Questions

Q1.

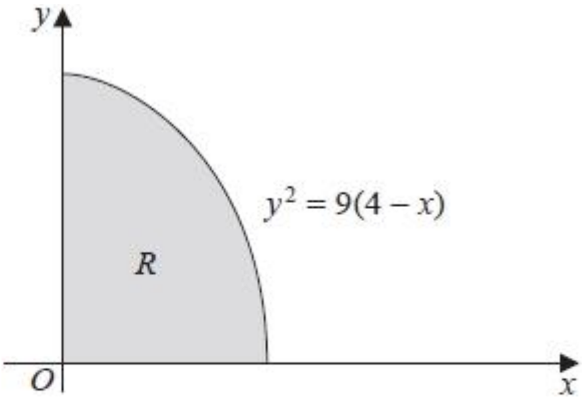

Figure 1

The shaded region R is bounded by the curve with equation $y^2 = 9(4 - x)$, the positive x-axis and the positive y-axis, as shown in Figure 1. A uniform solid S is formed by rotating R through 360° about the x-axis.
Use algebraic integration to find the x coordinate of the centre of mass of S.

(7)

(Total for question = 7 marks)

Q2.

The region enclosed by the curve with equation $y = \frac{1}{2}\sqrt{x}$, the x-axis and the lines $x = 2$ and $x = 4$, is rotated through 2π radians about the x-axis to form a uniform solid S. Use algebraic integration to find the exact value of the x coordinate of the centre of mass of S.

(6)

(Total for question = 6 marks)

Q3.

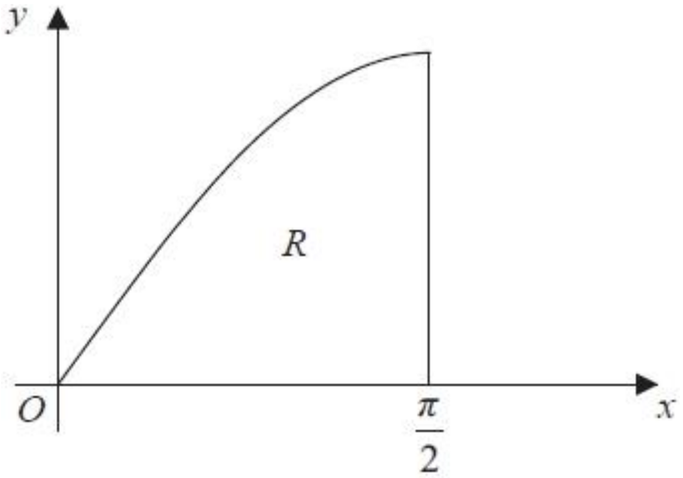

Figure 3

Figure 3 shows the finite region *R* which is bounded by part of the curve with equation $y = \sin x$, the x-axis and the line with equation $x = \frac{\pi}{2}$. A uniform solid *S* is formed by rotating *R* through 2π radians about the x-axis.

Using algebraic integration,

(a) show that the volume of *S* is $\frac{\pi^2}{4}$

(4)

(b) find, in terms of π, the x coordinate of the centre of mass of *S*.

(7)

(Total for question = 11 marks)

Q4.

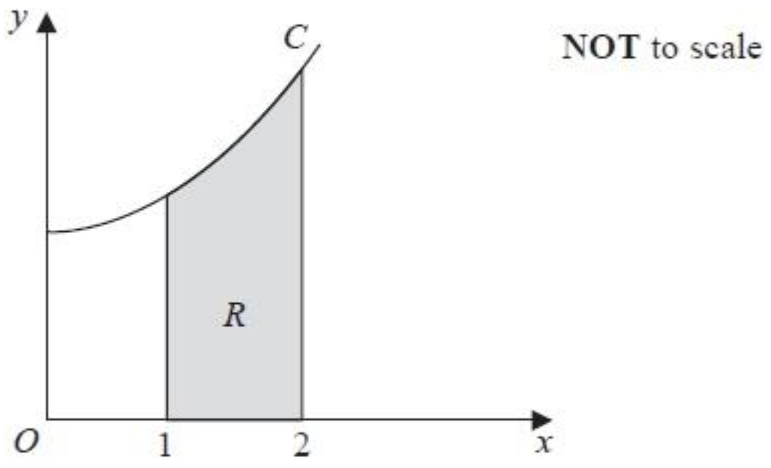

Figure 3

Figure 3 shows part of the curve C with equation $y = x^2 + 4$. The shaded region R is bounded by C, the line with equation $x = 1$, the x-axis and the line with equation $x = 2$

The unit of length on each axis is one centimetre.

A uniform wooden solid, S, is made in the shape formed by rotating the region R through 360° about the x-axis.

(a) Using algebraic integration,

(i) show that the volume of S is $\dfrac{613\pi}{15}\text{cm}^3$

(ii) find, to 3 significant figures, the distance of the centre of mass of S from O.

(8)

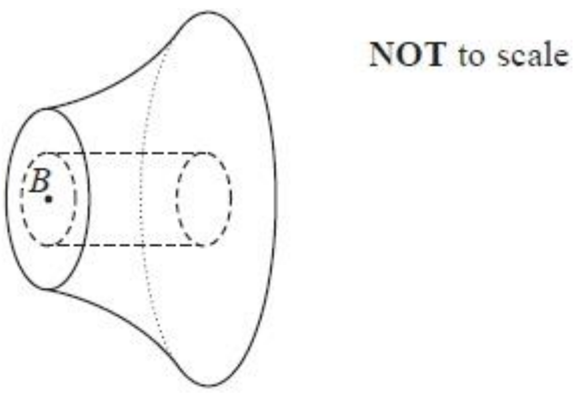

Figure 4

A solid, S_1, is formed by removing a solid cylinder of radius 3 cm and length 1 cm from S. A metal cylinder, of radius 3 cm and length 1 cm is placed in the resulting hole to form a new solid T, as shown in Figure 4. The axis of the metal cylinder coincides with the axis of symmetry of S_1. The point B is the centre of the smaller plane face of T. The mass per unit volume of S_1 is M and the mass per unit volume of the metal cylinder is $5M$.

(b) Find the distance of the centre of mass of T from B.

(5)

(Total for question = 13 marks)

Q5.

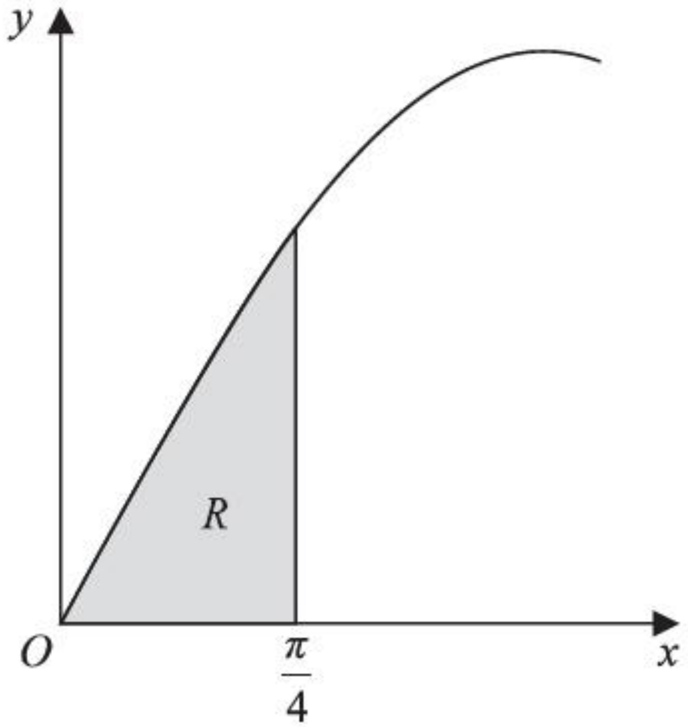

Figure 3

The region *R*, shown shaded in Figure 3, is bounded by part of the curve with equation $y = \sin x$, the line with equation $x = \frac{\pi}{4}$ and the *x*-axis. A uniform solid *S* is formed by rotating *R* through 2π radians about the *x*-axis.

(a) Use calculus to show that the volume of S is $\frac{\pi}{8}(\pi - 2)$.

(4)

(b) Use calculus to find, to 3 significant figures, the *x* coordinate of the centre of mass of *S*.

(8)

The point *A* lies on the circumference of the circular plane face of *S*. The solid *S* is freely suspended from *A* and hangs in equilibrium.

(c) Find, to the nearest degree, the size of the angle between *OA* and the vertical.

(4)

(Total for question = 16 marks)

Q6.

(a) Use algebraic integration to show that the centre of mass of a uniform solid hemisphere of radius r is at a distance $\frac{3}{8}r$ from the centre of its plane face.

