# Chapter 2: Variable Acceleration

Mr Faruk
Teacher of Mathematics
BSc/MSc/PGCE Mathematics
cieigcsesolutions@gmail.com

**2.** A particle $P$ moves in a straight line. At time $t = 0$, $P$ passes through a point $O$ on the line. At time $t$ seconds, the velocity of $P$ is $v$ m s$^{-1}$ where

$$v = (2t - 1)(1 - t)$$

(a) Find the acceleration of $P$ when $t = \frac{1}{2}$ **(3)**

(b) Find the distance travelled by $P$ in the interval $0 \leqslant t \leqslant 1$ **(6)**

a) $V = 2t - 2t^2 - 1 + t$

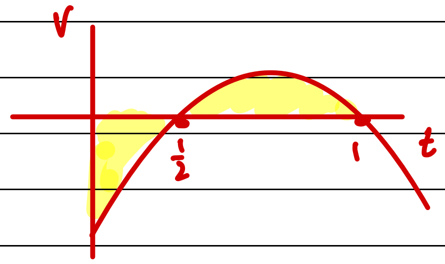
$= -2t^2 + 3t - 1$

$a = \frac{dv}{dt} = -4t + 3$

For $t = \frac{1}{2}$

$a = -4(\frac{1}{2}) + 3 = 1 \text{ ms}^{-2}$

b)




$-\int_0^{\frac{1}{2}} -2t^2 + 3t - 1 \, dt + \int_{\frac{1}{2}}^{1} -2t^2 + 3t - 1 \, dt$

$= -\left[-\frac{2}{3}t^3 + \frac{3}{2}t^2 - t\right]_0^{\frac{1}{2}} + \left[-\frac{2}{3}t^3 + \frac{3}{2}t^2 - t\right]_{\frac{1}{2}}^{1}$

$= \left(\frac{5}{24}\right) + \left(\left(-\frac{1}{6}\right) - \left(-\frac{5}{24}\right)\right)$

$= \frac{1}{4}$ m

**5.** A particle $P$ of mass 0.3 kg moves under the action of a single force **F** newtons. At time $t$ seconds ($t \geqslant 0$), $P$ has velocity **v** m s$^{-1}$, where

$$\mathbf{v} = (3t^2 - 4t)\mathbf{i} + (3t^2 - 8t + 4)\mathbf{j}$$

(a) Find **F** when $t = 4$

**(3)**

At the instants when $P$ is at the points $A$ and $B$, particle $P$ is moving parallel to the vector **i**.

(b) Find the distance $AB$.

**(9)**

a)

$a = \frac{dv}{dt} = (6t-4)i + (6t-8)j$

For $t=4$ , $a = 20i + 16j$

$F = ma$

$= 0.3\ (20i + 16j) = 6i + 4.8j$

b) $3t^2 - 8t + 4 = 0$

$\Rightarrow t = 2,\ t = \frac{2}{3}$

$s = \int v\, dt = (t^3 - 2t^2 + c_1)i + (t^3 - 4t^2 + 4t + c_2)j$

For $t = \frac{2}{3}$ , $s = (-\frac{16}{27} + c_1)i + (\frac{32}{27} + c_2)j$

$A(-\frac{16}{27} + c_1 , \frac{32}{27} + c_2)$

For $t = 2$ , $s = c_1 i + c_2 j$

$B(c_1 , c_2)$

$\vec{AB} = \frac{16}{27}i - \frac{32}{27}j$

$|\vec{AB}| = \frac{16}{27}\sqrt{5} = 1.33$ m

2.2

**3.** At time $t$ seconds ($t \geqslant 0$) a particle $P$ has position vector $\mathbf{r}$ metres, with respect to a fixed origin $O$, where

$$\mathbf{r} = (16t - 3t^3)\mathbf{i} + (t^3 - t^2 + 2)\mathbf{j}$$

Find

(a) the velocity of $P$ at the instant when it is moving parallel to the vector $\mathbf{j}$,

**(5)**

(b) the magnitude of the acceleration of $P$ when $t = 4$

**(4)**

a)

