Chapter 6: Statics of Rigid Bodies

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Chapter 6 Solutions

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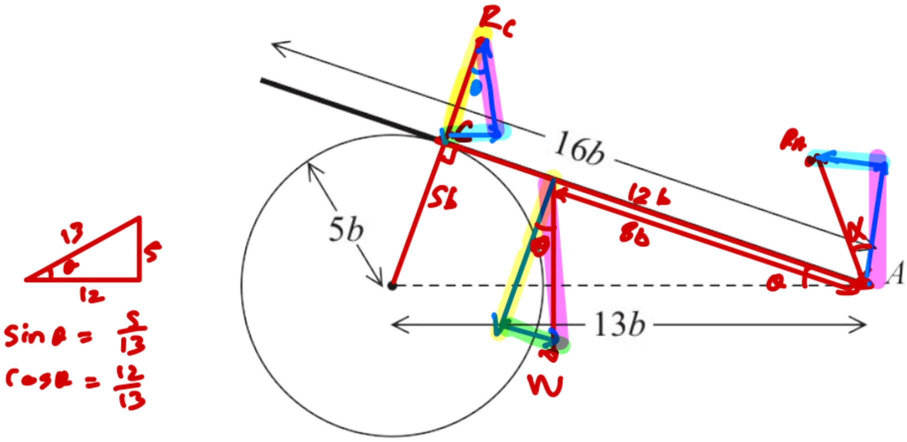

Figure 2

A uniform rod, of weight W and length $16b$, has one end freely hinged to a fixed point A. The rod rests against a smooth circular cylinder, of radius $5b$, fixed with its axis horizontal and at the same horizontal level as A. The distance of A from the axis of the cylinder is $13b$, as shown in Figure 2. The rod rests in a vertical plane which is perpendicular to the axis of the cylinder.

(a) Find, in terms of W, the magnitude of the reaction on the rod at its point of contact with the cylinder.

(4)

(b) Show that the resultant force acting on the rod at A is inclined to the vertical at an angle α where $\tan\alpha = \dfrac{40}{73}$

(6)

a) M(A)

$8b \times W\cos\theta = 12b \times R_C$

$8W\left(\frac{12}{13}\right) = 12R_C$

$\frac{8W}{13} = R_C$

Reaction at point of contact is $\frac{8W}{13}$

b) $R\rightarrow$:

$R_C \sin\theta = R_A \sin\alpha$

$\frac{8W}{13}\left(\frac{5}{13}\right) = R_A \sin\alpha$

$\Rightarrow R_A \sin\alpha = \frac{40W}{169}$ ①

$R\uparrow$:

$R_C \cos\theta + R_A\cos\alpha = W$

$$\left(\frac{8W}{13}\right)\left(\frac{12}{13}\right) + R_A \cos\alpha = W$$

$$R_A \cos\alpha = \frac{73W}{169} \quad (2)$$

$$(1) \div (2) \Rightarrow \frac{R_A \sin\alpha}{R_B \cos\alpha} = \tan\alpha = \frac{40W}{169} \div \frac{73W}{169} = \frac{40}{73}$$

June 2018

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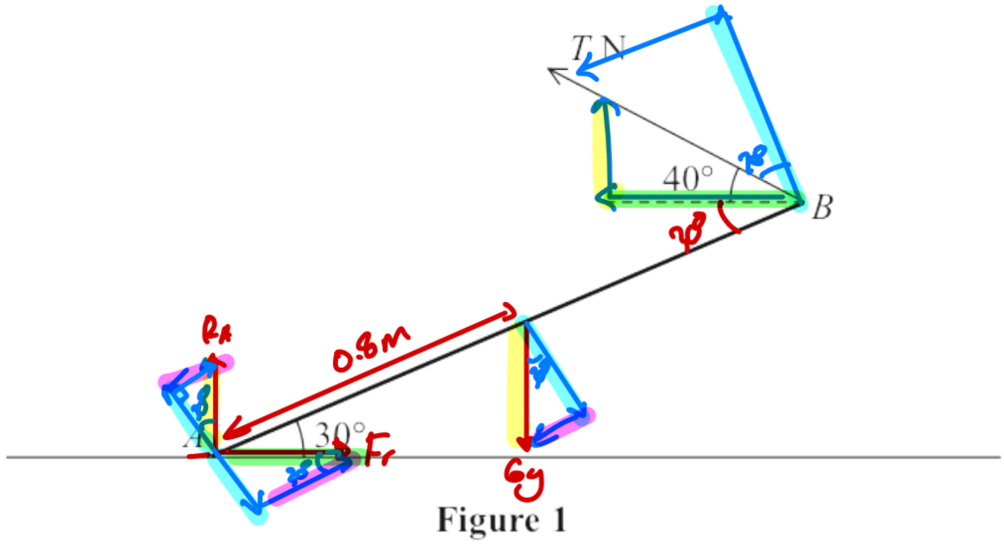