[*You may assume that the volume of a sphere of radius r is* $\frac{4}{3}\pi r^3$]

(5)

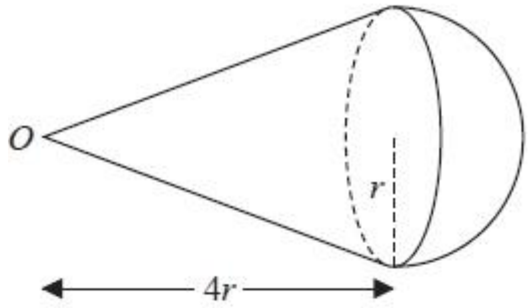

Figure 3

A uniform solid hemisphere of mass m and radius r is joined to a uniform solid right circular cone to form a solid S. The cone has mass M, base radius r and height $4r$. The vertex of the cone is O. The plane face of the cone coincides with the plane face of the hemisphere, as shown in Figure 3.

(b) Find the distance of the centre of mass of S from O.

(4)

The point A lies on the circumference of the base of the cone. The solid is placed on a horizontal table with OA in contact with the table. The solid remains in equilibrium in this position.

(c) Show that $M \geq \frac{1}{10}m$

(5)

(Total for question = 14 marks)

Q7.

(a) Use algebraic integration to show that the centre of mass of a uniform solid right circular cone of height h is at a distance $\frac{3}{4}h$ from the vertex of the cone.

[*You may assume that the volume of a cone of height h and base radius r is* $\frac{1}{3}\pi r^2 h$]

(5)

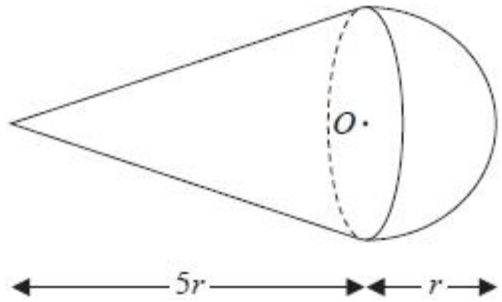

Figure 2

A uniform solid S consists of a right circular cone, of radius r and height $5r$, fixed to a hemisphere of radius r. The centre of the plane face of the hemisphere is O and this plane face coincides with the base of the cone, as shown in Figure 2.

(b) Find the distance of the centre of mass of S from O.

(5)

The point A lies on the circumference of the base of the cone. The solid is suspended by a string attached at A and hangs freely in equilibrium.

(c) Find the size of the angle between OA and the vertical.

(3)

The mass of the hemisphere is M. A particle of mass kM is fixed to the surface of the hemisphere on the axis of symmetry of S. The solid is again suspended by the string attached at A and hangs freely in equilibrium. The axis of symmetry of S is now horizontal.

(d) Find the value of k.

(4)

(Total for question = 17 marks)

Q8.

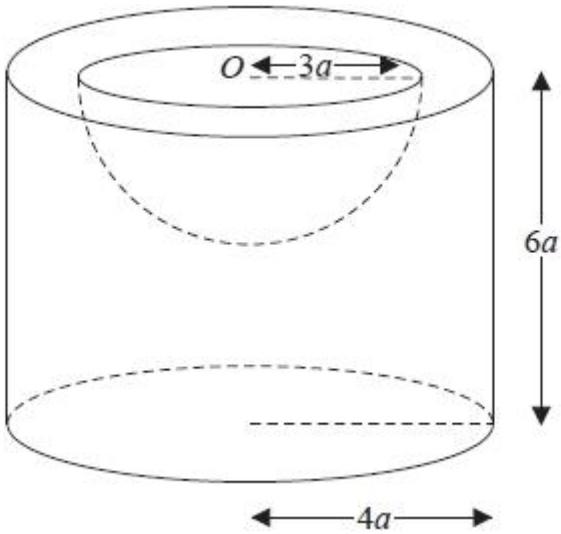

Figure 2

A uniform right circular solid cylinder has radius $4a$ and height $6a$. A solid hemisphere of radius $3a$ is removed from the cylinder forming a solid *S*. The upper plane face of the cylinder coincides with the plane face of the hemisphere. The centre of the upper plane face of the cylinder is *O* and this is also the centre of the plane face of the hemisphere, as shown in Figure 2. Find the distance from *O* to the centre of mass of *S*.

(6)

(Total for question = 6 marks)

Q9.

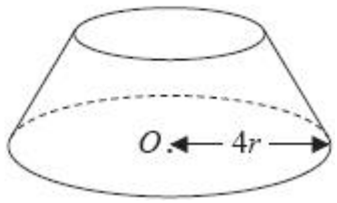

Figure 1

A uniform solid right circular cone *R*, with vertex *V*, has base radius $4r$ and height $4h$. A right circular cone *S*, also with vertex *V* and the same axis of symmetry as *R*, has base radius $3r$ and height $3h$. The cone *S* is cut away from the cone *R* leaving a solid *T*. The centre of the larger plane face of *T* is *O*. Figure 1 shows the solid *T*.

(a) Find the distance from *O* to the centre of mass of *T*.

(5)

The point *A* lies on the circumference of the smaller plane face of *T*. The solid is freely suspended from *A* and hangs in equilibrium. Given that $h = r$

(b) find the size of the angle between *OA* and the downward vertical.

(4)

(Total for question = 9 marks)

Q10.

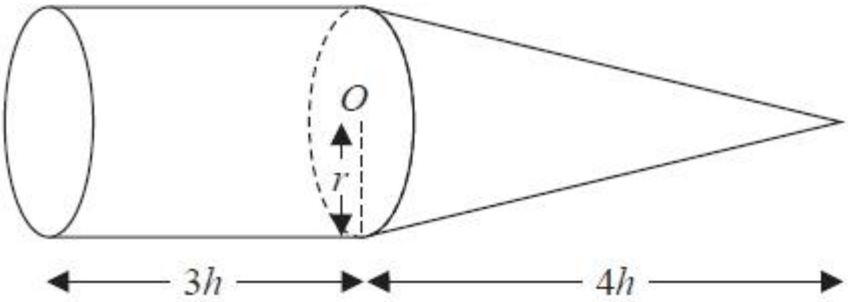

Figure 1

A uniform solid *S* consists of a right solid circular cone of base radius *r* and a right solid cylinder, also of radius *r*. The cone has height 4*h* and the centre of the plane face of the cone is *O*. The cylinder has height 3*h*. The cone and cylinder are joined so that the plane face of the cone coincides with one of the plane faces of the cylinder, as shown in Figure 1.

Find the distance from *O* to the centre of mass of *S*.

(Total for question = 5 marks)

Q11.

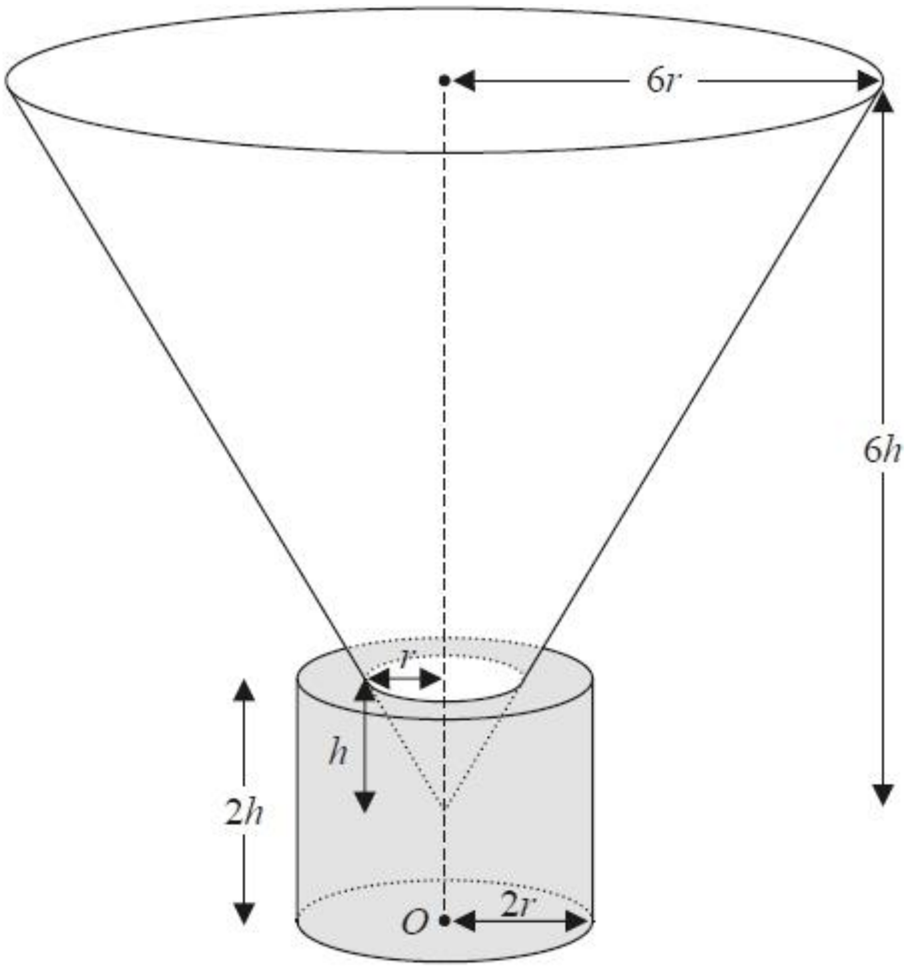

Figure 1

A cone is removed from a uniform solid right circular cylinder of mass M to form the solid with a conical hole shown shaded in Figure 1. A container is made by fixing a uniform conical shell of mass $\frac{1}{12}M$ in the hole. The cylinder has radius $2r$ and height $2h$, the conical hole has base radius r and height h and the conical shell has base radius $6r$ and height $6h$. The vertex of the shell coincides with the vertex of the hole. The axis of the shell, the axis of the hole and the axis of the cylinder are all vertical and coincide. The centre of the plane circular base of the container is O, as shown in Figure 1.