$v = \frac{dr}{dt} = (16 - 9t^2)i + (3t^2 - 2t)j$

$16 - 9t^2 = 0 \Rightarrow t^2 = \frac{16}{9}$

$\Rightarrow t = \frac{4}{3}$

For $t = \frac{4}{3}$, $v = \frac{8}{3}j$

b) $a = \frac{dv}{dt} = -18t\,i + (6t - 2)j$

For $t = 4$

$a = -72i + 22j$

$|a| = 2\sqrt{1417} = 75.3 \text{ ms}^{-2}$

**5.** A particle moves along the $x$-axis. At time $t$ seconds, $t \geqslant 0$, the velocity of the particle is $v\,\mathrm{m\,s^{-1}}$ in the direction of $x$ increasing, where $v = 2t^{\frac{3}{2}} - 6t + 2$

At time $t = 0$ the particle passes through the origin $O$. At the instant when the acceleration of the particle is zero, the particle is at the point $A$.

Find the distance $OA$.

**(8)**

a) $a = \frac{dv}{dt} = 3t^{\frac{1}{2}} - 6$

$3t^{\frac{1}{2}} - 6 = 0 \Rightarrow t^{\frac{1}{2}} = 2$

$\Rightarrow t = 4$ (at $t=4$, acceleration is zero)

$s = \int v\,dt = \frac{2t^{\frac{5}{2}}}{\frac{5}{2}} - \frac{6t^2}{2} + 2t + c$

$= \frac{4}{5}t^{\frac{5}{2}} - 3t^2 + 2t + c$

$c = 0$

$\Rightarrow s = \frac{4}{5}t^{\frac{5}{2}} - 3t^2 + 2t$

For $t = 4$

$s = -14.4$ (displacement is 14.4m at $t=4$)

So distance of OB is 14.4m

**5.** At time $t$ seconds ($t \geqslant 0$), a particle $P$ has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = (3t^2 - 4)\mathbf{i} + (2t - 4)\mathbf{j}$$

When $t = 0$, $P$ is at the fixed point $O$.

(a) Find the acceleration of $P$ at the instant when $t = 0$

**(2)**

(b) Find the exact speed of $P$ at the instant when $P$ is moving in the direction of the vector $(11\mathbf{i} + \mathbf{j})$ for the second time.

**(4)**

(c) Show that $P$ never returns to $O$.

**(4)**

a) $a = \frac{dv}{dt} = 6ti + 2j$

For $t=0$

$a = 2j$

b) $\frac{2t-4}{3t^2-4} = \frac{1}{11}$

$\Rightarrow 22t - 44 = 3t^2 - 4$

$\Rightarrow 3t^2 - 22t + 40 = 0$

$t = \frac{10}{3}$ , $t = 4$

For $t = 4$

$v = 44i + 4j$

$|v| = 4\sqrt{122} = 44.2 \ ms^{-1}$

c)

$s = \int (3t^2-4)i + (2t-4)j \, dt = (t^3 - 4t + c_1)i + (t^2 - 4t + c_2)j$

$c_1 = 0, \ c_2 = 0$

$\Rightarrow s = (t^3 - 4t)i + (t^2 - 4t)j$

2.5

$s = (t^3 - 4t)i + (t^2 - 4t)j$

$t^3 - 4t = t(t^2 - 4) = 0$

$\Rightarrow t = 0, 2$

$t^2 - 4t = t(t-4)$

$t = 0, 4$

Since two different $t$ value after $t=0$, it does not return to O.

**5.** At time $t$ seconds ($t \geqslant 0$), a particle $P$ has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = (3t^2 - 9t + 6)\mathbf{i} + (t^2 + t - 6)\mathbf{j}$$

(a) Find the acceleration of $P$ when $t = 3$

**(3)**

When $t = 0$, $P$ is at the fixed point $O$.

The particle comes to instantaneous rest at the point $A$.

(b) Find the distance $OA$.

