Edexcel International A Level (IAL) Maths: Mechanics 2

Your notes

Collisions

Contents

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Impulse in 2D

How does impulse work in 2D?

- Impulse and momentum can be used in 2D as they are vector quantities
  - Impulse in 2D essentially works the **same** way as impulse in 1D
- For a **constant force** given by the vector $\mathbf{F}\ N = (F_x\mathbf{i} + F_y\mathbf{j})$ N acting on a particle for t seconds the **impulse** is given by the **vector**:
  - $\mathbf{I} = \mathbf{F}t = (F_x\mathbf{i} + F_y\mathbf{j})t$
  - The units are still N s (equivalent to kg m s^{-1})
- For a particle of mass the impulse is still equal to the **change in momentum**
  - $\mathbf{I} = m(\mathbf{v} - \mathbf{u})$
  - where **u** m s^{-1} is the initial velocity vector and **v m s^{-1}** is the final velocity vector

How are questions different in 2D?

- You could be asked to work out the **magnitude** of the impulse
  - You would need to find the two components of the impulse vector and then use **Pythagoras**
- You could be asked to work out the **direction** of the impulse
  - You would need to find the two components of the impulse vector and then use **SOHCAHTOA (right-angled trigonometry)**
  - You might need to find the angle between the impulse and the vector $\mathbf{i}$ or $\mathbf{j}$ so always draw a sketch
- If two particles **collide in 2D** then for this specification they will always be **direct collisions**
  - This means that they will be travelling along the **same line** so the velocity vectors will be **scalar multiples** of each other
  - These problems are rare and would be solved using **conservation of momentum** in exactly the same way as in 1D
- If you know the magnitude and direction of the impulse or a velocity then you might have to **resolve** it into horizontal and vertical components

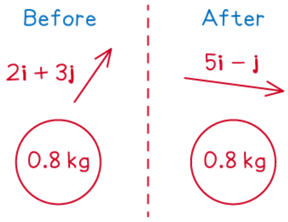
Worked Example

A ball of mass 0.8 kg is moving with velocity $(2\mathrm{i}+3\mathrm{j})\ \mathrm{m\ s^{-1}}$ when it receives an impulse Q N s . Immediately after receiving the impulse, the velocity of the ball is $(5\mathrm{i}-\mathrm{j})\ \mathrm{m\ s^{-1}}$.

(a) Find the magnitude of the impulse **Q** N .

(b) Find the angle between the direction of the impulse and the vector **j**.

Answer:

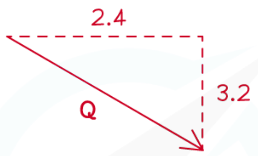


Use '$I = mv - mu$'

$\mathbf{Q} = 0.8(5\mathbf{i} - \mathbf{j}) - 0.8(2\mathbf{i} + 3\mathbf{j})$

$\mathbf{Q} = 2.4\mathbf{i} - 3.2\mathbf{j}$

Sketch the impulse vector

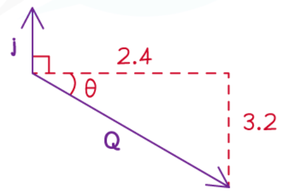


(a) Find magnitude using Pythagoras

$|\mathbf{Q}| = \sqrt{2.4^2 + 3.2^2}$

Magnitude = 4 Ns

(b) Find the angle by using trigonometry



$\tan\theta = \frac{3.2}{2.4}$

$\theta = \tan^{-1}\left(\frac{3.2}{2.4}\right) = 53.130...$

Add 90° to get angle between **Q** and **j**

$90 + 53.130... = 143.130...$

Angle between **Q** and **j** is 143° (3sf)

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Examiner Tips and Tricks

- Be careful with negatives, especially when adding and subtracting vectors.
- When finding angles and directions always sketch a diagram. Read the question carefully to help you decide where the angle should be measured from.

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Collisions – Multiple Collisions

Can there be multiple collisions?

- After **two objects collide** it is possible that one (or both) of them **collides with something else** such as
  - A **third object**
  - A **wall** (perpendicular to the motion)
- Deal with **each collision separately** and use the steps for direct collisions

Can there be a second collision between the original two objects?