Figure 1

A uniform rod AB, of mass 6 kg and length 1.6 m, rests with its end A on rough horizontal ground. The rod is held in equilibrium at 30° to the horizontal by a light string attached to the rod at B. The string is at 40° to the horizontal and lies in the same vertical plane as the rod, as shown in Figure 1. The tension in the string is T newtons. The coefficient of friction between the ground and the rod is μ.

(a) Show that, to 3 significant figures, $T = 27.1$

(4)

(b) Find the set of values of μ for which equilibrium is possible.

(5)

a) M(A):

$0.8 \times 6g\cos 30 = 1.6 \times T\cos 20$

$4.8g\cos 30 = 1.6\,T\cos 20$

$T = \dfrac{4.8g\cos 30}{1.6\cos 20} = 27.1$

b) R_A:

$R_A + T\sin 40 = 6g$

$R_A = 6g - T\sin 40$

$= 41.3835...$

$F_r = T\cos 40$

$= 20.7561$

$F_r \le \mu R \Rightarrow \mu \geq \dfrac{F_r}{R} \Rightarrow \mu \geq 0.502$

6.3

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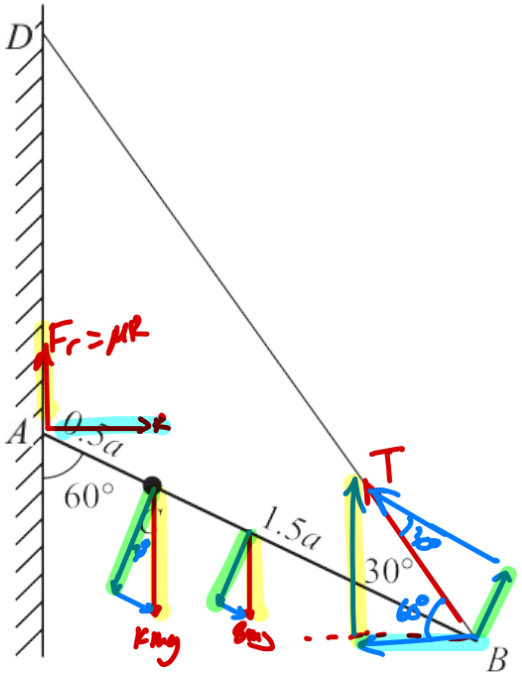

Figure 5

A uniform rod, AB, of mass $8m$ and length $2a$, has its end A resting against a rough vertical wall. One end of a light inextensible string is attached to the rod at B and the other end of the string is attached to the wall at the point D, which is vertically above A. The angle between the rod and the string is 30°. A particle of mass km is fixed to the rod at C, where $AC = 0.5a$. The rod is in equilibrium in a vertical plane perpendicular to the wall, and is at an angle of 60° to the wall, as shown in Figure 5. The tension in the string is T.

(a) Show that $T = \dfrac{\sqrt{3}}{4}(16 + k)mg$

(4)

The coefficient of friction between the wall and the rod is $\dfrac{2}{3}\sqrt{3}$.

Given that the rod is in limiting equilibrium,

(b) find the value of k.