Find, in terms of h, the distance of the centre of mass of the container from O.

(Total for question = 6 marks)

Q12.

A uniform solid right circular cone has base radius *r* and height *h*.

(a) Use algebraic integration to show that the distance of the centre of mass of the cone from its vertex is $\frac{3}{4}h$.

(5)

[*You may assume that the volume of a cone of base radius r and height h is* $\frac{1}{3}\pi r^2 h$]

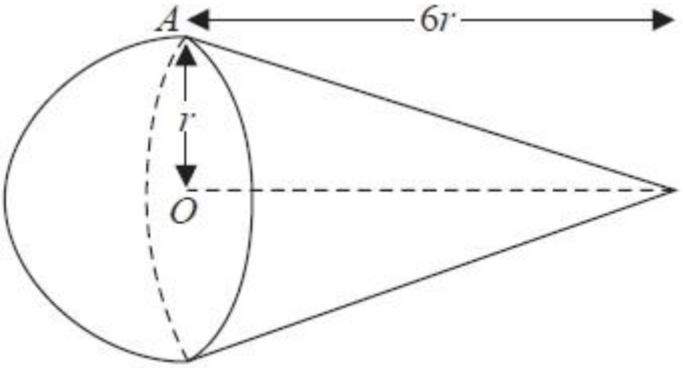

Figure 3

A solid *S* is formed by joining a uniform right circular solid cone of mass 5*m* to a uniform solid hemisphere, of radius *r* and mass *km* where *k* < 20. The cone has base radius *r* and height 6*r*. The plane face of the cone coincides with the plane face of the hemisphere. The centre of the plane face of the cone is *O* and the point *A* is on the circular edge of this plane face, as shown in Figure 3.

(b) Find the distance from *O* to the centre of mass of *S*.

(4)

The solid is suspended from *A* and hangs freely in equilibrium. The angle between the axis of the cone and the horizontal is 30°.

(c) Find, to the nearest whole number, the value of *k*.

(4)

(Total for question = 13 marks)

Q13.

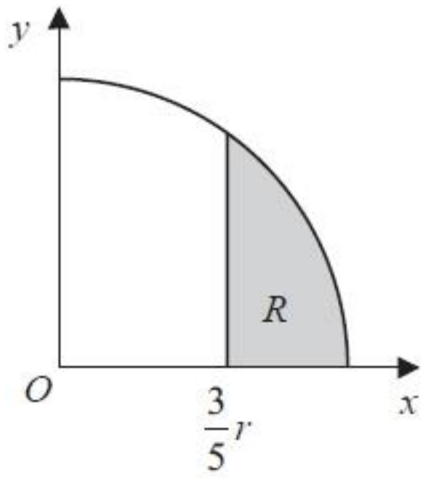

Figure 3

The region R, shown shaded in Figure 3, is bounded by the circle with centre O and radius r, the line with equation $x = \frac{3}{5}r$ and the x-axis. The region is rotated through one complete revolution about the x-axis to form a uniform solid S.

(a) Use algebraic integration to show that the x coordinate of the centre of mass of S is $\frac{48}{65}r$.

(8)

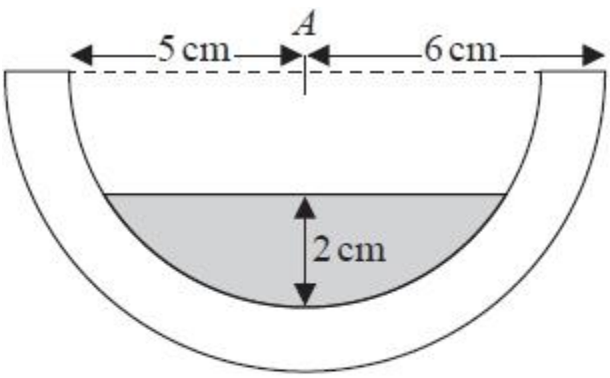

Figure 4

A bowl is made from a uniform solid hemisphere of radius 6 cm by removing a hemisphere of radius 5 cm. Both hemispheres have the same centre A and the same axis of symmetry. The bowl is fixed with its open plane face uppermost and horizontal. Liquid is poured into the bowl. The depth of the liquid is 2 cm, as shown in Figure 4. The mass of the empty bowl is $5M$ kg and the mass of the liquid is $2M$ kg.

(b) Find, to 3 significant figures, the distance from A to the centre of mass of the bowl with its liquid.

(8)

(Total for question = 16 marks)

Examiner's Report

Q1.

While the majority achieved full marks with little apparent difficulty, some lost all the marks by finding the centre of mass of an area rather than a volume, often appearing to be quoting halfremembered formulae rather than thinking about the mechanics of the situation. This question demanded the use of "algebraic integration" meaning that solutions which bypassed the two integrals but used calculators to find their correct numerical values were worthless. A very significant mistake seen quite a few times was due to carelessness was having initially written the given expression for y^2 correctly, some students squared it again and so attempted integrals involving y^4 instead of y^2.

Q2.

This proved to be a very straightforward question, with nearly all students scoring full marks. A small number treated the shape as a lamina, but these were very rare. All students who worked with a solid went on to show algebraic integration. Those who worked with a lamina lost all the marks and those who used a lamina formula for one integral and a solid for the other only gained the marks for the correct part. Another error was forgetting to square y when they substituted into their integral. The limits were almost invariably correct and there were few processing errors.

Q3.

This was the worst answered question on the paper and showed in many cases how poor the standard of integration was. Students caused themselves problems by not setting out their work clearly and, quite apart from the integration errors, there were missing halves and πs in many cases.

There were frequent sign errors in the integration of the sine and cosine functions and many could not integrate by parts. Several did not know the double angle formulae. Although the question asked for algebraic integration, some used their calculators and just wrote down an answer thus losing most of the marks.

The students who had a good level of Pure Mathematics generally did very well in this question.

Q4.

Part (a) was generally answered well. Most candidates did show algebraic integration as instructed. Those who did not lost most of the marks available. It was disappointing that many candidates did not show a clear substitution of limits to reach the given answer. Finding the centre of mass generally caused little difficulty and most candidates did give the answer to the specified 3s.f.

Part (b) was not generally answered well, with few candidates achieving full marks. It was rare for candidates to use the main method on the mark scheme, with the majority instead attempting to consider the hollowed out shape (S_1) and the additional cylinder, often without finding a new centre of mass for S_1. Other common mistakes were to mix up distances from O and distances from B, and to treat M as a mass, and not consider the volumes. Even when a fully correct method was used, candidates that used the rounded answer of 1.58 from (a) lost the final mark due to an inaccurate final answer.

Q5.

Part (a) was generally answered well, with most candidates adopting the correct approach and showing a good level of detailed working to arrive at the given result.

Whilst almost all knew what to do in (b), it was common for slips to occur in completing the integration and substitution of limits. The most popular approach was to split the integral and only use Integration by Parts for $\int_0^{\frac{\pi}{4}} x\cos 2x\,dx$, but many candidates realised that they could use the integral from (a) to do Integration by Parts for the whole integral. Either approach required care with multiple minus signs, as well as halves that were often taken out of integrals. It was also common for π to be cancelled incorrectly. In particular candidates often cancelled π from the top and bottom of the fractions before integrating, but then put in the result from (a), leading to an incorrect result.

Part (c) had many correct solutions, although some candidates were confused about exactly what angle they were trying to find. It was frustrating that even clearly very strong candidates sometimes lost the final mark in (c) and (b) by not giving the specified accuracy.

