Chapter 2: Variable Acceleration

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M2 Chapter 2 Solutions

2. A particle P moves in a straight line. At time $t = 0$, P passes through a point O on the line. At time t seconds, the velocity of P is v m s^{-1} where

$$v = (2t - 1)(1 - t)$$

(a) Find the acceleration of P when $t = \frac{1}{2}$

(3)

(b) Find the distance travelled by P in the interval $0 \leqslant t \leqslant 1$

(6)

5. A particle P of mass 0.3 kg moves under the action of a single force **F** newtons. At time t seconds ($t \geqslant 0$), P has velocity $\mathbf{v}$ m s^{-1}, where

$$\mathbf{v} = (3t^2 - 4t)\mathbf{i} + (3t^2 - 8t + 4)\mathbf{j}$$

(a) Find **F** when $t = 4$

(3)

At the instants when P is at the points A and B, particle P is moving parallel to the vector **i**.

(b) Find the distance AB.

(9)

3. At time t seconds ($t \geqslant 0$) a particle P has position vector $\mathbf{r}$ metres, with respect to a fixed origin O, where

$$\mathbf{r} = (16t - 3t^3)\mathbf{i} + (t^3 - t^2 + 2)\mathbf{j}$$

Find

(a) the velocity of P at the instant when it is moving parallel to the vector $\mathbf{j}$,

(5)

(b) the magnitude of the acceleration of P when $t = 4$

(4)

5. A particle moves along the x-axis. At time t seconds, $t \geqslant 0$, the velocity of the particle is $v\,\text{m s}^{-1}$ in the direction of x increasing, where $v = 2t^{\frac{3}{2}} - 6t + 2$

At time $t = 0$ the particle passes through the origin O. At the instant when the acceleration of the particle is zero, the particle is at the point A.

Find the distance OA.

(8)

5. At time t seconds ($t \geqslant 0$), a particle P has velocity $\mathbf{v}\,\text{m}\,\text{s}^{-1}$, where

$$\mathbf{v} = (3t^2 - 4)\mathbf{i} + (2t - 4)\mathbf{j}$$

When $t = 0$, P is at the fixed point O.

(a) Find the acceleration of P at the instant when $t = 0$

(2)

(b) Find the exact speed of P at the instant when P is moving in the direction of the vector $(11\mathbf{i} + \mathbf{j})$ for the second time.

(4)

(c) Show that P never returns to O.

(4)

5. At time t seconds ($t \geqslant 0$), a particle P has velocity $\mathbf{v}\,\text{m}\,\text{s}^{-1}$, where

$$\mathbf{v} = (3t^2 - 9t + 6)\mathbf{i} + (t^2 + t - 6)\,\mathbf{j}$$

(a) Find the acceleration of P when $t = 3$

(3)

When $t = 0$, P is at the fixed point O.

The particle comes to instantaneous rest at the point A.

(b) Find the distance OA.

(7)

5. At time t seconds, $t \geqslant 0$, a particle P has velocity $\mathbf{v}\,\text{m s}^{-1}$, where

$$\mathbf{v} = (5t^2 - 12t + 15)\mathbf{i} + (t^2 + 8t - 10)\mathbf{j}$$

When $t = 0$, P is at the origin O.

At time T seconds, P is moving in the direction of $(\mathbf{i} + \mathbf{j})$.

(a) Find the value of T.

(3)

When $t = 3$, P is at the point A.

(b) Find the magnitude of the acceleration of P as it passes through A.

(4)

(c) Find the position vector of A.

(4)

2. A particle P of mass 1.5 kg moves under the action of a single force **F** newtons.

At time t seconds, $t \geqslant 0$, P has velocity $\mathbf{v}\,\text{m s}^{-1}$, where

$$\mathbf{v} = (5t^2 - t^3)\mathbf{i} + (2t^3 - 8t)\mathbf{j}$$

(a) Find **F** when $t = 2$

(4)

At time $t = 0$, P is at the origin O.

(b) Find the position vector of P relative to O at the instant when P is moving in the direction of the vector **j**

(4)

3. A particle P moves on the x-axis.

At time $t = 0$, P is instantaneously at rest at O.

At time t seconds, $t > 0$, the x coordinate of P is given by

$$x = 2t^{\frac{7}{2}} - 14t^{\frac{5}{2}} + \frac{56}{3}t^{\frac{3}{2}}$$

Find

(a) the non-zero values of t for which P is at instantaneous rest

(3)

(b) the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(3)

(c) the acceleration of P when $t = 4$

(3)

3. A particle P of mass 0.25 kg is moving on a smooth horizontal surface under the action of a single force, **F** newtons.

At time t seconds $(t \geqslant 0)$, the velocity $\mathbf{v}\,\text{m s}^{-1}$ of P is given by

$$\mathbf{v} = (6\sin 3t)\mathbf{i} + (1 + 2\cos t)\mathbf{j}$$

(a) Find **F** in terms of t.