- Let A, B and C be three objects travelling in the **same straight line** and suppose **A and B collide directly** and subsequently **B and C collide directly**
- After the collisions between **A and B** and **B and C** there will be a **second collision** between **A and B if**:
  - **One** is **stationary** and the **other** is **travelling towards** it
  - Both are **travelling in opposite** directions **towards** each other
  - Both are **travelling in the same** direction and the **one in front is slower** than the one behind
- The process is **similar** if object **C is a wall**
  - After **B collides with the wall** its **direction will be reversed** so it will be travelling **towards A**
  - B will **collide with A** again if its **velocity in that direction** is **greater** than the velocity of A in that direction
  - To help you work out the speed of B after hitting the wall you will be given extra information such as the **impulse** exerted by the wall or the **coefficient of restitution**

Worked Example

Three uniform balls A, B and C of equal radius, and mass 0.1 kg, 0.2 kg and 0.3 kg respectively, can move along the same straight line on a smooth horizontal table with B in the middle of A and C. A and B are projected towards each other in opposite directions with speed $10\ \mathrm{m\,s^{-1}}$ and $2\ \mathrm{m\,s^{-1}}$ respectively while C is at rest. A and B collide directly which does not change the direction of motion of A and subsequently A moves with speed $1\ \mathrm{m\,s^{-1}}$.

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(a) Show that the speed of B immediately after it collides with A is $2.5\ \mathrm{m\,s^{-1}}$.

(b) In the subsequent motion, B collides directly with C. Immediately after this collision, C moves with speed $1.5\ \mathrm{m\,s^{-1}}$. Determine if there will be a second collision between A and B.

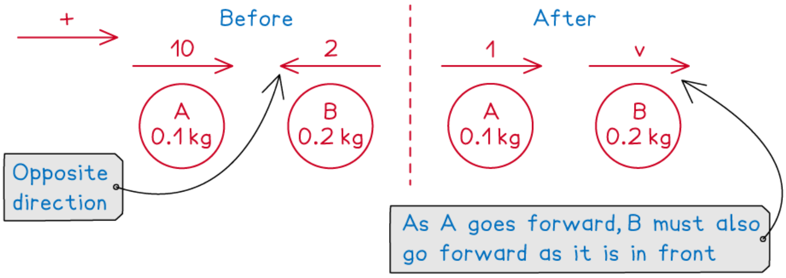
Answer:

a) Look at the collision between A and B

Step 1: Choose positive direction

Let the initial direction of A be positive

Step 2: Draw a before/after diagram



Step 3: Use conservation of momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$0.1(10) + 0.2(-2) = 0.1(1) + 0.2(v)$

$0.6 = 0.1 + 0.2v$

Step 4: Solve

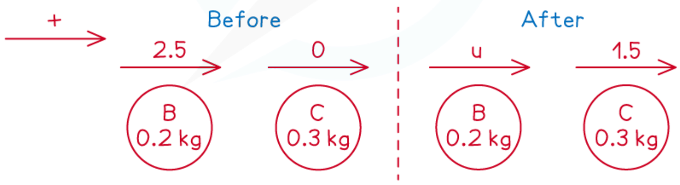
$v = 2.5$

Speed of B = 2.5 m s^{-1}

b) Look at the collision between B and C

Step 1: Choose positive direction

Step 2: Draw a before/after diagram



Step 3: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$0.2(2.5) + 0.3(0) = 0.2(u) + 0.3(1.5)$

$0.5 = 0.2u + 0.45$

Step 4: $u = 0.25$

Look at velocities of A and B after collisions

A and B are traveling in the same direction

A's speed is greater than B's speed

There will be another collision between A and B

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Examiner Tips and Tricks

- These questions can be difficult to visualise in your head so draw simple diagrams to show each collision.
- Use common sense, think how many possible (or impossible) ways there are for objects to move after the first collision. You will often have to consider the speed

of one or more objects to decide if a second or third collision is possible.

- These questions can involve lots of algebra, negatives and inequalities so do not rush them as you might make a silly mistake which can affect subsequent parts.

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Coefficient of Restitution

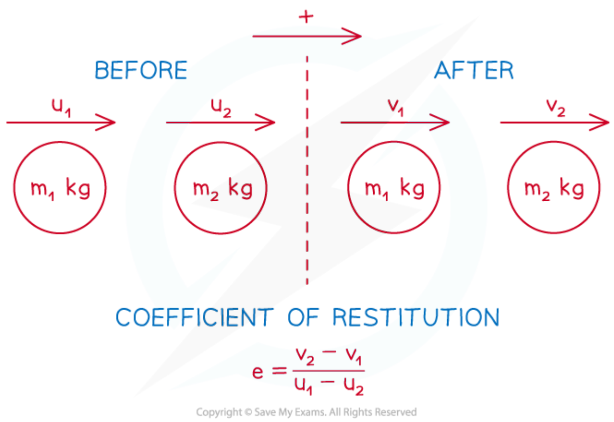
What is the coefficient of restitution?

