sin\theta=\sqrt{\frac{2gl}{3}}\times\frac{\sqrt{5}}{3}$	M1A1ft on v
	$0=\frac{2gl}{3}\times\frac{5}{9}-2gs$	M1
	$s=\frac{5l}{27}$	A1
	Height above $O=\frac{5l}{27}+l\cos(180-\theta)=\frac{5l}{27}+\frac{2l}{3}=\frac{23l}{27}$	A1 (5)

Question Number	Scheme	Marks
(a)		
M1	Forming an **energy** equation from start to the general position. 2 KE terms needed and a change in PE (with 1 or 2 terms)	
A1	Correct equation	
M1	Attempting an equation of motion along the radius at the general position. Weight must be resolved. Acceleration can be in either form.	
A1	Correct difference of forces	
A1	Acceleration as shown	
DM1	Eliminate v^2 between the 2 equations. Depends on both previous M marks. Working must be shown.	
A1cso	Obtain the given expression for T with no errors in the solution.	
(b)		
B1	$\cos\theta = -\frac{2}{3}$ (at point where string becomes slack)	
M1	Obtaining v^2 or v in terms of g and l	
A1	Correct expression for v, any equivalent form	
(c)		
M1	Resolve their speed at B to obtain the horizontal component	
A1ft	$\frac{2}{3}\times$ their speed	
M1	Attempting an energy equation from B to the highest point, 2 KE terms needed	
A1	Obtain the correct height above B	
A1cao	Obtain the vertical distance above B and complete to the correct height above O	
ALT(c)		
M1	Resolve their speed at B to obtain the vertical component	
A1ft	$\frac{\sqrt{5}}{3}\times$ their speed	
M1	Use $v^2 = u^2 + 2as$ with their (non-zero) u and $v = 0$ to obtain the height above B	
A1	Obtain the correct height above B	
A1	Complete to the correct height above O	

Q14.

Question Number	Scheme	Marks
(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mgr(1-\cos\alpha)$	M1A1A1
	$mg\cos\alpha \ (-R) = m\frac{v^2}{r}$	M1A1
	$gr\cos\alpha = u^2 + 2gr(1-\cos\alpha)$	dM1
	$\cos\alpha = \frac{1}{3gr}(u^2+2gr)$ *	A1 cso (7)
(b)	At top $mg - R_{top} = \frac{mu^2}{r}$	
	For circular motion, $R_{top} > 0 \ \therefore \frac{u^2}{r} < g,\ u < \sqrt{gr}$ *	M1,A1cso (2)
ALT:	$\cos\alpha < 1 \quad u^2 + 2gr < 3gr,\quad u < \sqrt{gr}$	M1,A1cso (2)
(c)	$\frac{1}{2}m \times \frac{9}{2}gr - \frac{1}{2}mu^2 = 2mgr$	M1A1 A1
	$u^2 = \frac{1}{2}gr$	
	$\cos\alpha = \frac{1}{3gr}\left(\frac{1}{2}gr + 2gr\right)$	
	$\cos\alpha = \frac{5}{6}$	dM1A1cao (5) **[14]**

(a)	
M1	Energy equation with a difference of KE terms and one PE term or a difference of PE terms, m in all terms or none but must be clear it is an energy equation and not $v^2 = u^2 + 2as$
A1A1	-1 each error
M1	Attempt an equation of motion along the radius at A. R need not be included. Weight must be resolved, acceleration in either form. Accept $v^2 = rg\cos\alpha$
A1	Correct equation with or without R. Acceleration in form shown
dM1	Eliminate v between the two equations (and set $R = 0$ if R was included) Depends on both previous M marks.
A1cso	Correct **given** result (in the form shown) with no errors in the working. At least one intermediate step between $gr\cos\alpha = u^2 + 2gr(1-\cos\alpha)$ and the answer must be shown.
(b)	
M1	Attempt an equation of motion along the radius at the top and use $R_{top} > 0$ to obtain an inequality for u^2 Must use $>$
A1cso	Correct **given** result
ALT	**M1** Use $\cos\alpha < 1$ in the result from (a) to obtain an inequality for u^2 **A1cso** As above