(3)

At time $t = 0$, the position vector of P relative to a fixed point O is $(4\mathbf{i} - \sqrt{3}\mathbf{j})\,\text{m}$.

(b) Find the position vector of P relative to O when P is first moving parallel to the vector **i**.

(6)

1. At time t seconds, $t \geqslant 0$, a particle P has position vector $\mathbf{r}$ metres with respect to a fixed origin O, where

$$\mathbf{r} = (t^3 - 8t)\mathbf{i} + \left(\frac{1}{3}t^3 - t^2 + 2t\right)\mathbf{j}$$

(a) Find the acceleration of P when $t = 4$

(5)

At time T seconds, $T \geqslant 0$, P is moving in the direction of $(2\mathbf{i} + \mathbf{j})$

(b) Find the value of T

(3)

4. At time t seconds ($0 \leqslant t < 5$), a particle P has velocity $\mathbf{v}\,\text{m}\,\text{s}^{-1}$, where

$$\mathbf{v} = \left(\sqrt{5-t}\right)\mathbf{i} + \left(t^2 + 2t - 3\right)\mathbf{j}$$

When $t = \lambda$, particle P is moving in a direction parallel to the vector $\mathbf{i}$.

(a) Find the acceleration of P when $t = \lambda$

(5)

The position vector of P is measured relative to the fixed point O
When $t = 1$, the position vector of P is $(-2\mathbf{i} + \mathbf{j})$m.

Given that $1 \leqslant T < 5$

(b) find, in terms of T, the position vector of P when $t = T$

(5)

4. [*In this question, the perpendicular unit vectors* **i** *and* **j** *are in a horizontal plane*.]

A particle Q of mass 1.5 kg is moving on a smooth horizontal plane under the action of a single force **F** newtons. At time t seconds $(t \geqslant 0)$, the position vector of Q, relative to a fixed point O, is **r** metres and the velocity of Q is **v** m s^{-1}
It is given that

$$\mathbf{v} = (3t^2 + 2t)\mathbf{i} + (t^3 + kt)\mathbf{j}$$

where k is a constant.

Given that when $t = 2$ particle Q is moving in the direction of the vector $\mathbf{i} + \mathbf{j}$

(a) show that $k = 4$

(2)

(b) find the magnitude of **F** when $t = 2$

(4)

Given that $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j}$ when $t = 0$

(c) find **r** when $t = 2$

(4)

[*In this question, the perpendicular unit vectors* **i** *and* **j** *are in a horizontal plane.*]

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

A particle P is moving on a smooth horizontal plane.

At time t seconds $(t \geqslant 0)$, the position vector of P, relative to a fixed point O, is **r** metres and the velocity of P is $\mathbf{v}\ \text{m s}^{-1}$ where

$$\mathbf{v} = \left(4t^2 - 5t\right)\mathbf{i} + \left(-10t - 12\right)\mathbf{j}$$

When $t = 0$, $\mathbf{r} = 2\mathbf{i} + 6\mathbf{j}$

(a) Find **r** when $t = 2$

(4)

When $t = T$ particle P is moving in the direction of the vector $\mathbf{i} - 2\mathbf{j}$

(b) Find the value of T

(3)

(c) Find the exact magnitude of the acceleration of P when $t = 2.5$

(3)

1. At time t seconds, $t > 0$, a particle P is at the point with position vector $\mathbf{r}$ m, where

$$\mathbf{r} = \left(t^4 - 8t^2\right)\mathbf{i} + \left(6t^2 - 2t^{\frac{3}{2}}\right)\mathbf{j}$$

(a) Find the velocity of P when P is moving in a direction parallel to the vector $\mathbf{j}$

(4)

(b) Find the acceleration of P when $t = 4$

(3)

1. A particle P moves along a straight line. The fixed point O is on the line. At time t seconds, $t > 0$, the displacement of P from O is x metres, where

$$x = 2t^3 - 21t^2 + 60t$$

Find

(a) the values of t for which P is instantaneously at rest

(4)

(b) the distance travelled by P in the interval $1 \leqslant t \leqslant 3$

(2)

(c) the magnitude of the acceleration of P at the instant when $t = 3$

(2)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A particle P is moving in a straight line.

At time t seconds, the speed, $v\,\text{m s}^{-1}$, of P is given by the continuous function

$$v = \begin{cases} \sqrt{2t+1} & 0 \leqslant t \leqslant k \\ \dfrac{3}{4}t & t > k \end{cases}$$

where k is a constant.

(a) Show that $k = 4$, explaining your method carefully.

(3)

(b) Find the acceleration of P when $t = 1.5$

(3)

At time $t = 0$, P passes through the point O

(c) Find the distance of P from O when $t = 8$

(7)