**(7)**

a)

$a = \frac{dv}{dt} = (6t-9)i + (2t+1)j$

For $t=3$

$a = 9i + 7j \ \ ms^{-2}$

b)

$v=0 \Rightarrow 3t^2 - 9t + 6 = 0$

$t=1, \ t=2$

$\Rightarrow t^2 + t - 6 = 0$

$t=-3, \ t=2$

$s = \int v\,dt = (t^3 - \frac{9}{2}t^2 + 6t + c_1)i + (\frac{1}{3}t^3 + \frac{1}{2}t^2 - 6t + c_2)j$

$c_1=0, \ c_2=0$

$\Rightarrow s = (t^3 - \frac{9}{2}t^2 + 6t)i + (\frac{1}{3}t^3 + \frac{1}{2}t^2 - 6t)j$

For $t=2$,

$s = 2i - \frac{22}{3}j$

$\vec{OA} = 2i - \frac{22}{3}j$

$|OA| = \frac{2}{3}\sqrt{130} = 7.60$

**5.** At time $t$ seconds, $t \geqslant 0$, a particle $P$ has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = (5t^2 - 12t + 15)\mathbf{i} + (t^2 + 8t - 10)\mathbf{j}$$

When $t = 0$, $P$ is at the origin $O$.

At time $T$ seconds, $P$ is moving in the direction of $(\mathbf{i} + \mathbf{j})$.

(a) Find the value of $T$.

**(3)**

When $t = 3$, $P$ is at the point $A$.

(b) Find the magnitude of the acceleration of $P$ as it passes through $A$.

**(4)**

(c) Find the position vector of $A$.

**(4)**

a)

$$\frac{t^2 + 8t - 10}{5t^2 - 12t + 15} = \frac{1}{1}$$

$\Rightarrow t^2 + 8t - 10 = 5t^2 - 12t + 15$

$\Rightarrow 4t^2 - 20t + 25 = 0$

$t = \frac{5}{2}$

$\Rightarrow T = \frac{5}{2}$ s

b)

$a = \frac{dv}{dt} = (10t - 12)i + (2t + 8)j$

For $t = 3$

$a = 18i + 14j$

$|a| = 2\sqrt{130}$

$= 22.8\ \mathrm{ms^{-2}}$

2.9

c)

$$s = \int v\,dt = \int (5t^2 - 12t + 15)i + (t^2 + 8t - 10)j \; dt$$

$$= \left(\tfrac{5}{3}t^3 - 6t^2 + 15t + c_1\right)i + \left(\tfrac{1}{3}t^3 + 4t^2 - 10t + c_2\right)j$$

$c_1 = 0 \;,\; c_2 = 0$

$$\Rightarrow s = \left(\tfrac{5}{3}t^3 - 6t^2 + 15t\right)i + \left(\tfrac{1}{3}t^3 + 4t^2 - 10t\right)j$$

For $t = 3$

$s = 36i + 15j \;\; m$

**2.** A particle $P$ of mass 1.5 kg moves under the action of a single force $\mathbf{F}$ newtons.

At time $t$ seconds, $t \geqslant 0$, $P$ has velocity $\mathbf{v}\,\text{m s}^{-1}$, where

$$\mathbf{v} = (5t^2 - t^3)\mathbf{i} + (2t^3 - 8t)\mathbf{j}$$

(a) Find $\mathbf{F}$ when $t = 2$

**(4)**

At time $t = 0$, $P$ is at the origin $O$.

(b) Find the position vector of $P$ relative to $O$ at the instant when $P$ is moving in the direction of the vector $\mathbf{j}$

**(4)**

a)

$a = \frac{dv}{dt} = (10t - 3t^2)i + (6t^2 - 8)j$

For $t = 2$

$a = 8i + 16j$

$F = ma$

$= 1.5\,(8i + 16j) = 12i + 24j$

b)

parallel to vector $j \Rightarrow 5t^2 - t^3 = 0$

$\Rightarrow t^2(5 - t) = 0$

$\Rightarrow t = 0,\ t = 5$

$s = \int v\,dt = \left(\frac{5}{3}t^3 - \frac{1}{4}t^4 + c_1\right)i + \left(\frac{1}{2}t^4 - 4t^2 + c_2\right)j$

$c_1 = 0,\ c_2 = 0$

$\Rightarrow s = \left(\frac{5}{3}t^3 - \frac{1}{4}t^4\right)i + \left(\frac{1}{2}t^4 - 4t^2\right)j$

For $t = 5$

$s = \frac{625}{12}i + \frac{425}{2}j \Rightarrow r = \frac{625}{12}i + \frac{425}{2}j$

**3.** A particle $P$ moves on the $x$-axis.

At time $t = 0$, $P$ is instantaneously at rest at $O$.