(c) **M1** **A1 A1** **dM1** **A1cao**	Energy equation from top to reaching the plane. Difference of KE terms and a PE term needed m in all terms or none (see (a)) -1 each error Use their expression for u^2 in the result given in (a) to obtain a numerical value for $\cos\alpha$ Complete to 5/6. Accept 0.83 or better.	
 ALT 1 **M1** **A1A1** **M1** **A1cao**	**Alternatives for (c)** Use energy from A to the plane $\frac{1}{2}m\times\frac{9}{2}gr-\frac{1}{2}mv^2=mgr(1+\cos\alpha)$ M1 equation with correct no of terms and dimensionally correct A1A1 -1 each error Use their equation of motion from (a) to find an expression for v^2 and complete to a numerical value for $\cos\alpha$ Complete to 5/6. Accept 0.83 or better.	
ALT 2	Using SUVAT equations: Let speed at A be v_A where $v_A{}^2=u^2+2gr(1-\cos\alpha)$ (from the energy equation in (a)) Horiz comp = $v_A\cos\alpha$ Vert comp = $v_A\sin\alpha$ Vert comp at the plane $=V$ where $V^2=(v_A\sin\alpha)^2+2gr(1+\cos\alpha)$ $(\text{speed at plane})^2=V^2+(v_A\cos\alpha)^2$ $=(v_A\sin\alpha)^2+2gr(1+\cos\alpha)+(v_A\cos\alpha)^2$ $\frac{9gr}{2}=v_A{}^2+2gr(1+\cos\alpha)$ $\frac{9gr}{2}=u^2+2gr(1-\cos\alpha)+2gr(1+\cos\alpha)$ $\frac{9gr}{2}=u^2+4gr\Rightarrow u^2=\frac{1}{2}gr$ $\cos\alpha=\frac{1}{3gr}\left(\frac{1}{2}gr+2gr\right)\Rightarrow\cos\alpha=\frac{5}{6}$	 M1A1A1 M1A1cao

   M1 **A1A1** **M1** **A1cao**	Find the speed at A, obtain horizontal and vertical components of this speed. Obtain the vertical component of the speed at the plane using SUVAT Obtain the resultant speed at the plane, equate to $3\sqrt{\frac{gr}{2}}$ and eliminate v_A. For the equation obtained following the steps described above -1 each error in this equation. Equation need not be simplified. Deduct one mark if the horizontal and vertical components at A have been interchanged As main scheme

Q15.

Question Number	Scheme	Marks
	$\omega = \frac{2\pi}{0.5} (= 4\pi)$	B1
	$R = m\omega^2 \times 0.35 \quad \left(= \frac{28m\pi^2}{5}\right)$	M1
	$F_r = mg$	B1
	$F_r \leq \mu R$	M1
	$mg \leq \mu \frac{28m\pi^2}{5}$	DM1
	$\mu \geq 0.18 \quad (\mu \geq 0.177)$	A1
		[6]

B1 Use of period to find ω.

M1 Equation of motion horizontally. Must be considering R. Acceleration can be in either circular motion form.

B1 Resolve vertically. If only seen within friction inequality, then must be correct way round.

M1 Use of $F_r \leq \mu R$. Condone use of strict inequality for this mark.

DM1 Substitute in F_r and R. Could be an equation. Dependent on first M mark.

A1 $\mu \geq 0.18 \quad (\mu \geq 0.177)$ **cao**

Note: If equation used throughout and correct inequality added for final answer, full marks are available if no incorrect working seen.

Q16.

Question Number	Scheme	Marks
(a)	R$(\uparrow)$ $R\cos\theta = mg$	B1
	R$(\rightarrow)$ $R\sin\theta = m\dfrac{v^2}{50}$	M1A1
	$\tan\theta = \dfrac{3}{4} = \dfrac{v^2}{50g}$	
	$v^2 = \dfrac{150g}{4}$ $v = 19.17... = 19$ or 19.2 m s^{-1}	dM1A1 (5)
(b)	R$(\uparrow)$ $R\cos\theta - F\sin\theta = mg$	M1A1
	R$(\rightarrow)$ $R\sin\theta + F\cos\theta = m\dfrac{v^2}{50}$	M1A1A1
	$F = \dfrac{1}{4}R$	B1
	$\dfrac{4}{5}R - \dfrac{1}{4}R \times \dfrac{3}{5} = mg$	
	$\dfrac{3}{5}R + \dfrac{1}{4}R \times \dfrac{4}{5} = m\dfrac{v^2}{50}$	
	$\dfrac{v^2}{50g} = \dfrac{4}{5} \div \dfrac{13}{20}$, $v = 24.55.... = 25$ or 24.6 m s^{-1}	dM1,A1 (8) [13]