Q6.

Part (a): many candidates had no idea how to find the centre of mass algebraically. Some of those who did know the formula used the given formula for the volume of a sphere for their hemisphere and adjusted the result of their integral accordingly. Others used the formula for a lamina or tried to find the centre of mass of a hollow hemisphere.

Part (b): this was the easiest part of the question but some made it harder by using either volumes or volume x given mass and ended up with the wrong answer. Many found the distance from the common face and then forgot to adjust their answer.

Part (c) was rarely done correctly. Different methods were attempted but the most successful was to use the distance of the centre of mass from the centre and to consider the critical position using the semi vertical angle of the cone. A common mistake was to incorrectly assume that $OA = 4r$. Many of those who found the relationship between M and m at the critical position moved from an equality to the given inequality with no reason given for the direction of the inequality, thereby forfeiting the final mark.

Q7.

There was a disappointing response to the standard piece of bookwork in part (a). Quite a few simply failed to attempt the question or gave up after a few lines but there were many neat, well argued solutions. However with a given answer it is crucial that no steps are omitted which in other circumstances would not be penalised, and as a consequence of missed steps some candidates lost one or two marks. There were a number of responses which ignored the given request for a general cone and proved the result for the cone involved in Figure 2 and many used an incorrect equation for the line. Most candidates made a sensible attempt at setting up the moments equation in part (b) and generally the appropriate masses and distances were used. A significant number chose to take moments about the vertex, but they nearly always went on to convert to the correct distance. Although not many ran into problems with all of the π's and 1/3's, candidates could have made their lives a lot easier by simplifying to ratios before forming their equations. In part (c), candidates generally struggled to find correct masses, with the most common error being to use M as the mass of S. In general candidates understood the mechanical implication and put the final centre of mass at O. A good number again took moments about the vertex, but this was generally done correctly (albeit with usually incorrect masses).

Q8.

The method required here was well known and most errors were avoidable. The most common of these was giving the final answer as 3.43 instead of 3.43*a*, or its fraction equivalent. More significant mark losses were incurred by those who either used wrong formulae for the volumes involved or who, unwisely, had maybe just miscalculated from correct formulae that they had not written first. Sign errors were not uncommon; either the volumes or the moments were added rather than subtracted and those who had used negative displacements in their working sometimes incorrectly gave the final answer for the distance as negative.

Q9.

Part (a) was answered very well, with most students arriving at the correct answer by the mark scheme method. Some students measured their distances from the top of the cone, but they generally did go on to find the required distance. The most likely source of error was to forget/make a slip adding on *h* to the distance for the top cone. This often resulted in a centre of mass outside the object; surprisingly this didn't seem to alert them to a problem. Part (b) caused more problems, although a pleasingly large number did work out the correct angle. Most students realised that they needed to use *r* −... for the numerator and used 3*r* for the denominator. Many stopped there, although most who continued found the correct angle, although this was often given as 8.09° rather than 8.10°.

Q10.

This was generally answered very well, with the majority of students scoring full marks. Work was usually set out in a way that made the mass ratios and distances clear, with almost all students taking distances from *O* and including a negative value in their moments equation. The only mark that caused any significant difficulties was the final mark, with students either dropping an *h* (which was always present in the moments equation) or leaving a minus sign.

Q11.

This question proved to be far more discriminating, with few candidates presenting completely correct solutions. In particular candidates struggled to use the information about the masses to correctly formulate a mass ratio. Common mistakes were to attempt to compare all masses using their volumes, rather than in terms of M, or to assume that M was the mass of the cylinder **after** the cone had been cut away.

Candidates seemed to be evenly divided between forming a single 4 term equation, or finding the centre of mass of the cylinder with the cone removed $\left(\frac{89}{92}h\right)$ and then attempting to find the CoM with the shell added. Whilst most candidates had correct distances for the cylinder and cut away cone, a common mistake was to use the centre of mass of a solid cone for the shell, so candidates should take great care when using the formula book for standard shapes.

Q12.

Part (a) This was standard bookwork but was often very difficult to mark. When the main method was used solutions were generally not too hard to follow, although clear substitution of limits was not always shown. Many students realised that they could do plenty of cancelling before integration, which meant that the given result was reached very neatly, and the lack of working became justified. When the alternative equation was used, however, the substitution became far more of an issue. Very few clear solutions of 1/12 were seen, with students presumably deciding it was too messy and therefore failing to realise that they needed an extra final step. A few students had incorrect starting equations, for instance $y = \frac{x}{6}$, but they could score the method marks if they proceeded "correctly".

Part (b) Some students ignored the masses given in the question and worked with the volumes of the shapes which lost all but the first mark. The final answer was often left with fractions within fractions, so that a correct answer lost the final A mark.

Part (c) This was not answered well, mainly because students often failed to identify the required angle correctly and then sometimes had an equation which was upside down. A whole number answer was required but was not always given.

Q13.

Part (a) was generally done well and full marks were common. Most realised the necessity of showing detailed substitution of their limits, but a few went directly from their 2 integrated expressions to the given answer. Candidates should realise that when asked to “show” a result, far more explicit working is required and these candidates lost 3 of the available 8 marks.

Part (b) proved challenging and so a good discriminator. There were some excellent solutions but, although the general method was clearly known by almost all, the extra step of needing to deal with the bowl before involving the given masses proved too difficult for some. Failed attempts fell into 3 main categories:

- those who assumed that they should know where the centre of mass of the bowl was – usually at half or $\frac{3}{8}$ of its depth – and made no attempt to calculate it
- those who found the centre of mass of the bowl successfully but failed to realise the connection with (a) when finding the centre of mass of the liquid. Some guessed this value, others realised that 48*r*/65 was relevant but used r = 2 and then added 3.
- those who attempted to do the whole calculation for the bowl and liquid in one go. These very rarely attempted to calculate relative masses as shown in the mark scheme alternative. More commonly, the volumes of the various elements were used as relative masses with the 5*M* and 2*M* either ignored completely or used as densities.

Mark Scheme

Q1.

Question Number	Scheme	Marks
	Vol $=(\pi)\int y^2\mathrm{d}x=(\pi)\int_0^4 9(4-x)\,\mathrm{d}x=9(\pi)\left[4x-\frac{1}{2}x^2\right]_0^4$	M1
	$=9(\pi)(16-8)=(72(\pi))$	A1
	$\pi\int_0^4 9x(4-x)\,\mathrm{d}x=9\pi\left[2x^2-\frac{1}{3}x^3\right]_0^4$	M1A1
	$=9\pi\left(32-\frac{64}{3}\right)\ (=96\pi)$	dM1
	$\bar{x}=\frac{96\pi}{72\pi}=\frac{4}{3}$ (Accept 1.3 or better)	dM1A1cao
		[7]

M1	Attempting the required vol integral, with or without π Power to increase by 1 in at least one term and neither to decrease. Limits not needed.
A1	Correct result after substitution, with or without π provided same as previous line; need not be simplified.
M1	Attempting $(\pi)\int y^2x\,\mathrm{d}x$; with or without π, limits not needed
A1	Correct integration and correct limits seen; with or without π provided same as previous line.
dM1	Substitute their limits, with or without π provided same as previous line; need not be simplified. This may be implied by subsequent working. Depends on second M mark
dM1	Dividing the results of their integration. π to be in both numerator and denominator or in neither. First 2 M marks needed.
A1cao	Correct answer, exact or decimal
NB	Inclusion of ρ should be treated in the same way as π ie ignore for the separate integrals but must be in both integral or neither for the 4th M mark.

Q2.

Question Number	Scheme	Marks
	$\text{Vol} = \int_2^4 (\pi)\times\frac{1}{4}x\,dx$	
	$=(\pi)\left[\frac{1}{8}x^2\right]_2^4$	M1
	$=(\pi)\left[2-\frac{1}{2}\right]=\frac{3(\pi)}{2}$	A1
	$\int_2^4(\pi)\times\frac{1}{4}x^2\,dx$	
	$=(\pi)\left[\frac{1}{12}x^3\right]_2^4$	M1
	$=(\pi)\frac{1}{12}[64-8]=\frac{56}{12}(\pi)$	A1
	$\bar{x}=\frac{56}{12}\pi\times\frac{2}{3\pi}=\frac{28}{9}$	M1A1 (6)

NB Centre of mass of a LAMINA scores 0/6
For the first 4 marks π not needed in either integral. For the third M mark, π must be included in both integrals or neither.

M1 Use $\text{Vol}=(\pi)\int_2^4 y^2\,dx$ and attempt the integration. Limits not needed.

A1 Correct volume following substitution of correct limits. Can be decimal or implied by a correct final answer.

M1 Use $(\pi)\int_2^4 xy^2\,dx$ and attempt the integration. Limits not needed.