At time $t$ seconds, $t > 0$, the $x$ coordinate of $P$ is given by

$$x = 2t^{\frac{7}{2}} - 14t^{\frac{5}{2}} + \frac{56}{3}t^{\frac{3}{2}}$$

Find

(a) the non-zero values of $t$ for which $P$ is at instantaneous rest **(3)**

(b) the total distance travelled by $P$ in the interval $0 \leqslant t \leqslant 4$ **(3)**

(c) the acceleration of $P$ when $t = 4$ **(3)**

a) $v = \frac{dx}{dt} = 7t^{\frac{5}{2}} - 35t^{\frac{3}{2}} + 28t^{\frac{1}{2}}$

$= t^{\frac{1}{2}}(7t^2 - 35t + 28)$

$v = 0 \Rightarrow t^{\frac{1}{2}} = 0 \Rightarrow = 0$

$\Rightarrow 7t^2 - 35t + 28 = 0$

$\Rightarrow t = 1, \quad t = 4$

b) For $t = 0$, $x = 0$

For $t = 1$, $x = \frac{20}{3}$

$\Rightarrow$ distance travelled $\frac{20}{3}$m from $t=0$ to $t=1$

For $t = 1$, $x = \frac{20}{3}$

For $t = 4$, $x = -\frac{128}{3}$

$\frac{20}{3} - -\frac{128}{3} = \frac{148}{3}$ $\Rightarrow$ distance travelled $\frac{148}{3}$ m from $t=1$ and $t=4$

$\Rightarrow$ Total distance travelled is 56 m

c) $a = \frac{dv}{dt} = \frac{35}{2}t^{\frac{3}{2}} - \frac{105}{2}t^{\frac{1}{2}} + 14t^{-\frac{1}{2}}$

For $t = 4$

$a = 42 \text{ ms}^{-2}$

**3.** A particle $P$ of mass 0.25 kg is moving on a smooth horizontal surface under the action of a single force, $\mathbf{F}$ newtons.

At time $t$ seconds $(t \geqslant 0)$, the velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ of $P$ is given by

$$\mathbf{v} = (6\sin 3t)\mathbf{i} + (1 + 2\cos t)\mathbf{j}$$

(a) Find $\mathbf{F}$ in terms of $t$.

**(3)**

At time $t = 0$, the position vector of $P$ relative to a fixed point $O$ is $(4\mathbf{i} - \sqrt{3}\mathbf{j})\,\mathrm{m}$.

(b) Find the position vector of $P$ relative to $O$ when $P$ is first moving parallel to the vector $\mathbf{i}$.

**(6)**

a)

$a = \frac{dv}{dt} = (18\cos 3t)i - (2\sin t)j$

$F = ma$

$= 0.25(18\cos 3t)i - 0.25(2\sin t)j$

$= (\frac{9}{2}\cos 3t)i - (\frac{1}{2}\sin t)j$

b)

$s = \int v\,dt = \int (6\sin 3t)i + (1+2\cos t)j\,dt$

$= (-\frac{6}{3}\cos 3t + c_1)i + (t + 2\sin t + c_2)\,dt$

For $t=0$

$(-2 + c_1)i + c_2 j = 4i - \sqrt{3}j$

$\Rightarrow c_1 = 6, \ c_2 = -\sqrt{3}$

$\Rightarrow s = (-2\cos 3t + 6)i + (t + 2\sin t - \sqrt{3})j$

parallel to vector i $\Rightarrow$ $1 + 2\cos t = 0 \Rightarrow \cos t = -\frac{1}{2} \Rightarrow t = \frac{2}{3}\pi$

