Edexcel International A Level (IAL) Maths: Mechanics 2

Your notes

Work & Energy

Contents

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Work

What does the term work mean in Mechanics?

- In Mechanics the word work refers to the **work done** by a force when it causes an object to move
  - Mechanical work happens when both a **force** is applied and the object **moves**
  - A force that is holding an object stationary is not producing any mechanical work
  - The object **gains energy** due to the work done by forces acting in the **direction of motion**
  - The object **loses energy** due to the work done **against resistive forces**
- The **line of action** of a force refers to the **point of application** of the force and the **direction** the force was applied in
- Work is a **scalar quantity,** it has size without direction

How do we calculate work done by a force?

- If the **point of application** of a force of magnitude ***F*** **N**, which moves an object a distance, ***d*** **metres**, is in the same direction as the **line of action** of the force, then the **work done**, W , by the force is

$$W = Fd$$

- If the line of action of the force is at an **angle** to the direction of motion, then the **component** of the force in the direction travelled is multiplied by the distance instead
- If the object moves **vertically upwards** then the work is done against gravity and this calculation becomes

$$W = mgh$$

- The units for the work done by a force are **Newton metres (N m)**, but it is more common to use **Joules**
  - **1 Joule = 1 N m**
  - 1 Joule is equal to the amount of work done by a force of 1 Newton moving an object 1 metre along the line of action of the force
  - **1 Kilojoule** is equal to **1000 Joules** (**1 kJ = 1000 J**)
- If the line of action of the force is different to the direction of motion, start by **resolving** the force into components **parallel** and **perpendicular** to the direction of motion
  - When a force of ***F*** **N** is acting at an angle of $\theta°$ to the direction of motion, the component of *F* acting in the direction of motion will be $F\cos\theta$ and the **work done** by the force will be $Fd\cos\theta$

- The **perpendicular** component of the force will produce no work

- The **net work done** on an object will be equal to the work done by the force that moves the object forwards, in addition to any work done against resistive forces

How do we use work done with N2L (F = ma)?

- If the **work done by a force** is known it can often be used along with **N2L** to find one of the components in the formula
- Often the object will be moving at constant speed so the **acceleration** of the object will be zero so the forces acting on the object in the **direction of motion** will be in equilibrium

STEP 1: Draw a diagram or add all the forces to the diagram given in the question

STEP 2: If the line of action of the force is at an angle to the direction of motion, find the component of the force parallel to the direction of motion

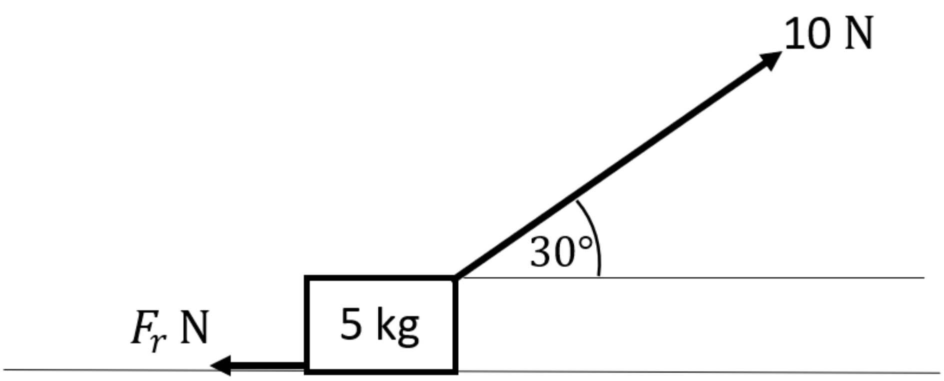
STEP 3: If necessary, resolve the forces acting in the direction of motion to find the magnitude of the force acting on the object

STEP 4: Use ***W = Fs*** to find the work done by the force on the object

- If the problem involves **friction**, you may have to resolve perpendicular to the plane to find the value of the **normal reaction force, *R***
    - If the force overcomes friction to cause an object to move, it is said to do **work done against friction**
    - The object will be moving, so **friction will be limiting** and $F_{\text{MAX}} = \mu R$

Worked Example

A child pulls a box of mass 5 kg along a rough horizontal surface at a constant speed by a force of magnitude 10 N inclined at 30° to the horizontal. The only resistance force is from friction, F_r N as shown in the diagram below.



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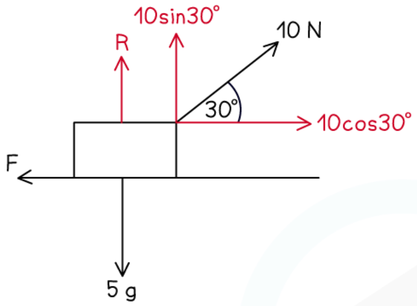
Calculate the work done against friction as the child pulls the box 5 metres along the floor.