A1 Correct result following substitution of correct limits. Can be decimal or implied by a correct final answer.

M1 Use $\dfrac{\int \pi xy^2 dx}{\int \pi y^2 dx}$ (with their values for the integrals)

A1 Correct x coordinate. Must be exact. Give A0 if decimal equivalent of 56/12 is given.

Q3.

Question Number	Scheme	Marks
(a)	$\text{Vol} = \pi\int_0^{\frac{\pi}{2}} y^2\,\mathrm{d}x = \pi\int_0^{\frac{\pi}{2}} \sin^2 x\,\mathrm{d}x$	M1
	$= (\pi)\int_0^{\frac{\pi}{2}} \frac{1}{2}(1-\cos 2x)\,\mathrm{d}x$	M1
	$= \frac{\pi}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}} = \frac{\pi^2}{4}$ *	DM1A1cso (4)
(b)	$\pi\int_0^{\frac{\pi}{2}} y^2 x\,\mathrm{d}x = \pi\int_0^{\frac{\pi}{2}} x\sin^2 x\,\mathrm{d}x$	M1
	$= \pi\int_0^{\frac{\pi}{2}} \frac{1}{2}x(1-\cos 2x)\,\mathrm{d}x = \frac{\pi}{2}\left[\frac{x^2}{2}\right]_0^{\frac{\pi}{2}} - \frac{\pi}{2}\int_0^{\frac{\pi}{2}} x\cos 2x\,\mathrm{d}x$	
	$= -\frac{\pi}{2}\left[x \times \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}} + \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{1}{2}\sin 2x\,\mathrm{d}x,\ + \frac{\pi^3}{16}$	M1,B1
	$= 0 - \frac{\pi}{2}\left[\frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{2}} + \frac{\pi^3}{16}$	DM1
	$= -\frac{\pi}{8}[-1-1] + \frac{\pi^3}{16} = \frac{\pi^3}{16} + \frac{\pi}{4}$	A1
	$\bar{x} = \frac{\pi^3 + 4\pi}{16} \div \frac{\pi^2}{4} = \frac{\pi^2 + 4}{4\pi}$	M1A1cso (7) [11]

ALT(b)	$\pi\int_0^{\frac{\pi}{2}} y^2 x\,\mathrm{d}x = \pi\int_0^{\frac{\pi}{2}} x\sin^2 x\,\mathrm{d}x$	M1
	$= \pi\left[\frac{x}{2}\left(x - \frac{1}{2}\sin 2x\right)\right]_0^{\frac{\pi}{2}} - \pi\int \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right)\mathrm{d}x$	M1
	$= \frac{\pi^3}{8}, -\frac{\pi}{2}\left[\frac{x^2}{2} + \frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{2}}$	B1,DM1
	$= \frac{\pi^3}{8} - \frac{\pi}{2}\left[\frac{\pi^2}{8} - \frac{1}{4} - \frac{1}{4}\right] = \frac{\pi^3}{16} + \frac{\pi}{4}$	A1
	$\bar{x} = \frac{\pi^3 + 4\pi}{16} \div \frac{\pi^2}{4} = \frac{\pi^2 + 4}{4\pi}$	M1A1cso (7)

Question Number	Scheme	Marks
	NB: Beware of incorrect signs for trig functions which become 0 on substitution of the limits.	
(a)		
M1	Forming the required integral, limits not needed. May be seen at end (see NB below)	
M1	Using $\sin^2\theta = k(1\pm\cos 2\theta)$ with $k=\pm\frac{1}{2}$ or ± 2 limits not needed, π not needed	
DM1	Integration of their function and substitution of correct limits. π may be missing. Depends on the first M mark (but not the second).	
A1cso	Correct **given** result with no errors in the solution.	
NB:	If π missing, but appears suddenly at the end with no explanation given, only M0M1M1A0 available. If explanation for inc. π is given, all marks are available.	
(b)	The first 5 marks are available for the integration w/wo π	
M1	Use $\int y^2 x\,\mathrm{d}x$ limits not needed	
M1	Attempt integration by parts (first stage) for $x\cos 2x$ or by using their result for $\int \sin^2 x\,\mathrm{d}x$ from (a), limits not needed	
B1	For $\frac{\pi^3}{16}$ (or $\frac{\pi^2}{16}$ if working the integration without π) Using the result from (a) gets this mark for $\frac{\pi^3}{8}$ or $\frac{\pi^2}{8}$	
DM1	Completing the integration. Depends on the second M mark.	
A1	Substituting the correct limits to obtain $\frac{\pi^3}{16}+\frac{\pi}{4}$ or $\frac{\pi^2}{16}+\frac{1}{4}$	
M1	Using $\bar{x} = (\pi)\int y^2 x\,\mathrm{d}x \div (\pi)\int y^2\mathrm{d}x$. π must be included for both integrals or neither. Denominator: to be the result given in (a). Numerator: Some attempt to obtain (by algebraic integration) a value for this integral must have been made and their value used here.	
A1cso	$(\pi^2+4)/4\pi$ any equivalent (in terms of π) accepted	

Q4.

Question Number	Scheme	Marks
(a)(i)	$V=\pi\int_1^2\left(x^2+4\right)^2\mathrm{d}x=\pi\int_1^2\left(x^4+8x^2+16\right)\mathrm{d}x$	
	$=\pi\left[\frac{1}{5}x^5+\frac{8}{3}x^3+16x\right]_1^2=\frac{613\pi}{15}\ (\mathrm{cm}^3)\quad *$	M1A1 A1cso
(ii)	$(\pi)\int_1^2 x\left(x^2+4\right)^2\mathrm{d}x=(\pi)\int_1^2\left(x^5+8x^3+16x\right)\mathrm{d}x$	
	$=(\pi)\left[\frac{1}{6}x^6+2x^4+8x^2\right]_1^2\qquad alt\,(\pi)\left[\frac{\left(x^2+4\right)^3}{6}\right]_1^2$	M1A1
	$\bar{x}=\frac{(\pi)\left[\frac{1}{6}x^6+2x^4+8x^2\right]_1^2}{\frac{613}{15}(\pi)}=\frac{\frac{129}{2}}{\frac{613}{15}}=1.578...=1.58\ (\mathrm{cm})$	M1dM1A1 (8)
(b)	Mass $\quad\frac{613\pi}{15}M\quad 9\pi M\quad 45\pi M\quad\left(36\pi+\frac{613\pi}{15}\right)M=\frac{1153\pi}{15}M$	B1
	Dist from B $\quad 0.578\quad 0.5\quad 0.5\quad \bar{y}$	B1ft
	$\frac{613\pi}{15}\times0.578-9\pi\times0.5+45\pi\times0.5=\left(36\pi+\frac{613\pi}{15}\right)\bar{y}$	M1A1ft
	$\bar{y}=\frac{1249}{2306}=0.5416...=0.54\ (\mathrm{cm})$	A1 (5)
		[13]

(a)(i)M1	Attempt the squaring and integrating (at least one power going up). Allow w/o π
A1	Correct integration allow w/o π
A1*cso	Correct volume, with no errors seen. (Must include π and no $V = ...$ w/o π must have been seen.)
(ii)M1	Attempt $\int x\left(x^2+4\right)^2 dx$. Must either expand or obtain $k\left(x^2+4\right)^3$. π not needed. Limits not needed
A1	Correct algebraic integration, π not needed. Limits not needed
M1	Substitute the (correct) limits in their integrated function. Independent, but must have been attempting $\int xy^2 dx$
M1	Divide the two integrals (correct way up). Depends on the 1st and 2nd M marks. π and ρ in both or neither.
A1	Correct final result. Must be 3 sf.
	(SC Correct answer with no algebraic integration shown can score M0A0 M1 M0A0)
(b)	
B1	Correct masses seen explicitly or in an equation.
B1ft	Correct distances from B (or any vertical axis). Follow through distance from (a).
M1	Form a moments equation, with lighter cylinder subtracted and the heavier one added
A1ft	Correct equation, follow through their distance from (a).
A1	Correct distance from B, 2 sf or better

Alt (b) Find mass and CoM of S_1 first $\quad \text{Mass} = \dfrac{478\pi}{15}M \quad CoM = \dfrac{287}{478} \approx 0.6004$

Award B1B1 when all component masses and distances are seen. Complete method needed for M1.
Award first A1 for correct masses/distances initially used in forming both equations.

(Note: Use of 0.58 leads to $\bar{x} = 0.603$ (cm) for S_1. This gives a final answer 0.543. If they give 0.54, award full marks, as premature approximation does not affect final answer, but penalise 0.543)

SC – If the use M and $5M$ for the masses, award max B0B1 M1A0A0

Q5.