(6)

M(A):

$0.5a \times kmg\cos 30 + a \times 8mg\cos 30 = 2a \times T\sin 30$

$\frac{\sqrt{3}}{4}kmg + 4\sqrt{3}\,mg = T$

$\frac{\sqrt{3}}{4}mg(k + 16) = T$

$T = \frac{\sqrt{3}}{4}(16 + k)mg$

$R = T\cos 60$

$\mu R + T\sin 60 = kmg + 8mg$

$\frac{2}{3}\sqrt{3}\, T\cos 60 + T\sin 60 = kmg + 8mg$

$\left(\frac{2}{3}\sqrt{3}\cos 60 + \sin 60\right)T = mg(k+8)$

$\frac{5}{6}\sqrt{3} \times \frac{\sqrt{3}}{4}(16+k)mg = mg(k+8)$

$\frac{5}{8}(16+k) = k+8$

$10 + \frac{5}{8}k = k+8$

$2 = \frac{3}{8}k \Rightarrow k = \frac{16}{3}$

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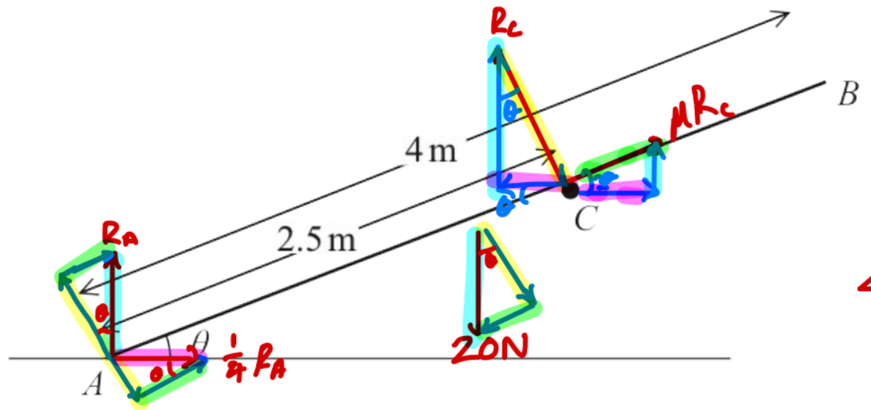
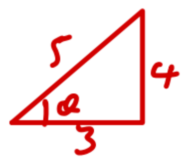


Figure 2



$\sin\theta = \frac{3}{5}$

A plank AB rests in equilibrium against a fixed horizontal pole. The plank has length 4 m and weight 20 N and rests on the pole at C, where $AC = 2.5$ m. The end A of the plank rests on rough horizontal ground and AB makes an angle θ with the ground, as shown in Figure 2. The coefficient of friction between the plank and the ground is $\frac{1}{4}$.

The plank is modelled as a uniform rod and the pole as a rough horizontal peg that is perpendicular to the vertical plane containing AB.

Given that $\cos\theta = \frac{4}{5}$ and that the friction is limiting at both A and C,

(a) find the magnitude of the normal reaction on the plank at C,

(3)

(b) find the coefficient of friction between the plank and the pole.

(8)

a) $M(A)$:

$2 \times 20\cos\theta = 2.5 R_C$

$\Rightarrow R_C = \frac{40(\frac{4}{5})}{2.5} = 12.8N$

b) $R\uparrow$:

$R_A + R_C\cos\theta + \mu R_C\sin\theta = 20$

$R_A = 20 - 12.8(\frac{4}{5}) - 12.8(\frac{3}{5})\mu$

$= 9.76 - 7.68\mu$

$R\rightarrow$:

$R_A\sin\theta + \frac{1}{4}R_A\cos\theta + \mu R_C = 20\sin\theta$

$R_A(\frac{3}{5}) + \frac{1}{4}R_A(\frac{4}{5}) + \mu R_C = 20(\frac{3}{5})$

$0.8R_A + \mu R_C = 12$

$$0.8\,(9.76 - 7.68\mu) + 12.8\mu = 12$$

$$7.808 - 6.144\mu + 12.8\mu = 12$$

$$6.656\mu = 4.192$$

$$\mu = 0.630$$

January 2020

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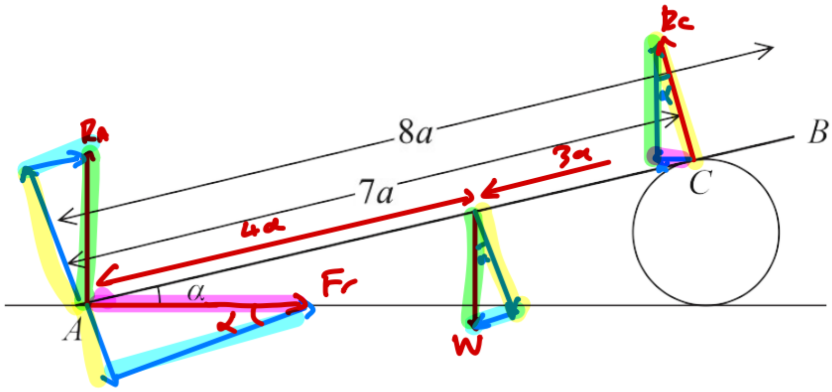