- The coefficient of restitution (also known as **Newton's Experimental Law**) is the **ratio** of the **relative speed of separation** and the **relative speed of approach** when two objects collide
  - Essentially this just means the speed of separation divided by the speed of approach
- The coefficient of restitution is **denoted e**
  - e is **dimensionless** as it is a ratio
  - The value of e in a particular situation will depend on the materials that the two particles are made from
- In this course e can take the values in the range **$0 \le e \le 1$**
  - **e =1**: These are called **perfectly elastic** collisions and in these collisions there is **no loss in kinetic energy**
  - **e =0**: These are called **perfectly inelastic** collisions and in these collisions the objects **coalesce**

How do I calculate the coefficient of restitution between two objects?

- $e = \dfrac{\text{Speed of seperation of the objects}}{\text{Speed of approach of the objects}}$
- The speed of approach depends on whether the objects are travelling towards each other or in the same direction. For example, if the speeds of the two objects **before** the collision are 5 m s^{-1} and 2 m s^{-1} then the **speed of approach** is:
  - 3 m s^{-1} if they are moving in the **same direction** (each second the objects approach each other by a further 3 metres)
  - 7 m s^{-1} if they are moving in **opposite directions** (each second the objects approach each other by a further 7 metres)
- The speed of separation depends on whether the objects are travelling away from each other or in the same direction. For example, if the speeds of the two objects **after** the collision are 5 m s^{-1} and 2 m s^{-1} then the **speed of separation** is:
  - 3 m s^{-1} if they are moving in the **same direction** (each second the objects separate by a further 3 metres)
  - 7 m s^{-1} if they are moving in **opposite directions** (each second the objects separate by a further 7 metres)
- If the velocities of the two objects before the collision are u_1 m s^{-1} and u_2 m s^{-1} and the velocities after the collision are v_1m s^{-1} and v_2 m s^{-1} then:

- $e = \frac{v_2 - v_1}{u_1 - u_2}$
- Note that **velocities can be negative** so be careful with signs



How do I calculate the coefficient of restitution between an object and a wall?

- As the wall does not have any velocity
  - $e = \frac{\text{Speed of rebound of the object}}{\text{Speed of approach of the object}}$
- If the **speed** of the object **before** the hitting the wall is u m s^{-1} and the **speed after** is v m s^{-1} then the formula above simplifies to:
  - $e = \frac{0 - (-v)}{u - 0}$
  - $e = \frac{v}{u}$

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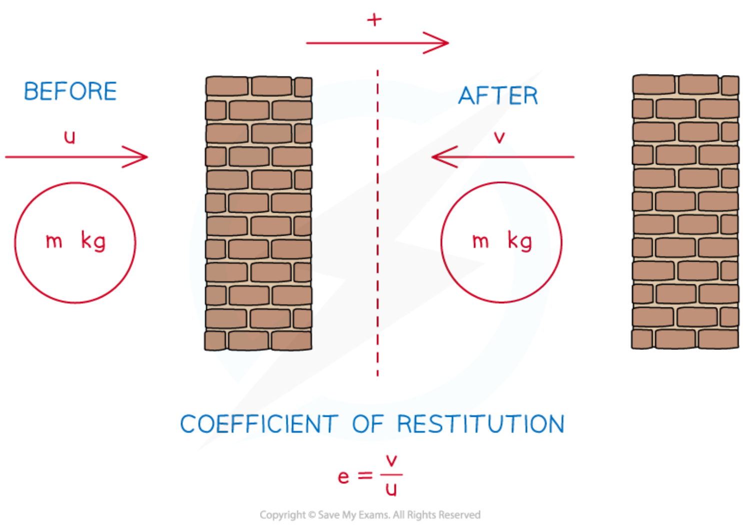

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How do I solve collision problems involving the coefficient of restitution?