Answer:

Step 1: Draw a diagram and add the forces



Step 2: Resolve the angled force into components parallel and perpendicular to the direction of motion

Step 3: Acceleration is zero so forces in the horizontal direction balance

$$F = 10\cos 30° = 5\sqrt{3}\ \text{N}$$

Step 4: Use $W = Fs$ to find the work done

Work done against friction = Work done by the force

$$= (10\cos 30)(5)$$

$$= 25\sqrt{3}\ \text{N m}$$

$$= 43.30...\ \text{J}$$

WD = 43.3 J (3sF)

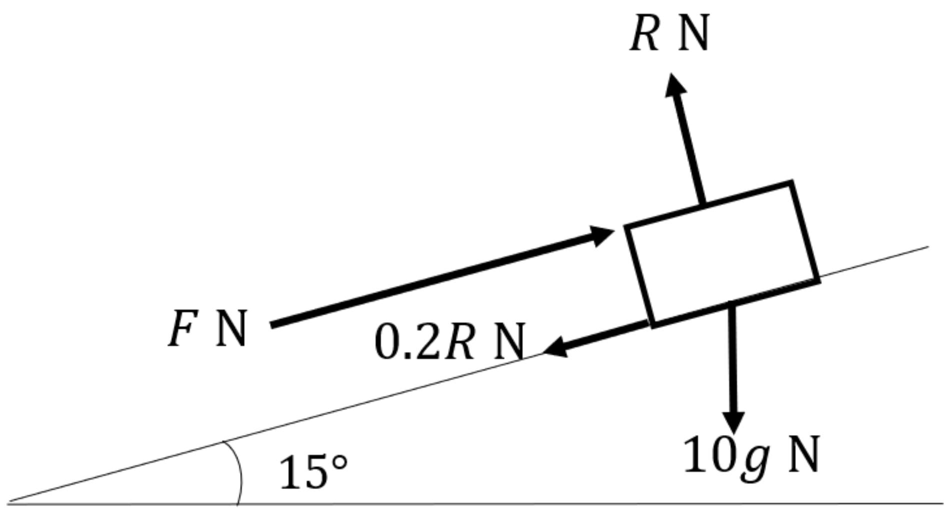
How do we use work done on an inclined plane?

- The object will be said to travel up the **line of greatest slope** on an **inclined plane**
- If the **coefficient of friction** is involved then the weight will need to be resolved to find the value of the **normal reaction force, *R***
  - There is no work done by the normal reaction force as it acts perpendicular to the direction of motion
- The **work done against gravity** can be found by using right – angled trigonometry to find the change in vertical height of the object

Worked Example

A crate of mass 10 kg is pushed 6 metres up a rough ramp inclined at 15° to the horizontal by a force F N. The crate moves with constant speed along the line of greatest slope. The coefficient of friction between the container and the ramp is 0.2 as shown in the diagram below.

Your notes

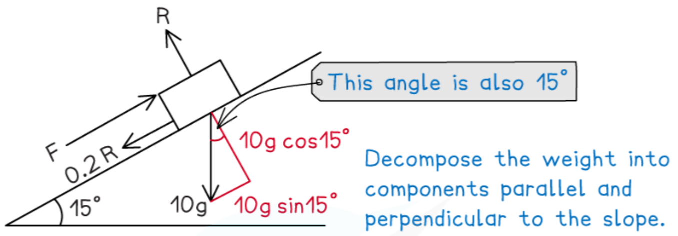


(i) Calculate the work done against friction.

(ii) Calculate the work done against gravity.

Answer:

(i)



Find the value of R: $R = 10g\cos 15°$

$= (10)(10)(\cos 15) = 96.592\ldots$ N

Use $Fr = \mu R$ as the crate is moving

$Fr = 0.2 \times 96.592\ldots$

$= 19.318\ldots$

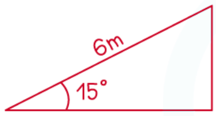
Use $W = Fs$ to find the work done.

Work done against friction $= Fs = (19.318)(6)$

$= 115.911\ldots$ Nm

Work done against friction = 116 J (3.s.f.)

(ii) Use right-angled trigonometry to find the gain in vertical height



$h = 6\sin 15° = 1.5529...\text{ m}$

Use work done against gravity = mgh

$$\text{work done against gravity} = (10)(10)(1.5529...)$$
$$= 155.291...\text{ Nm}$$

Work done against gravity = 155J (3 s.f.)