For $t = \frac{2}{3}\pi$

$s = 4i + \frac{2}{3}\pi j$ m

**1.** At time $t$ seconds, $t \geqslant 0$, a particle $P$ has position vector $\mathbf{r}$ metres with respect to a fixed origin $O$, where

$$\mathbf{r} = (t^3 - 8t)\mathbf{i} + \left(\frac{1}{3}t^3 - t^2 + 2t\right)\mathbf{j}$$

(a) Find the acceleration of $P$ when $t = 4$

**(5)**

At time $T$ seconds, $T \geqslant 0$, $P$ is moving in the direction of $(2\mathbf{i} + \mathbf{j})$

(b) Find the value of $T$

**(3)**

a) $v = \frac{dr}{dt} = (3t^2 - 8)i + (t^2 - 2t + 2)j$

$a = \frac{dv}{dt} = (6t)i + (2t - 2)j$

For $t = 4$

$a = 24i + 6j \ ms^{-2}$

b)

$\frac{t^2 - 2t + 2}{3t^2 - 8} = \frac{1}{2}$

$\Rightarrow 2t^2 - 4t + 4 = 3t^2 - 8$

$\Rightarrow t^2 + 4t - 12 = 0$

$t = -6, \ t = 2$

$\Rightarrow T = 2$

**4.** At time $t$ seconds ($0 \leqslant t < 5$), a particle $P$ has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = \left(\sqrt{5-t}\right)\mathbf{i} + \left(t^2 + 2t - 3\right)\mathbf{j}$$

When $t = \lambda$, particle $P$ is moving in a direction parallel to the vector $\mathbf{i}$.

(a) Find the acceleration of $P$ when $t = \lambda$

**(5)**

The position vector of $P$ is measured relative to the fixed point $O$
When $t = 1$, the position vector of $P$ is $(-2\mathbf{i} + \mathbf{j})$m.

Given that $1 \leqslant T < 5$

(b) find, in terms of $T$, the position vector of $P$ when $t = T$

**(5)**

a) parallel to vector i

$\Rightarrow \quad t^2+2t-3=0$

$t=-3, \; t=1$

$\Rightarrow \lambda = 1$

$v = \left((5-t)^{\frac{1}{2}}\right)i + (t^2+2t-3)j$

$a = \frac{dv}{dt} = \left(\frac{1}{2}\times -1\,(5-t)^{-\frac{1}{2}}\right)i + (2t+2)j$

$= \left(-\frac{1}{2}(5-t)^{-\frac{1}{2}}\right)i + (2t+2)j$

For $t=1$

$a = -\frac{1}{4}i + 4j$

b)

$s = \int v\,dt = \left((5-t)^{\frac{1}{2}}\right)i + (t^2+2t-3)j$

$= \left(\frac{1}{\frac{3}{2}}\times -1(5-t)^{\frac{3}{2}} + c_1\right)i + \left(\frac{1}{3}t^3 + t^2 - 3t + c_2\right)j$

$= \left(-\frac{2}{3}(5-t)^{3/2} + c_1\right)i + \left(\frac{1}{3}t^3 + t^2 - 3t + c_2\right)j$

When $t=1$, $s=-2i+j$

$$\Rightarrow \left(-\frac{16}{3}+c_1\right)i+\left(-\frac{5}{3}j+c_2\right)j = -2i+j$$

$$c_1 = \frac{10}{3}$$

$$c_2 = \frac{8}{3}$$

$$s = \left(-\frac{2}{3}(5-t)^{\frac{3}{2}}+\frac{10}{3}\right)i + \left(\frac{1}{3}t^3+t^2-3t+\frac{8}{3}\right)j$$

$$\Rightarrow s = \left(-\frac{2}{3}(5-T)^{\frac{3}{2}}+\frac{10}{3}\right)i + \left(\frac{1}{3}T^3+T^2-3T+\frac{8}{3}\right)j$$

**4.** [*In this question, the perpendicular unit vectors* **i** *and* **j** *are in a horizontal plane.*]

A particle $Q$ of mass 1.5 kg is moving on a smooth horizontal plane under the action of a single force **F** newtons. At time $t$ seconds $(t \geqslant 0)$, the position vector of $Q$, relative to a fixed point $O$, is **r** metres and the velocity of $Q$ is **v** m s$^{-1}$
It is given that

$$\mathbf{v} = (3t^2 + 2t)\mathbf{i} + (t^3 + kt)\mathbf{j}$$

where $k$ is a constant.