Figure 4

A uniform rod, AB, of weight W and length $8a$, rests in equilibrium with the end A on rough horizontal ground. The rod rests on a smooth cylinder. The cylinder is fixed to the ground with its axis horizontal. The point of contact between the rod and the cylinder is C, where $AC = 7a$, as shown in Figure 4. The rod is resting in a vertical plane that is perpendicular to the axis of the cylinder. The rod makes an angle α with the horizontal.

(a) Show that the normal reaction of the ground on the rod at A has magnitude $W\left(1 - \frac{4}{7}\cos^2\alpha\right)$ **(6)**

Given that the coefficient of friction between the rod and the ground is μ and that $\cos\alpha = \frac{3}{\sqrt{10}}$

(b) find the range of possible values of μ.

(5)

a)

M(A):

$4a \times W\cos\alpha = 7a \times R_c$

$4W\cos\alpha = 7R_c$

$\frac{4}{7}W\cos\alpha = R_c$

R_A:

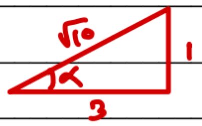
$R_A + R_c\cos\alpha = W$

$R_A = W - R_c\cos\alpha$

$= W - \frac{4}{7}W\cos\alpha\cos\alpha$

$= W\left(1 - \frac{4}{7}\cos^2\alpha\right)$

b)



$\sin\alpha = \frac{1}{\sqrt{10}}$

$F_r = R_C \sin\alpha$

$F_r = \frac{4}{7} W\cos\alpha \sin\alpha$

$= \frac{4}{7} W\left(\frac{3}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{10}}\right) = \frac{12}{70} W$

$F_r \leq \mu R_A$

$\frac{12}{70} W \leq \mu \times W\left(1 - \frac{4}{7}\cos^2\alpha\right)$

$\frac{12}{70} \leq \mu\left(1 - \frac{4}{7}\left(\frac{3}{\sqrt{10}}\right)^2\right)$

$\frac{12}{70} \leq \frac{17}{35}\mu$

$\frac{6}{17} \leq \mu$

$\mu \geqslant \frac{6}{17}$

May 2020

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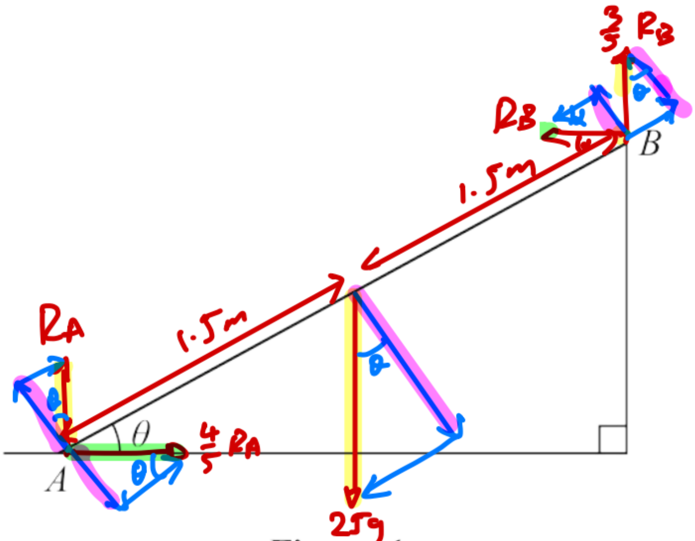

Figure 1

A uniform rod AB, of mass 25 kg and length 3 m, has end A resting on rough horizontal ground. The end B rests against a rough vertical wall.

The rod is in a vertical plane perpendicular to the wall.