(a)B1 Resolve vertically, equation must be fully correct.

M1 Form an equation of motion horizontally. R must be resolved; acceleration can be in either form.

A1 Correct equation, acceleration $v^2/50$ or v^2/r.

ALT Equation parallel to the track: $mg\sin\theta = m\dfrac{v^2}{50}\cos\theta$

M1 weight and acceleration both resolved A1A1 one mark each term (50 or r)
No B mark here.

dM1 Use $\tan\theta = 3/4$ with the eqns (or eqn) to reach $v^2 = ...$ or $v = ...$ Depends on the M1 above.

A1 Correct value of v. Must be 2 or 3 sf

(b)M1 Resolve vertically, R and F both resolved. Treat μmg as F for the first 5 marks.

A1 Correct equation. Treat μmg as F

M1 Resolve horizontally, R and F both resolved, acceleration in either form. Treat μmg as F

A1 For the lhs of the equation. Treat μmg as F

A1 For the rhs with acceleration as shown with 50 or r

ALT Parallel to the track: $F + mg\sin\theta = m\left(v^2/50\right)\cos\theta$ M1A1 50 or r

Perpendicular to the track: $R - mg\cos\theta = m\left(v^2/50\right)\sin\theta$ M1A1A1 50 or r

B1 $F = \dfrac{1}{4}R$ (μmg scores B0)

dM1 Eliminate R and F to obtain an equation for v^2. Depends on the two previous M marks, but if μmg used for F award M0 here.

A1 Correct value of v. Must be 2 or 3 sf.

Q17.

Question Number	Scheme	Marks
	R$(\uparrow)$ $T_A\cos 60° = T_B\cos 60° + mg$	M1A1
	$T_A = T_B + 2mg$	
	NL2 along radius: $T_A\cos 30° + T_B\cos 30° = ma\cos 30°\omega^2$	M1A1A1
	$T_A + T_B = ma\omega^2$	
	$T_A = \frac{1}{2}\left(ma\omega^2 + 2mg\right)$	dM1A1
	$T_B = \frac{1}{2}\left(ma\omega^2 - 2mg\right)$	A1
	$T_A = \frac{1}{2}\left(ma\omega^2 + 2mg\right) < 3mg$	
	$\omega^2 < \frac{4g}{a}$	M1
	$T_B = \frac{1}{2}\left(ma\omega^2 - 2mg\right) > 0$	
	$\omega^2 > \frac{2g}{a}$	M1
	$S = \frac{2\pi}{\omega} \Rightarrow \pi\sqrt{\frac{a}{g}} < S < \pi\sqrt{\frac{2a}{g}}$ $k = 2$	dM1A1cso (12) **[12]**

M1	Resolving vertically. Both tensions resolved but can be sin or cos of 30 or 60. Omission of g is an accuracy error.
A1	Fully correct equation
M1	Attempt NL2 along the radius. Both tensions resolved but can be sin or cos of 30° or 60°. Acceleration in either form. Allow with r instead of $a\cos 30°$
A1	Both forces correct. (r or $a\cos 30°$)
A1	Fully correct equation with acceleration in $a\cos 30° \times \omega^2$ form
dM1	Solve the equations for either tension in terms of m, a, ω (g may be missing). Depends on both M marks above.
A1	Either tension correct
A1	Second tension correct
M1	Use their $T_A < 3mg$ to obtain an inequality for ω^2 (or ω) in terms of g and a Use of $\leqslant$ scores M0
M1	Use their $T_B > 0$ to obtain an inequality for ω^2 (or ω) in terms of g and a Use of $\geqslant$ scores M0
dM1	Use $S = \frac{2\pi}{\omega}$ with **both** inequalities to obtain a final result. Depends on the two M marks for the inequalities.
A1cso	Correct final result as shown in the question from fully correct working. Value of k need not be shown explicitly.