(a)	$\text{Vol} = \pi\int_0^{\frac{\pi}{4}} y^2 \mathrm{d}x = \pi\int_0^{\frac{\pi}{4}} \sin^2 x\,\mathrm{d}x$	M1
	$= \pi\int_0^{\frac{\pi}{4}} \frac{1}{2}(1-\cos 2x)\,\mathrm{d}x$	dM1
	$= \frac{\pi}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}} = \frac{\pi}{2}\left(\frac{\pi}{4}-\frac{1}{2}\right) = \frac{\pi}{8}(\pi-2)$ *	dM1A1 (4)
(b)	$\pi\int_0^{\frac{\pi}{4}} y^2 x\,\mathrm{d}x = \pi\int_0^{\frac{\pi}{4}} x\sin^2 x\,\mathrm{d}x$	M1
	$= \pi\int_0^{\frac{\pi}{4}} \frac{1}{2}x(1-\cos 2x)\,\mathrm{d}x$	dM1
	$= \frac{\pi}{2}\left[\frac{x^2}{2}\right]_0^{\frac{\pi}{4}} - \frac{\pi}{2}\int_0^{\frac{\pi}{4}} x\cos 2x\,\mathrm{d}x$	
	$\frac{\pi^3}{64}, -\frac{\pi}{2}\left[x\times\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}} + \frac{\pi}{2}\int_0^{\frac{\pi}{4}} \frac{1}{2}\sin 2x\,\mathrm{d}x$	B1,dM1
	$= \frac{\pi^3}{64} \quad -\frac{\pi}{2}\times\frac{\pi}{8}\times 1 - \frac{\pi}{2}\left[\frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{4}}$	dM1
	$= \frac{\pi^3}{64} - \frac{\pi^2}{16} - \frac{\pi}{8}(0-1) = \frac{\pi^3}{64} - \frac{\pi^2}{16} + \frac{\pi}{8} \quad (=0.2603...)$	A1
	$\overline{x} = \left(\frac{\pi^3}{64} - \frac{\pi^2}{16} + \frac{\pi}{8}\right) \div \frac{\pi}{8}(\pi-2) \quad = 0.5806... = 0.581$	M1A1 (8)

(c)	$\tan\theta = \dfrac{\frac{\pi}{4} - \overline{x}}{\sin\frac{\pi}{4}}$	M1
	$\tan\alpha = \dfrac{\frac{\pi}{4}}{\sin\frac{\pi}{4}}$	B1
	$\text{reqd angle} = \alpha - \theta = 48.00... - 16.12... = 31.8... = 32°$	M1A1 (4) [16]

(a)	
M1	Set up the volume integral in terms of x. π and limits not needed (ignore any shown).
dM1	Attempt to use double angles to prepare integral. Allow $\frac{1}{2}(\pm 1 \pm \cos 2x)$
dM1	Attempt integration and substitute of correct limits. Depends on previous M mark.
A1	Arrive at given result, with no errors seen. If π only appears at end, there must be some justification.

(b)	**Note:** If correct algebraic integration seen, but no evidence of use of limits, a correct final answer implies the B1 and A marks.
M1	Attempt $\int xy^2\,dx$ in terms of x, π and limits not needed (ignore any shown)
dM1	Use double angle identity and split integral into two parts, ready for integration. Dependent on first M1.
B1	$\frac{\pi^3}{64}$ or $\frac{\pi^2}{64}$ if π cancelled
dM1	Attempt to start Integration by Parts. Dependent on previous M1.
dM1	Complete integration and substitute limits. Dependent on previous M1.
A1	Correct result (exact or decimal). Accept $\frac{\pi^2}{64} - \frac{\pi}{16} + \frac{1}{8}$
M1	Use $\bar{x} = \frac{\int xy^2 dx}{\int y^2 dx}$ π and ρ must be in both or neither.
A1cao	0.581 **must be 3s.f.**
Alt	Integral from (a) can be used, rather than splitting the integral
	$\int \frac{1}{2}x(1-\cos 2x)\,dx = \frac{1}{2}x\left(x - \frac{\sin 2x}{2}\right) - \frac{1}{2}\int\left(x - \frac{\sin 2x}{2}\right)dx$
	For second M1 we just need the use of the double angle identity and clear evidence of the use of the result from (a).
M1	Use their $\bar{x}$ correctly to find $\tan\theta$
B1	Correct expression for $\tan\alpha$ seen.
M1	Correct strategy (based on their $\bar{x}$) to find the required angle.
A1	32° cao

Q6.

Question Number	Scheme	Marks
(a)	$(\pi\rho)\int_0^r xy^2\,\mathrm{d}x$	
	$=(\pi\rho)\int_0^r x\left(r^2-x^2\right)\mathrm{d}x$	M1
	$=(\pi\rho)\left[\frac{1}{2}x^2r^2-\frac{x^4}{4}\right]_0^r$	A1
	$=(\pi\rho)\frac{r^4}{4}$	A1
	$M\bar{x}=\pi\rho\int xy^2\,\mathrm{d}x$	M1
	$\bar{x}=\frac{\pi\rho r^4}{4}\div\frac{2\pi\rho r^3}{3}=\frac{3}{8}r$ *	A1 (5)
(b)	Mass m M $(m+M)$	
	Dist from O $\frac{3}{8}r+4r$ $\frac{3}{4}\times 4r$ $\bar{x}$	B1
	$\frac{35}{8}rm+3rM=(m+M)\bar{x}$	M1A1ft
	$\bar{x}=\frac{(35m+24M)r}{8(m+M)}$	A1 (4)
(c)	$\bar{x}\cos\theta\leqslant OA$	M1
	$\cos\theta=\frac{4r}{OA}$	B1
	$\bar{x}\leqslant\frac{OA^2}{4r}$	A1
	$\bar{x}=\frac{(35m+24M)r}{8(m+M)}\leqslant\frac{17r^2}{4r}$	M1
	$35m+24M\leqslant 34(m+M)$	
	$M\geqslant\frac{m}{10}$ *	A1 (5) [14]

Question Number	Scheme	Marks
(a)		
M1	Using $\int xy^2\mathrm{d}x$ π and/or ρ may be missing limits need not be shown	
A1	Correct integration. π, ρ and limits need not be shown	
A1	Use limits to obtain $\frac{r^4}{4}$	
M1	Use $M\bar{x} = \pi\rho\int xy^2\mathrm{d}x$ with their previous result. π and ρ must be seen in both sides or neither.	
A1cso	Obtain $\frac{3}{8}r$	
(b)		
B1	Correct distances from O or centre of plane face (can both be positive).	
M1	Construct a moments equation with their distances using their masses (which may be volumes)	
A1ft	Correct moments equation, follow through their distances (but not their masses). If working from centre of plane face one term must be negative now.	
A1	Correct result for $\bar{x}$ (any equivalent) (inc fractions within fractions)	
(c)		
M1	Attempting an inequality with $\bar{x}$ and OA (Sign either way round or =)	
B1	A correct trig function connecting the angle used and OA	
A1	Obtain $\bar{x} \leqslant \frac{OA^2}{4r}$	
M1	Use their expression for $\bar{x}$ in their inequality/equality.	
A1cso	Obtain the given result.	
ALT for (c)		
	Use distance from centre of common face and tangents M is min when $\frac{\bar{x}-4r}{r} = \frac{r}{4r}$ $(= \tan\theta)$	

Q7.

Question Number	Scheme	Marks
(a)	$y = \frac{rx}{h}$	
	$(\pi)\int_0^h xy^2\mathrm{d}x = (\pi)\int_0^h \frac{r^2x^3}{h^2}\mathrm{d}x$	M1A1
	$(\pi)\left[\frac{r^2x^4}{4h^2}\right]_0^h = (\pi)\frac{r^2h^2}{4}$	A1
	$\bar{x} = \frac{\int xy^2\mathrm{d}x}{\int y^2\mathrm{d}x} = \frac{r^2h^2}{4} \div \frac{r^2h}{3} = \frac{3}{4}h$ *	M1A1cso (5)
(b)	Mass $\quad \frac{1}{3}\pi \times 5r^3 \quad \frac{2}{3}\pi r^3 \quad \frac{7}{3}\pi r^3$ $\quad 5 \quad 2 \quad 7$	B1
	Dist from O $\quad \frac{1}{4}\times 5r \quad -\frac{3}{8}r \quad \bar{x}$	B1
	$\frac{25r}{4} - \frac{3r}{4} = 7\bar{x}$	M1A1ft
	$\bar{x} = \frac{11r}{14}$	A1 cao (5)
(c)	$\tan\theta = \frac{\frac{11r}{14}}{r} = \frac{11}{14}$	M1A1ft
	$\theta = 38.15...^\circ$ (38 or better) (0.6659... rad; 0.67 or better)	A1 cao (3)
(d)	Mass of cone $= \frac{5}{2}M$ or Mass of $S = \frac{7}{2}M$ or mass of the particle $= \frac{2}{3}\pi r^3\rho$ and work with masses in volume form	B1
	$kMr(g) = \frac{7}{2}M \times \frac{11r}{14}(g)$	M1A1ft
	$k = \frac{11}{4}$ or 2.75	A1 cao (4) [17]