Your notes

Examiner Tips and Tricks

- Read the question carefully to decide if the point of application of the force is acting in the direction of motion. Always draw a diagram or add to the diagram given in the question. Check to see if there are any resisting forces.

Energy

There are many different forms of energy including, but not limited to, heat energy, light energy, chemical energy and nuclear energy. In this course we deal with a type of energy called **mechanical energy** which exists in two forms**, kinetic energy (KE)** and **gravitational potential energy (GPE)**.

Kinetic Energy

What is kinetic energy?

- Kinetic Energy is the energy formed by an object's movement.
- Kinetic energy is a **scalar quantity**, it can not be negative
- The **work done** by a resultant force that acts to move an object in a particular direction will be equal to the **change in kinetic energy** of the object

How is kinetic energy calculated?

- A particle can only have kinetic energy when it is moving
- If a particle with mass, *m* kg is moving with speed v m s^{-1} then its kinetic energy can be calculated using the formula

$$KE = \frac{1}{2}mv^2$$

- If the particle is moving in two dimensions with the velocity vector **v** then kinetic energy can be calculated in two ways
  - Using the formula on each component individually and finding the sum of the KE in each component
  - Finding the magnitude of the velocity to get the speed and then using the formula for KE
- Kinetic energy is measured in **joules (J)**
  - 1 Kilojoule = 1000 joules (1 kJ = 1000 J)

How do we derive the formula for the change in kinetic energy?

- The **work done** by the resultant force is equal to the **change in kinetic energy**
  - This is the **final kinetic energy** minus the **initial kinetic energy**
  - The formula for the **change in kinetic energy** is

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

where ***u*** is the initial velocity and ***v*** is the final velocity

- This is often written as $\frac{1}{2}m(v^2 - u^2)$
- Newton's Second Law ($F = ma$) and the '*suvat*' equation $v^2 = u^2 + 2as$ can be used to derive the formula for the change in kinetic energy

NEWTON'S SECOND LAW: $F = ma$ ①

'SUVAT' EQUATION: $v^2 = u^2 + 2as$ ②

REARRANGE EQUATION ② TO MAKE 'a' THE SUBJECT:

$$v^2 = u^2 + 2as$$

$$v^2 - u^2 = 2as$$

$$a = \frac{v^2 - u^2}{2s}$$

SUBSTITUTE 'a' INTO EQUATION ①:

$$F = m\left(\frac{v^2 - u^2}{2s}\right)$$

$$= \frac{m(v^2 - u^2)}{2s}$$

MULTIPLY BOTH SIDES BY 's'

$$Fs = \frac{m(v^2 - u^2)}{2}$$

$$\therefore \ WD = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$\therefore$ WORK DONE = CHANGE IN KINETIC ENERGY

Worked Example

A jogger increases her speed from 2 m s^{-1} to 3 m s^{-1} and her change in kinetic energy is 150 J, find the mass of the jogger.

Answer:

Your notes

Final kinetic energy

Initial kinetic energy

Change in kinetic energy $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2}m(v^2 - u^2)$

$u = 2 ms^{-1}$

$v = 3 ms^{-1}$

Substitute values into equation:

$\frac{1}{2}m(3^2 - 2^2) = 150$

$m(9 - 4) = 300$

$5m = 300$

$m = 60$

Jogger's mass is 60 kg

Examiner Tips and Tricks

- Always double check the units are in kg for mass and m s^{-1} for velocity before carrying out any calculations.
- Be careful not to make the mistake of using the difference between the velocities with the equation, remember it should be the difference between the squares of the speeds.
- Make sure you are familiar with the method for the derivation of the formula for the change in kinetic energy.

Gravitational Potential Energy

What is potential energy?

- Potential energy is the energy stored in a **stationary** object
- **Gravitational potential energy (GPE)** is the energy a particle possesses when it is at a **fixed height** and **gravity** is acting on it
- There are other types of potential energy but in this module only **GPE** is dealt with and so sometimes GPE is referred to as **potential energy (PE)**
- GPE will change as the vertical height of an object changes
  - The **work done against gravity** on a particle as it moves **upwards** is equal to its **increase** in GPE
  - The **work done by gravity** on a particle as it moves **downwards** is equal to its **decrease** in GPE

How is gravitational potential energy calculated?