Question Number	Scheme	Marks
	Solutions using $\omega = \frac{2\pi}{S}$ $\left(\text{or } \frac{2\pi}{T}\right)$	
	$\text{R}(\uparrow)$ $T_A \cos 60° = T_B \cos 60° + mg$	M1A1
	$T_A = T_B + 2mg$	
	$T_A + T_B = ma\left(\frac{2\pi}{S}\right)^2$	M1A1A1
	$T_A = \frac{1}{2}\left(ma\left(\frac{2\pi}{S}\right)^2 + 2mg\right)$	dM1A1
	$T_B = \frac{1}{2}\left(ma\left(\frac{2\pi}{S}\right)^2 - 2mg\right)$	A1
	$T_A = \frac{1}{2}\left(ma\left(\frac{2\pi}{S}\right)^2 + 2mg\right) < 3mg$	
	$S^2 > \frac{\pi^2 a}{g}$	M1
	$T_B = \frac{1}{2}\left(ma\left(\frac{2\pi}{S}\right)^2 - 2mg\right) > 0$	
	$S^2 < \frac{\pi^2 a}{2g}$	M1
	$\Rightarrow \pi\sqrt{\frac{a}{g}} < S < \pi\sqrt{\frac{2a}{g}}$ $\quad k = 2$	dM1A1cso (12)
NB	The final M mark is for using $S = \frac{2\pi}{\omega}$ and must only be awarded when both inequalities have been used to obtain the final result.	
	Solutions using T_A=3mg **and** $T_B = 0$: If **2** cases are considered, (i) with T_A=3mg and (ii) with $T_B = 0$, first 8 marks are available but no more. If equations are formed including T_A=3mg and $T_B = 0$ **in the same equation**, there may be marks gained before the sub is made but once the sub is made there are no further marks available.	

Q18.

Question Number	Scheme	Marks
(a)	$\frac{1}{2}mv^2 - \frac{1}{2}m\times 10ag = mag$ $v^2 = 12ag$	M1A1
	After impact $V = e\sqrt{12ag}$	B1
	Energy to top: $\frac{1}{2}me^2(12ag) - \frac{1}{2}mW^2 = mag$	M1
	At top $T + mg = m\frac{W^2}{a}$	M1A1
	$T \geqslant 0 \quad W^2 \geqslant ag$	dM1
	$\frac{1}{2}me^2(12ag) - mag\left(=\frac{1}{2}mW^2\right) \geqslant \frac{1}{2}mag$	
	$e^2 \geqslant \frac{1}{4}, \quad \therefore e \geqslant \frac{1}{2}$.... *	dM1,A1cso (9)

(b)	Energy to string breaking: $\frac{1}{2}m\frac{3}{4}(12ag) - \frac{1}{2}mX^2 = mag\cos 30°$	M1
	$X^2 = 9ag - 2ag\cos 30°$	A1
	Horiz speed $= X\cos 30° = \cos 30°\sqrt{ag(9-2\cos 30°)}$	M1
	By energy, speed at $D =$ speed at B (after rebounding) $= \frac{\sqrt{3}}{2}\sqrt{12ag} \quad \left(=3\sqrt{ag}\right)$ OR use vert speed at C, $\left(X\sin 30° = \sin 30°\sqrt{ag(9-2\cos 30°)}\right)$, to find the vert speed at D by SUVAT	M1
	$\cos\theta = \frac{\cos 30°\sqrt{ag(9-2\cos 30°)}}{3\sqrt{ag}}$ OR $\tan\theta = \frac{\text{vert speed}}{\text{horiz speed}}$	M1
	$\theta = \cos^{-1}\left(\frac{\cos 30°\sqrt{(9-2\cos 30°)}}{3}\right) = 38.89...°$	A1 (6)
	Accept 39°, 38.9° or better; or 0.68, 0.679, radians or better	[15]