- **STEP 1**: **Draw** a before/after diagram and label the **positive direction**
  - There may be multiple diagrams if there are multiple collisions
- **STEP 2**: Form an equation using the **coefficient of restitution**
  - The unknown(s) could be the coefficient of restitution or any of the speeds or directions
- **STEP 3**: Form an equation using the principle of **conservation of momentum**
  - In the case of a collision with a wall you may be given the impulse or some other information instead
- **STEP 4**: **Solve** and give your answer in **context**
  - You may have to solve simultaneous equations
  - You may have to solve an inequality
  - You may have to form an inequality using $0 \le e \le 1$ or using the fact that velocity is positive (or negative) if the object is going forwards (or backwards)

Worked Example

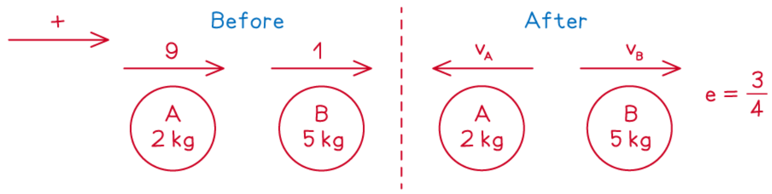
Two balls A and B are of equal radius and have masses 2 kg and 5 kg respectively. A and B collide directly. Immediately before the collision, A and B are moving in the same direction along a straight line on a smooth horizontal surface with speeds 9

$\mathrm{m\ s^{-1}}$ and $1\ \mathrm{m\ s^{-1}}$ respectively. Immediately after the collision, the direction of motion of A is reversed. The coefficient of restitution between A and B is $\frac{3}{4}$.

Find the speeds of A and B immediately after the collision.

Answer:

Step 1: Draw a before/after diagram



Step 2: Coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

v_A is in opposite direction

$$\frac{3}{4} = \frac{v_B - (-v_A)}{9 - 1} = \frac{v_B + v_A}{8}$$

$$6 = v_B + v_A \quad (1)$$

Step 3: Conservation of momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$2(9) + 5(1) = 2(-v_A) + 5(v_B)$$

$$23 = 5v_B - 2v_A \quad (2)$$

Step 4: Solve

Rearrange (1) $v_A = 6 - v_B$

Substitute into (2) $23 = 5v_B - 2(6 - v_B)$

$23 = 7v_B - 12$

$\therefore v_B = 5$

Substitute back into (1) $v_A = 6 - 5$

$\therefore v_A = 1$

After collision:
Speed of A is $1\ \mathrm{ms^{-1}}$
Speed of B is $5\ \mathrm{ms^{-1}}$

Worked Example

A sphere A is projected with speed $5\ \mathrm{m\ s^{-1}}$ along a straight line towards another sphere B, of equal radius, which is at rest on a smooth horizontal surface. After the

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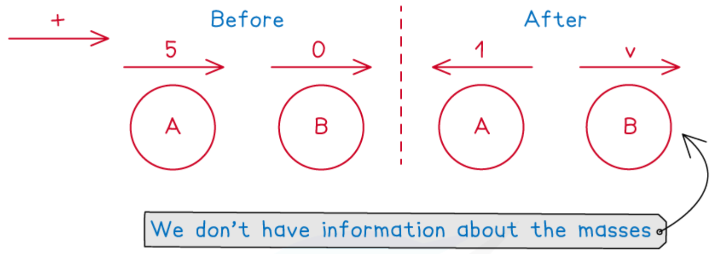
collision, B moves with speed $v \text{ m s}^{-1}$ and A moves in the opposite direction with speed 1 m s^{-1}. The coefficient of restitution between A and B is e.

(a) Show that $v = 5e - 1$.

(b) Hence, find the range of possible values of e.

Answer:

Step 1: Draw a before/after diagram



Step 2: Coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$e = \frac{v - (-1)}{5 - 0}$$ 1 is in opposite direction

$$e = \frac{v + 1}{5}$$

We now have enough information to do part (a)

a) $v + 1 = 5e$

$v = 5e - 1$

b) After the collision B moves forward so $v > 0$

$5e - 1 > 0$

$e > \frac{1}{5}$

Your notes

Examiner Tips and Tricks

- Exam questions refer to spheres as having equal radii, this just means the objects are the same size so that dimensions don't affect the collision. Exam questions often leave velocities in terms of e. If you know the direction of the object then you know whether the velocity is positive or negative. This can be used to form an inequality for the range of possible values of e for that scenario.