Given that when $t = 2$ particle $Q$ is moving in the direction of the vector $\mathbf{i} + \mathbf{j}$

(a) show that $k = 4$

**(2)**

(b) find the magnitude of **F** when $t = 2$

**(4)**

Given that $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j}$ when $t = 0$

(c) find **r** when $t = 2$

**(4)**

a) For $t=2$, $V = 16i + (8+2k)j$

$\frac{8+2k}{16} = \frac{1}{1} \Rightarrow 8+2k = 16$
$\Rightarrow 2k = 8$
$\Rightarrow k = 4$

b) $a = \frac{dv}{dt} = (6t+2)i + (3t^2+4)j$
For $t=2$
$a = 14i + 16j$
$|a| = 2\sqrt{113}$

$F = ma$
$F = 1.5 \times 2\sqrt{113} = 3\sqrt{113} = 31.9$ N

2.21

c)

$r = \int v\,dt = \int (3t^2+2t)i + (t^3+4t)j\,dt$

$= (t^3+t^2+c_1)i + (\frac{1}{4}t^4+2t^2+c_2)j$

For $t=0$, $r = 3i + 4j$

$\Rightarrow c_1 = 3$ and $c_2 = 4$

$\Rightarrow r = (t^3+t^2+3)i + (\frac{1}{4}t^4+2t^2+4)j$

For $t=2$

$r = 15i + 16j$

**2.** [*In this question, the perpendicular unit vectors* **i** *and* **j** *are in a horizontal plane.*]

**In this question you must show all stages of your working.**
**Solutions relying on calculator technology are not acceptable.**

A particle $P$ is moving on a smooth horizontal plane.

At time $t$ seconds $(t \geqslant 0)$, the position vector of $P$, relative to a fixed point $O$, is $\mathbf{r}$ metres and the velocity of $P$ is $\mathbf{v}\,\mathrm{m\,s^{-1}}$ where

$$\mathbf{v} = (4t^2 - 5t)\mathbf{i} + (-10t - 12)\mathbf{j}$$

When $t = 0$, $\mathbf{r} = 2\mathbf{i} + 6\mathbf{j}$

(a) Find $\mathbf{r}$ when $t = 2$

**(4)**

When $t = T$ particle $P$ is moving in the direction of the vector $\mathbf{i} - 2\mathbf{j}$

(b) Find the value of $T$

**(3)**

(c) Find the exact magnitude of the acceleration of $P$ when $t = 2.5$

**(3)**

a) $r = \int v\,dt = \int (4t^2 - 5t)i + (-10t - 12)j\ dt$

$= (\frac{4}{3}t^3 - \frac{5}{2}t^2 + c_1)i + (-5t^2 - 12t + c_2)j$

For $t = 0$

$c_1 i + c_2 j = 2i + 6j$

$c_1 = 2$, $c_2 = 6$

$r = (\frac{4}{3}t^3 - \frac{5}{2}t^2 + 2)i + (-5t^2 - 12t + 6)j$

$t = 2$

$r = \frac{8}{3}i - 38j$

b)

$$\frac{-10t-12}{4t^2-5t} = \frac{-2}{1}$$

$\Rightarrow -10t-12 = -8t^2+10t$

$\Rightarrow 8t^2-20t-12=0$

$t=-\frac{1}{2}$ , $t=3$

$\Rightarrow T=3$

c) $a = \frac{dv}{dt} = (8t-5)i + (-10)j$

For $t=2.5$

$a = 15i - 10j$

$|a| = 5\sqrt{13}\ ms^{-2}$

**1.** At time $t$ seconds, $t > 0$, a particle $P$ is at the point with position vector $\mathbf{r}$ m, where

$$\mathbf{r} = \left(t^4 - 8t^2\right)\mathbf{i} + \left(6t^2 - 2t^{\frac{3}{2}}\right)\mathbf{j}$$