(a)M1	Attempting $(\pi)\int_0^h xy^2\mathrm{d}x$ π not needed. No integration needed for this mark but must move to a function of x only. limits not needed.
A1	Correct integral $\int_0^h \frac{r^2x^3}{h^2}\mathrm{d}x$ with or without π limits needed
A1	Integrate correctly and substitute correct limits
M1	Using $\bar{x} = \frac{\int xy^2\mathrm{d}x}{\int y^2\mathrm{d}x}$ π in both or neither integral. Denominator need not be in integral form. If mass/unit volume included, must appear in both or neither.
A1cso	Correct GIVEN answer. cso so $y = \frac{rx}{h}$ must have been used. If $h = r$ is assumed: M marks only available. If $h = 5r$ is assumed max available is 4/5 (Deduct final A mark)
(b)B1	Correct mass ratio
B1	Correct distances from O or any other point. Both known distances may be positive here, but one must have a minus both here and in the equation or just in the equation.
M1	Attempt a moments equation using their mass ratio and distances (signs may be incorrect)
A1ft	Correct equation, follow through their mass ratio and distances but signs to be correct
A1cao	Correct distance for the c of m
(c)M1	Using tan to find the required angle. Ratio can be either way up.
A1ft	Tan ratio to be correct way up and have correct numbers, follow through their ans to (b)
A1cao	Correct size of angle in degrees or radians, 2 sf or better
(d)B1	Correct mass for cone or for S
M1	Moments equation about O or another point. This may include an $\bar{x}$ for the c of m of S + particle
A1ft	Correct nos in the equation, follow through their ans to (b) and mass of $S \neq M$ $\bar{x}$ to be 0 now
A1cao	Correct value, 11/4 or 2.75
ALTs	**For (d)**
1	Use $M = \frac{2}{3}\pi r^3$ so mass of particle is $\frac{2}{3}\pi r^3 k$ and use volumes in the work. Mass of the particle scores B1 provided all the work is using volumes.
2	Can work with the cone, hemisphere and particle so the equation will have 3 terms (or 4 if $\bar{x}$ for the c of m of S + particle is not assumed to be zero.

Q8.

Question Number	Scheme	Marks
	ratio of masses $\quad 96\pi a^3\rho \quad 18\pi a^3\rho \quad 78\pi a^3\rho$	M1A1
	$\quad 16 \quad 3 \quad 13$ oe	
	Distance from O: $\quad 3a \quad \frac{9a}{8} \quad \bar{x}$	B1
	$16\times 3a - 3\times\frac{9a}{8} = 13\bar{x}$	M1A1ft
	$\bar{x} = \frac{357}{104}a = 3\frac{45}{104}a$ or $3.432a$....(accept $3.4a$ or better)	A1 [6]

M1	Attempt the ratio of masses. Terms to be consistent in use of π, a^3, ρ and formulae must be correct.
A1	Correct ratio, any equivalent form. Simplification not required.
B1	Correct distances from O or any other point
M1	Form a moments equation using their mass ratio and distances. Must be dimensionally correct and Hemisphere must be subtracted on LHS.
A1ft	Correct equation ft their mass ratio and distances.
A1	Correct answer, exact or decimal. Must be from O and must be positive.

Q9.

Question Number	Scheme	Marks
(a)	Mass $\frac{1}{3}\pi\times16r^2\times4h$ $\quad\frac{1}{3}\pi\times9r^2\times3h$ $\quad\frac{1}{3}\pi\times37r^2h$	B1
	(Ratio 64 27 37)	
	Dist h $\quad h+\frac{3}{4}h$ $\quad\bar{x}$	B1
	$64h-27\times\frac{7}{4}h=37\bar{x}$	M1A1ft
	$\bar{x}=\frac{67}{148}h$ accept $0.45(270.....)h$ or better	A1 (5)
(b)	$\tan\theta=\frac{h-\bar{x}}{3r},=\frac{1-\frac{67}{148}}{3}$	M1,A1ft
	$\theta=\tan^{-1}\left(\frac{148-67}{3\times148}\right)\ (=10.34...)$	
	Reqd angle $=\tan^{-1}\left(\frac{r}{3r}\right)-\theta=8.096...^\circ$	M1A1 (4)
	Accept 8.1 or better	[9]
ALT:	$\tan\theta=\frac{3r}{h-\bar{x}},=\frac{3}{1-\frac{67}{148}}$	M1A1
	Reqd angle $=\theta-\tan^{-1}(3)=8.096...^\circ$	M1A1

(a)

B1 Correct ratio of masses, any equivalent. (Mark the ratio, not formulae.)

B1 Correct distances from O or any other point

M1 Use their mass ratio and distances to form a moments equation with 3 terms.

A1ft Correct equation, follow through their mass ratio and distances.

A1 Correct distance from O. Exact or min 2 sf

(b)

M1 Use their $\bar{x}$ to form an expression for $\tan\theta$. Can include h and r. $h-\bar{x}$ needed but fraction can be either way up. Denom (numerator in ALT) can be r, $3r$ or $4r$.

A1ft Fraction either way up and $h=r$ used. Follow through their $\bar{x}$

M1 Complete the method to obtain the required angle

A1 Correct size of the angle, 2 sf min. Radians accepted - 0.14 or better.

NB If solid is hung from a point on the rim of the base give M1A0M0A0

Q10.

Question Number	Scheme	Marks
	cone cylinder S	
	Mass $\frac{1}{3}\pi r^2 \times 4h$ $\pi r^2 \times 3h$ $\frac{13}{3}\pi r^2 h$	B1
	4 9 13	
	Dist $(-)h$ $\frac{3}{2}h$ $\overline{x}$	B1
	$-4h + \frac{27}{2}h = 13\overline{x}$	M1A1ft
	$\overline{x} = \frac{19}{26}h \ \ (= 0.73h \text{ or better})$	A1
		[5]
B1	Correct mass ratio, any equivalent form	
B1	Correct distances from any point	
M1	Form a moments equation with their mass ratios and distances. If distances are measured from O there must be a difference of mass x distance terms	
A1ft	Correct equation, follow through their mass ratios and distances but must be dimensionally correct	
A1	Correct answer. Must be positive.	

Q11.

Question Number	Scheme	Marks
	Vol of cylinder $= 8\pi r^2 h$ Vol cut away $= \frac{1}{3}\pi r^2 h$	
	Cylinder, part cut away, conical shell, container	
	mass: M, $(\pm)\frac{1}{24}M$, $\frac{1}{12}M$, $\frac{25}{24}M$	M1A1
	dist from O: h, $\frac{7}{4}h$, $5h$, $\bar{x}$	B1
	$Mh - \frac{7}{96}Mh + \frac{5}{12}Mh = \frac{25}{24}M\bar{x}$	M1A1
	$\bar{x} = \frac{129}{100}h$ or $1.29h$	A1
		[6]

M1	Attempt at mass ratios. Must have 4 masses. M may be consistently omitted. Condone omission of 1/3 in volume of cone. Must be using the given masses, not volumes.
A1	Correct mass ratios. Sign not needed for cut away part.
B1	Correct distances (from a consistent point – see below)
M1	Attempt at moments equation. Condone missing minus. Accept as part of a vector equation.
A1	Correct equation.
A1	$\bar{x} = \frac{129}{100}h$ or $1.29h$

ALT distances

	Cylinder	Cut away	Shell
Vertex	0	$\frac{3}{4}h$	$4h$
Top of cylinder	$(-)h$	$(-)\frac{1}{4}h$	$3h$
Top of cone	$6h$	$\frac{21}{4}h$	$2h$

Note: If completing in 2 stages, centre of mass with cut-away cylinder without shell is $\frac{89}{92}h$.

Marks can only be awarded when they give a complete method, including the conical shell.

Special Case: If they first find the CoM of cut-away cylinder, then use M for its mass, award max M0A0 B1 M1A1 A0. Their equation must be $M \times \frac{89}{92}h + \frac{M}{12} \times 5h = \frac{13M}{12}\bar{x}$

Q12.