(a)

M1	Energy equation from A to B. Must have 2 KE terms and 1 PE term.
A1	Fully correct equation
B1	Correct speed immediately after impact.
M1	Energy equation from B to A, with their speed after impact at B
M1	Equation of motion along the radius at A
A1	Correct equation at A
dM1	Use tension at $A \geqslant 0$ to obtain a inequality for (speed at A)^2 Depends on the third M mark
	ALT for the last 3 marks:
	$m\dfrac{W^2}{a} - mg \geqslant 0$
dM1	Use the energy equation to obtain $e^2 \geqslant ....$ Depends on second and third M marks
A1cso	Reach the **given** answer with no errors seen

(b)

M1	Energy equation from B to C or from A to C $\left(\dfrac{3}{4} \text{ or } e^2 \text{ used}\right)$
A1	Correct expression for (speed at C)^2 Must have 3/4 now.
M1	Attempt horizontal component of speed at C with their speed at C.
M1	Obtain the speed at D or vertical speed at D by finding the vertical speed at C and using SUVAT . **NB: This is an A mark on e-PEN**
M1	Use horizontal speed of P with (resultant) speed or vertical speed of P at D to form an expression for the cos or tan of the required angle.
A1	Correct size of the angle in degrees or radians. 2 sf minimum (g cancels).

Q19.

Question Number	Scheme	Marks
(a)	Energy to B: $\frac{1}{2}m\times\frac{7ag}{2}-\frac{1}{2}mv^2=mga$	M1A1
	NL2 along rad at B: $R=m\frac{v^2}{a}$	M1A1
	$R=\frac{3mg}{2}$	A1cao (5)

(b)	Energy A to C: $\frac{1}{2}m\times\frac{7ag}{2}-\frac{1}{2}mV^2=mga\left(1+\cos\theta\right)$	M1A1
	OR energy B to C: $\frac{1}{2}m\times\frac{3ag}{2}-\frac{1}{2}mV^2=mga\cos\theta$	
	NL2 along rad at C: $mg\cos\theta=m\frac{V^2}{a}$	M1A1
	Solve for θ: $\frac{1}{2}m\times\frac{7ag}{2}-\frac{1}{2}mga\cos\theta=mga\left(1+\cos\theta\right)$	dM1
	$\cos\theta=\frac{1}{2}\quad\left(\theta=60^\circ\right)$	A1
	Horiz motion: $s=a\sin\theta$, speed $=V\cos\theta$	
	$t=\frac{a\sin\theta}{V\cos\theta}=\sqrt{\frac{2}{ag}}\times a\sqrt{3}=\sqrt{\frac{6a}{g}}$	M1
	Vert motion: $s=-V\sin\theta\times t+\frac{1}{2}gt^2$	M1
	$s=-\sqrt{\frac{ag}{2}}\times\frac{\sqrt{3}}{2}\times\sqrt{\frac{6a}{g}}+\frac{1}{2}g\times\frac{6a}{g},\ =-\frac{3a}{2}+3a=\frac{3a}{2}$	A1,A1
	$\left(\begin{array}{l}\text{Horiz dist } A \text{ to } C\text{: } s=a\sin 60^\circ=\frac{a\sqrt{3}}{2}\\ \text{sufficient that this was used to find the time}\end{array}\right)$	
	Vert dist A to C: $s=\frac{3a}{2}$	
	$\therefore$ Strikes surface at A	A1cso (11) [16]