- Gravitational potential energy is equal to the **product** of the force of the **weight** of an object and its vertical **height, *h*** , above a fixed point

Your notes

$$GPE = mgh$$

- If the object is sitting on the ground or the point chosen as the fixed base level, the object will have no gravitational potential energy
- As the object moves upwards, its GPE will increase
- As the object moves downwards again, its GPE will decrease
- When **mass** is measured in **kg**, **acceleration due to gravity** is measured in **m s^{-2}** and height is in metres, m, gravitational potential energy is measured in **joules (J)**
  - 1 kilojoule = 1000 joules (1 kJ = 1000 J)

Your notes

Worked Example

A ball of mass 400 grams is thrown vertically upwards from a height of 1 metre above the ground. It reaches a maximum height of 4 metres before falling to the ground again. Stating clearly whether is represents a gain or a loss, write down the change in the gravitational potential energy of the ball

(i) between the instant it is thrown and the instant it reaches its maximum height,

(ii) between the instant it is thrown and the instant it hits the ground.

Answer:

Convert any non-SI units into SI units

400 g = 0.4 kg

Take the ground to be the base level, then

At start: $m = 0.4,\ g = 9.8,\ h = 1$

$GPE = mgh = (0.4)(9.8)(1)$

$= 3.92$ J

At maximum height: $m = 0.4,\ g = 9.8,\ h = 4$

$GPE = mgh = (0.4)(9.8)(4)$

$= 15.68$ J

(i) Change in GPE = 15.68 − 3.92 = 11.76 J

The ball's height has increased, so its GPE has increased

Gain in GPE = 11.8 J (3sf)

(ii) When the ball hits the ground it is at base level so has no GPE

Change in GPE = 3.92 − 0 = 3.92 J

The ball's height has decreased, so its GPE has decreased

Loss in GPE = 3.92 J (3sf)

Examiner Tips and Tricks

- Always double check the units are in **kg** for mass and **m** for height before carrying out any calculations.
- Remember that it is the **vertical height** that must be used within the calculations for **GPE**, if you are given the slant height you must use trigonometry to find the vertical height first.

Your notes

Your notes

Work-Energy Principle

What is the Work-Energy Principle?

- The **Work-Energy Principle** has many forms, but is in essence an **energy balance**
  - the **final** amount is the **initial** amount plus any energy put in (or minus any taken out)
  - it's a bit like money in a bank account!
- The principle can be written as:
  - **total final energy = total initial energy ± work done** by **non-gravitational** forces
    - or, using subscripts for final and initial, $E_f = E_i \pm WD$
  - "total energy" here means the **sum of Gravitational Potential Energy and Kinetic Energy**
    - e.g. $E_i = GPE_i + KE_i = mgh_i + \frac{1}{2}mv_i^2$
  - **Non-gravitational** forces are any external forces that are not related to gravity
    - e.g. frictions, tensions, driving forces, etc
    - but **wouldn't** include weight, mg, because work done against gravity has **already** been considered in the **GPE** part
  - **Use +** for work done by forces that **"help"** the object to move forwards
    - e.g. **tension** in a string pulling **forwards**, a **driving** force, etc
  - **Use –** for work done by forces that **"hinder"** (resist) the object from moving forwards
    - e.g. **friction**, **tension** in a string pulling **backwards**, a **resistance** force, air resistance, etc
- Some situations may have **more than one** form of work done
  - add or subtract each one, depending whether they help or hinder

How else can the Work-Energy Principle be written?

- You can write the Work-Energy Principle in terms of **gains** or **losses** in KE and GPE
  - but this method can cause a lot of **sign errors**!
- Write out each term in the original version, "**total final energy = total initial energy ± work done** by **non-gravitational** forces"
  - use subscripts for **final** and **initial**

- $mgh_f + \frac{1}{2}mv_f^2 = mgh_i + \frac{1}{2}mv_i^2 \pm WD$

- **group** together KEs and GPEs as an overall **change** ("final - initial")
  - $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -\left(mgh_f - mgh_i\right) \pm WD$
- **change in KE = - change in GPE ± WD**
  - you can read this as **gain** in KE = **loss** in GPE ± WD
  - but some situations **lose** KE, so the gain is **negative** (you need to be really careful with the **signs** and what "loss" means!)
- The first method (**energy balance**) works for **every situation**, but this "**gain-loss**" method needs **adapting** for **each** situation

Can I use the Work-Energy Principle on an inclined plane?

- Yes, and remember to calculate **work done** as "the **component** of force in the **direction of motion**" × "distance moved in direction of motion"
  - in this case, the direction of motion is **parallel** to the **slope**
- You may also need to calculate the **final vertical height** for GPE
  - e.g. using trigonometry, with the distance in the direction of the slope as the hypotenuse

Examiner Tips and Tricks

- If a question asks you to find something "using the work-energy principle", don't use Newton's 2nd Law with SUVAT!