Question Number	Scheme	Marks
(a)	$(\pi)\int y^2x\,dx=(\pi)\int_0^h\left(\frac{r}{h}\right)^2x^3dx$ OR $(\pi)\int y^2x\,dx=(\pi)\int_0^h\left(r-\frac{r}{h}x\right)^2x\,dx$	M1
	$=(\pi)\left(\frac{r}{h}\right)^2\left[\frac{x^4}{4}\right]_0^h,\ =(\pi)\frac{r^2h^4}{4h^2}\ \left(=(\pi)\frac{r^2h^2}{4}\right)$	dM1,A1
	$\bar{x}=(\pi)\frac{r^2h^2}{4}\div\frac{1}{3}(\pi)r^2h=\frac{3}{4}h$ *	M1A1cso (5)
(b)	cone, hemisphere, S	
	Mass: cone $5m$, hemisphere km, S $m(5+k)$	
	Distance from O: cone $\frac{1}{4}\times 6r$, hemisphere $(-)\frac{3}{8}r$, S $\bar{x}$	B1
	$5m\times\frac{3}{2}r-km\times\frac{3}{8}r=m(5+k)\bar{x}$	M1A1ft
	$\bar{x}=\frac{3r(20-k)}{8(5+k)}$	A1 (4)
(c)	Angle between AO and vertical $=30°$ or angle between axis and vertical $=60°$	B1
	$\tan 30=\frac{\bar{x}}{r},\ =\frac{3(20-k)}{8(5+k)}$ OR $\tan 60=\frac{r}{\bar{x}},\ =\frac{8(5+k)}{3(20-k)}$	M1,A1ft
	$k=4.844...=5$	A1cso (4) **[13]**

(a)	
M1	Use $(\pi)\int y^2x\,dx$ to obtain an integral in x. Integral should be one of the forms shown. π and limits need not be shown
dM1	Attempt the integration; π and limits need not be shown. Depends on the previous M mark.
A1	Substitute (correct) limits to obtain correct result. π not needed, no need to simplify. Second integral gives $\frac{r^2h^2}{12}$ or $\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right)r^2h^2$ on substitution of limits
M1	Divide their result by the volume (given formula or an integral), π to be included in numerator and denominator or in neither. Can have the distance from the vertex or the distance from the base. Must reach $\bar{x}=...$
A1cso	Correct **given** result; no errors in the working. If the distance from the base has been found $h-\frac{1}{4}h$ must be seen to justify the answer. **Special Case:** Cone base radius r and height $6r$ Equation is $y=\frac{1}{6}x$ Integral is $(\pi)\int_0^{6r}\frac{x^3}{36}dx$ or $(\pi)\int_0^{6r}\left(r-\frac{x}{6}\right)^2x\,dx$ Award M marks only (if earned)

(b)	
B1	Correct distances shown explicitly or used. Negative sign may be missing here.
M1	Moments equation including a minus sign. Must use the given masses (or ratio of them), not volumes.
A1ft	Fully correct equation follow through their distances
A1	Correct expression for the required distance. Must be positive but can include modulus sign. $(k-20)$ in the numerator without modulus signs scores A0. Equivalents accepted but not fractions within fractions.
(c)	
B1	For either of the angles, shown explicitly or used. May be seen on a diagram.
M1	$\tan 30$ or $60=\frac{\bar{x}}{r}$ or $\frac{r}{\bar{x}}$ (ie 30 or 60 and fraction either way up)
A1ft	Fully correct equation. Follow through their $\bar{x}$
A1cso	$k=5$

Q13.

Question Number	Scheme	Marks
(a)	$\text{Vol} = (\pi)\int_{\frac{3}{5}r}^{r}\left(r^2 - x^2\right)\mathrm{d}x = (\pi)\left[r^2x - \frac{1}{3}x^3\right]_{\frac{3}{5}r}^{r}$	M1A1
	$= (\pi)\left(r^3 - \frac{1}{3}r^3 - \left(\frac{3}{5}r^3 - \frac{9}{125}r^3\right)\right)\ \left(= \frac{52}{375}(\pi)r^3\right)$	M1
	$(\pi)\int_{\frac{3}{5}r}^{r} x\left(r^2 - x^2\right)\mathrm{d}x = (\pi)\left[\frac{1}{2}r^2x^2 - \frac{1}{4}x^4\right]_{\frac{3}{5}r}^{r}$	M1A1
	$= (\pi)\left(\frac{1}{2}r^4 - \frac{1}{4}r^4 - \left(\frac{9}{50}r^4 - \frac{81}{2500}r^4\right)\right)\ \left(= \frac{64}{625}(\pi)r^4\right)$	M1
	$\bar{x} = \dfrac{\int xy^2\mathrm{d}x}{\int y^2\mathrm{d}x} = \dfrac{\frac{64}{625}}{\frac{52}{375}}r$	M1
	$= \frac{48}{65}r$ *	A1cso (8)

(b)	Bowl alone: Mass ratio 6^3 5^3 91	
	Dist from A: $\frac{3}{8}\times 6$ $\frac{3}{8}\times 5$ $\bar{y}$	
	$216\times\frac{3}{8}\times 6 - 125\times\frac{3}{8}\times 5 = 91\bar{y}$	M1A1A1
	$\bar{y} = 2.7651...$ $\left(\frac{2013}{728},\ 2\frac{557}{728}\right)$	A1
	Bowl and liquid: Mass ratio 5 2 7	
	Dist from A: 2.7651... $\frac{48}{13}$ $\bar{z}$	B1 (48/13)
	$7\bar{z} = 5\times 2.7651 + \frac{48}{13}\times 2$	M1A1ft
	$\bar{z} = 3.030... = 3.03$ cm	A1 (8) [16]
ALT	Find mass of whole hemisphere and part cut away in terms of M and use a single moments equation (see end)	

Question Number	Scheme	Marks
(a)	**Lamina scores 0/8.** **If no evidence of algebraic integration seen, only the last M mark is available.**	
M1	Attempt the volume integral, π and limits not needed (ignore any shown)	
A1	Correct integration, π and limits not needed (ignore any shown)	
dM1	Substitute the correct limits in their result. Evidence of substitution must be seen. Depends on previous M mark	
M1	Attempt $\int xy^2\,\mathrm{d}x$, π and limits not needed (ignore any shown)	
A1	Correct integration, π and limits not needed (ignore any shown)	
dM1	Substitute the correct limits in their result. Evidence of substitution must be seen. Depends on previous M mark	
M1	Use $\bar{x} = \dfrac{\int xy^2\mathrm{d}x}{\int y^2\mathrm{d}x}$ with their previous results (need not be simplified results). π in both or neither integral	
A1cso	Correct final (**given**) result obtained from fully correct working.	
(b)		
M1	Attempt a moments equation with the *difference* of two hemispheres. Dimensions for the hemispheres must be correct.	
A1	Correct masses or ratio of masses	
A1	Correct distances	
A1	Correct distance for the bowl – exact or decimal	
B1	For the correct distance of the c of m of the liquid from A	
M1	Attempt a moments equation – bowl and liquid added. Must attempt the distance for the liquid ie we are looking for a numerical distance , not just a letter and must have shown evidence of calculating the c of m of the bowl (M mark for this may have been lost)	
A1ft	Correct equation, follow through their distances (ie 48/13 and c of m of bowl)	
A1	Correct answer from correct working. Must be 3 sf	

Question Number	Scheme	Marks
ALT (b)	Vol of bowl $= \frac{2}{3}\pi\left(6^3 - 5^3\right) = \frac{2}{3}\pi \times 91$	
	$\frac{2}{3}\pi\rho \times 91 = 5M$	B1
	Mass ratio $\quad 6^3 \quad\quad 5^3$	
	$6^3 \times \frac{5}{91}M \quad\quad 5^3 \times \frac{5}{91}M \quad\quad 2M \quad\quad 7M$	M1A1A1
	Dist from A: $\frac{3}{8}\times 6 \quad\quad \frac{3}{8}\times 5 \quad\quad \frac{48}{13} \quad\quad \bar{y}$	B1(48/13)
	$6^3 \times \frac{5}{91}M \times \frac{3}{8}\times 6 - 5^3 \times \frac{5}{91}M \times \frac{3}{8}\times 5 + 2M \times \frac{48}{13} = 7M\,\bar{y}$	M1A1ft
	$\bar{y} = 3.030... = 3.03$	A1 (8)
B1	For a correct equation connecting the mass of the bowl and $5M$. Award if $\frac{5}{91}M$ or $\frac{5}{91}$ is seen used correctly in at least one term in their equation. Enter as the first A mark on e-PEN	
M1	For attempting the mass ratio for the 4 parts needed including their "5/91"	
A1A1	Deduct one per error	
B1	For 48/13	
M1	Attempt a moments equation with 4 terms and correct signs. An attempt at the mass ratio of the parts based on the mass of the bowl being 5M must have been seen even if this attempt failed to qualify for the first M mark.	
A1ft	Correct equation, follow through their masses and distances (ie 48/13 and c of m of bowl)	
A1	Correct answer from correct working. Must be 3 sf	