Question Number	Scheme	Marks
(a)M1	Attempt an energy equation from start to B. Must have 2 KE terms and one PE term	
A1	Fully correct equation	
M1	Attempt an equation of motion along the radius at B. Acceleration can be in either form. Only force to be the reaction.	
A1	Fully correct equation, acceleration as shown.	
A1cao	Eliminate v^2 to obtain the expression for the reaction at B.	
(b)		
M1	Attempt an energy equation from start to C. Must have 2 KE terms and a PE term which includes a trig function. PE may be expressed as 2 separate terms	
A1	Fully correct equation	
M1	Attempt an equation of motion along the radius at C. The reaction may be included initially but must become 0 before this mark can be awarded. Weight must be resolved; acceleration can be in either form.	
A1	Fully correct equation, acceleration as shown.	
dM1	Eliminate V and obtain a value for $\cos\theta$. Depends on the 2 previous M marks of (b)	
A1	Correct value for $\cos\theta$. Award if seen explicitly or implied by subsequent working.	
M1	Use the horizontal motion to obtain the time to travel a horizontal distance $= a\sin\theta$, with their θ. Speed must be resolved. Time obtained must be a function of a and g only.	
M1	Use $s = ut + \frac{1}{2}at^2$ to obtain an expression for the vertical distance at time t. Acceleration to be g and initial speed to be a component of their V. (trig function or its value allowed here)	
A1	Correct equation in a, g and s	
A1	Correct vertical distance	
A1cso	State that or use the horizontal distance A to C is $a\sin 60° = \frac{a\sqrt{3}}{2}$ and the vertical distance A to C is $(3a)/2$ so the particle strikes the surface at A. All work must be correct.	

ALT	**For the last 4 marks:** Find time to travel $3a/2$ vertically down from C:	
	Vert motion: $s = -V\sin\theta \times t + \frac{1}{2}gt^2$	M1
	$\frac{3a}{2} = -\sqrt{\frac{ag}{2}} \times \frac{\sqrt{3}}{2}t + \frac{1}{2}gt^2, \Rightarrow t = \sqrt{\frac{6a}{g}}$	A1,A1
	Same time horiz and vertically so strikes surface at A	A1cso
M1	Use $s = ut + \frac{1}{2}at^2$ to obtain an expression for the time to travel $\frac{3a}{2}$ vertically. Acceleration to be g and initial speed to be a component of their V. (trig function or its value allowed here)	
A1	Correct equation in a, g and t	
A1	$t = \sqrt{\frac{6a}{g}}$	
A1cso	State that the horizontal and vertical times are the same, so the particle strikes the surface at A. All work must be correct.	
NB	θ is defined in the question as the angle with the vertical. If the angle with the horiz is called θ but otherwise *totally* correct, deduct A mark for $\cos\theta$ and the final a mark. If there are errors in the working, mark as scheme.	

Q20.

Question Number	Scheme	Marks
	$(4a)^2 + r^2 = (8a - r)^2$, Radius $= 3a$	M1,A1
	$T\cos\theta = mg - R$	M1A1
	$T + T\sin\theta = m \times \text{rad} \times \omega^2$	M1A1A1
	$\frac{8}{5}T = 3ma\omega^2$	
	$2mg - 2R = 3ma\omega^2$	DM1
	$R \geq 0 \quad 2mg - 3ma\omega^2 \geq 0 \quad \omega^2 \leq \frac{2g}{3a}$	M1A1
	$S \geqslant 2\pi\sqrt{\frac{3g}{2a}}, = \pi\sqrt{\frac{6a}{g}}$ *	DM1,A1cso
		[12]

M1	Attempt to obtain the radius, probably using Pythagoras. Can be done by justifying eg $3 + 5 = 8$ so 3,4,5 triangle
A1	Correct radius seen here or used *providing* the M mark has been awarded.
M1	Resolve vertically, 3 forces in the equation with T resolved
A1	Correct equation with $\cos\theta$ or $\frac{4}{5}$
M1	Equation of motion along the horizontal radius, T must be resolved, their radius or r allowed, acceleration in either form. Allow if T_A and T_B used
A1	Forces correct, $\sin\theta$ or $\frac{3}{5}$ Both tensions to be the same
A1	Acceleration correct in form shown, their radius or r
DM1	Eliminate T and replace trig functions with their values if not done earlier. This is a M mark, so trig functions need not be correct. Depends on 2nd and 3rd M marks but **not the first**.
M1	Use to obtain an inequality for ω^2
A1	Correct inequality Must use $r = 3a$ now or later.
DM1	Use period $= \frac{2\pi}{\omega}$ to obtain an inequality for S. Depends on previous M mark.
A1cso	Correct inequality obtained from correct and complete working **NB Candidates who assume a 3,4,5 triangle without showing any working or justification will lose the first 2 marks and this one.**