The coefficient of friction between the rod and the ground is $\frac{4}{5}$

The coefficient of friction between the rod and the wall is $\frac{3}{5}$

The rod rests in limiting equilibrium.

The rod is at an angle of θ to the ground, as shown in Figure 1.

Find the exact value of $\tan\theta$.

(9)

M(A):

$1.5 \times 25g\cos\alpha = 3\times R_B\sin\theta + 3\times\frac{3}{5}R_B\cos\theta$

$37.5g\cos\alpha = 3R_B\sin\theta + \frac{9}{5}R_B\cos\theta$

$37.5g = 3R_B\tan\alpha + \frac{9}{5}R_B$

R(↑): $R_A + \frac{3}{5}R_B = 25g$

R(→): $\frac{4}{5}R_A = R_B \Rightarrow R_A = \frac{5}{4}R_B$

6.11

$$\Rightarrow \frac{5}{4}R_B + \frac{3}{5}R_B = 25g$$

$$\frac{37}{20}R_B = 25g$$

$$R_B = \frac{500}{37}g$$

$$37.5g = 3 \times \frac{500}{37}g\tan\alpha + \frac{9}{5} \times \frac{500}{37}g$$

$$37.5 = \frac{1500}{37}\tan\alpha + \frac{900}{37}$$

$$\frac{1500}{37}\tan\alpha = \frac{975}{74}$$

$$\tan\alpha = \frac{13}{40}$$

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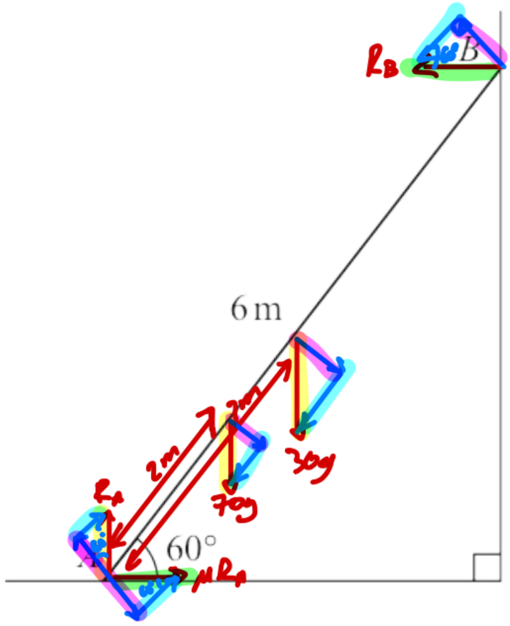

Figure 3

A ladder AB has length 6 m and mass 30 kg. The ladder rests in equilibrium at 60° to the horizontal with the end A on rough horizontal ground and the end B against a smooth vertical wall, as shown in Figure 3.

A man of mass 70 kg stands on the ladder at the point C, where $AC = 2$ m, and the ladder remains in equilibrium. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall. The man is modelled as a particle.

(a) Find the magnitude of the force exerted on the ladder by the ground.

(6)

The man climbs further up the ladder. When he is at the point D on the ladder, the ladder is about to slip.

Given that the coefficient of friction between the ladder and the ground is 0.4

(b) find the distance AD.

(4)

(c) State how you have used the modelling assumption that the ladder is a rod.

(1)

M(A):

$2 \times 70g\cos 60 + 3 \times 30g\cos 60 = 6 \times R_B \sin 60$

$115g = 3\sqrt{3}\, R_B$

$R_B = \frac{115g}{3\sqrt{3}}$

$\Rightarrow \mu R_A = \frac{115g}{3\sqrt{3}}$

R(↑): $R_A = 70g + 30g = 100g = 980$ N

$\sqrt{R_A^2 + (\mu R_A)^2} = 1000$ N

b)

$\mu R_A = 0.4 \times 980$

$= 392$

$\Rightarrow R_B = 392$

M(A):

$3 \times 30g\cos 60 + x \times 70g\cos 60 = 6 \times 392 \sin 60$

$$x = \frac{6 \times 392 \sin 60 - 90g\cos 60}{70g\cos 60}$$

$= 4.65\,m$

OR

M(B):

$3 \times 30g\cos 60 + x \times 70g\cos 60 + 6 \times 392 \sin 60 = 6 \times 980\cos 60$