(a) Find the velocity of $P$ when $P$ is moving in a direction parallel to the vector $\mathbf{j}$

**(4)**

(b) Find the acceleration of $P$ when $t = 4$

**(3)**

a)

$v = \frac{dr}{dv} = (4t^3 - 16t)i + (12t - 3t^{\frac{1}{2}})j$

parallel to vector $j$

$\Rightarrow \quad 4t^3 - 16t = 0$

$4t(t^2 - 4) = 0$

$t = 0, \; t = -2, \; t = 2$

For $t = 2$

$v = 24 - 3\sqrt{2} \; ms^{-1}$

b) $a = \frac{dv}{dt} = (12t^2 - 16) + (12 - \frac{3}{2}t^{-\frac{1}{2}})j$

For $t = 4$

$a = 176i + \frac{45}{4}j \; ms^{-2}$

**1.** A particle $P$ moves along a straight line. The fixed point $O$ is on the line. At time $t$ seconds, $t > 0$, the displacement of $P$ from $O$ is $x$ metres, where

$$x = 2t^3 - 21t^2 + 60t$$

Find

(a) the values of $t$ for which $P$ is instantaneously at rest **(4)**

(b) the distance travelled by $P$ in the interval $1 \leqslant t \leqslant 3$ **(2)**

(c) the magnitude of the acceleration of $P$ at the instant when $t = 3$ **(2)**

a) $v = \frac{dx}{dt} = 6t^2 - 42t + 60$

$6t^2 - 42t + 60 = 0$

$t = 2$ , $t = 5$

b)

For $t = 1$ , $x = 41$

For $t = 2$ , $x = 52$

$52 - 41 = 11$

For $t = 2$ , $x = 52$

For $t = 3$ , $x = 45$

$52 - 45 = 7$

$11 + 7 = 18$

distance travelled is 18 m

c) $a = \frac{dv}{dt} = 12t - 42$

For $t = 3$

$a = 6 \text{ ms}^{-2}$

**2.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

A particle $P$ is moving in a straight line.

At time $t$ seconds, the speed, $v\,\mathrm{m\,s^{-1}}$, of $P$ is given by the continuous function

$$v = \begin{cases} \sqrt{2t+1} & 0 \leqslant t \leqslant k \\ \frac{3}{4}t & t > k \end{cases}$$

where $k$ is a constant.

(a) Show that $k = 4$, explaining your method carefully.

**(3)**

(b) Find the acceleration of $P$ when $t = 1.5$

**(3)**

At time $t = 0$, $P$ passes through the point $O$

(c) Find the distance of $P$ from $O$ when $t = 8$

**(7)**

a) $\sqrt{2t+1} = \frac{3}{4}t$

$2t+1 = \frac{9}{16}t^2$

$\Rightarrow \frac{9}{16}t^2 - 2t - 1 = 0$

$t = -\frac{4}{9},\ t = 4$

But $t \geqslant 0 \Rightarrow k = 4$

b) $v = (2t+1)^{\frac{1}{2}}$

$a = \frac{dv}{dt} = \frac{1}{2} \times 2(2t+1)^{-\frac{1}{2}}$

For $t = 1.5$

$a = 0.5\ \mathrm{ms^{-2}}$

a)

$$\int_0^4 (2t+1)^{\frac{1}{2}}\,dt = \left[\frac{1}{\frac{3}{2}} \times \frac{1}{2}(2t+1)^{\frac{3}{2}}\right]_0^4$$

$$= \left[\frac{1}{3}(2t+1)^{\frac{3}{2}}\right]_0^4$$

$$= (9) - \left(\frac{1}{3}\right) = \frac{26}{3}$$

$$\int_4^8 \frac{3}{4}t\,dt = \left[\frac{\frac{3}{4}t^2}{2}\right]_4^8 = \left[\frac{3}{8}t^2\right]_4^8$$

$$= (24) - (6) = 18$$

Distance from O to P is $\frac{26}{3} + 18 = \frac{80}{3}$ m