Worked Example

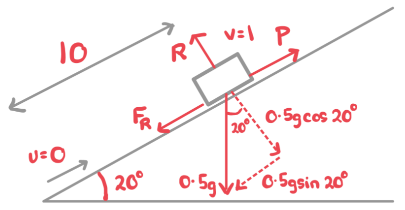
A dog uses a constant force of P N to push its toy of mass 0.5 kg up a rough slope inclined at 20° to the horizontal. The force, P, acts parallel to the slope. The toy starts from rest and, after moving 10 metres up the line of greatest slope, it is travelling at 1 ms^{-1}. The coefficient of friction between the toy and the slope is 0.1.

Use the work-energy principle to find P.

Answer:

Your notes

Draw a diagram and include important information



$\mu = 0{\cdot}1$

First find friction, $F_R = \mu R$
Resolving perpendicular to the plane

$R = 0{\cdot}5g\cos 20°$
$\therefore F_R = 0{\cdot}1(0{\cdot}5g\cos 20°) = 0{\cdot}05g\cos 20°$

Work-energy principle ("$E_f = E_i \pm WD$")
Work done, P 'helps' toy move, friction (F_R) 'hinders' it
("$WD = Fd$")

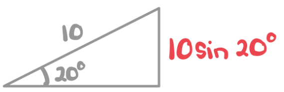
$\therefore WD = (P - F_R) \times 10 = 10P - 0{\cdot}5g\cos 20°$

Consider kinetic energy ("$KE = \frac{1}{2}mv^2$")

$KE_i = 0$ (initial)
$KE_f = \frac{1}{2} \times 0{\cdot}5 \times 1^2 = 0{\cdot}25$ (final)

Gravitational potential energy ("mgh")

$GPE_i = 0$
$GPE_f = 0{\cdot}5g(10\sin 20°)$
$= 5g\sin 20°$



Your notes

Your notes

Now apply $E_f = E_i + WD$

$0.25 + 5g\sin 20° = 10P - 0.5g\cos 20°$

Using $g = 9.8$

$P = 2.161\ 348\ldots$

$\boxed{\therefore P = 2.2\text{ N} \quad (2\text{ s.f.})}$

2 s.f. is appropriate since we used $g = 9.8$ which is 2 s.f.

Conservation of Energy

What is Conservation of Energy?

- **Conservation of Energy** is a **special case** of the **Work-Energy Principle** when there is **no work done** by **non-gravitational forces**
  - so **total final energy = total initial energy**
  - total energy here means **GPE + KE**
  - or, using subscripts for **final** and **initial**:
    - $mgh_f + \frac{1}{2}mv_f^2 = mgh_i + \frac{1}{2}mv_i^2$
- This could be because there are **no non-gravitational forces**
  - e.g. a particle falling **freely under gravity** from a fixed height above the ground
- Or this could be because all non-gravitational forces are always **perpendicular** to the direction of motion
  - e.g. the vertical reaction force on an object moving along a horizontal surface contributes no work done (the force has no horizontal component)
    - recall work done is the "**component of force** in the **direction of motion**" × "distance moved in direction of motion"

Examiner Tips and Tricks

- In practice, you can apply the Work-Energy Principle as before, but it'll be slightly easier as there's no work done by external forces

Your notes

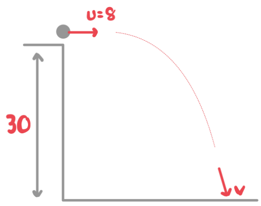
Worked Example

A particle of mass m kg is fired horizontally off a vertical cliff at 8 ms^{-1} and travels freely under gravity. The cliff is 30 metres high and the ground below is horizontal.

Use energy considerations to find the speed of impact of the particle with the horizontal ground below.

Answer:

Draw a diagram



The work-energy principle is $E_f = E_i \pm WD$

There are no non-gravitational forces, so energy is conserved

$$E_f = E_i$$

Kinetic energy ("$\frac{1}{2}mv^2$"): $KE_i = \frac{1}{2}m(8)^2 = 32m$

$KE_f = \frac{1}{2}mv^2$

Gravitational potential energy ("mgh") $GPE_i = mg(30) = 30mg$

$GPE_f = 0$

Applying $E_f = E_i$

$$\frac{1}{2}\cancel{m}v^2 = 32\cancel{m} + 30\cancel{m}g$$

Using $g = 9.8$

$$v^2 = 652$$

$$v = 25.534\ldots$$

$\boxed{\therefore \text{Speed of impact is } 26 \text{ m s}^{-1} \text{ (2 s.f.)}}$