$x = 1.35$

$6 - 1.35 = 4.65\,m$

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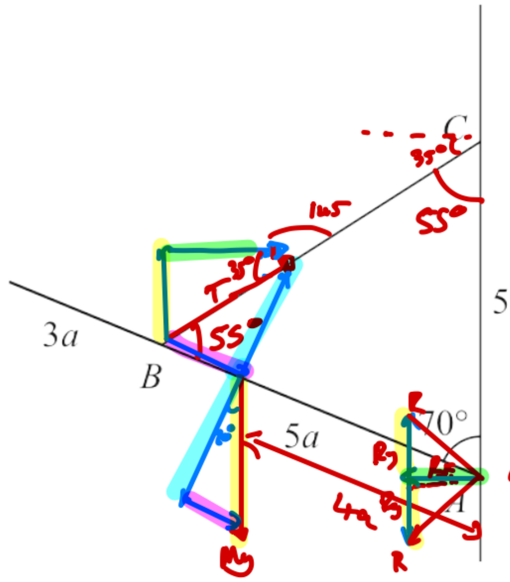

Horizontal component of R must act to the left since the horizontal component of T is acting to the right. Vertical component of R, set it up or down.

Figure 3

A uniform rod, of length $8a$ and mass M, has one end freely hinged to a fixed point A on a vertical wall. One end of a light inextensible string is attached to the rod at the point B, where $AB = 5a$. The other end of the string is attached to the wall at the point C, where $AC = 5a$ and C is vertically above A. The rod rests in equilibrium in a vertical plane perpendicular to the wall with angle $BAC = 70°$, as shown in Figure 3.

(a) Find, in terms of M and g, the tension in the string. **(3)**

The magnitude of the force acting on the rod at A is λMg, where λ is a constant.

(b) Find, to 2 significant figures, the value of λ. **(6)**

a)

M(A) :

$5a \times T\sin 55 = 4a \times Mg\cos 20$

$5T \sin 55 = 4Mg\cos 20$

$T = \frac{4}{5} Mg \frac{\cos 20}{\sin 55}$

$= 0.918 Mg$

b) R↑:

$T\sin 35 + R_y = Mg$

$R_y = Mg - T\sin 35$

$= Mg - 0.918Mg \sin 35 = 0.473$

$R(\rightarrow)$:

$$R_x = T\cos 35$$
$$= 0.918 Mg \cos 35$$
$$= 0.752 Mg$$

$$R = \sqrt{R_x^2 + R_y^2} = 0.884$$

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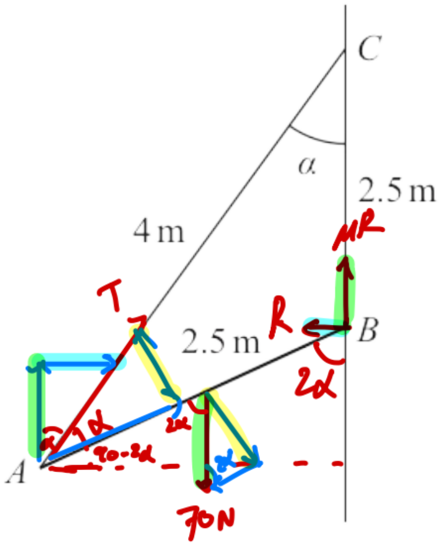


Figure 2

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

A pole AB has length 2.5 m and weight 70 N.
The pole rests with end B against a rough vertical wall. One end of a cable of length 4 m is attached to the pole at A. The other end of the cable is attached to the wall at the point C.
The point C is vertically above B and $BC = 2.5$ m.
The angle between the cable and the wall is α, as shown in Figure 2.
The pole is in a vertical plane perpendicular to the wall.
The cable is modelled as a light inextensible string and the pole is modelled as a uniform rod.

Given that $\tan\alpha = \frac{3}{4}$

(a) show that the tension in the cable is 56 N.

(4)

Given also that the pole is in limiting equilibrium,

(b) find the coefficient of friction between the pole and the wall.

(6)

a) M(B):

$1.25 \times 70\sin 2\alpha = 2.5 \times T\sin\alpha$

$87.5 \times 2\sin\alpha\cos\alpha = 2.5T\sin\alpha$

$175\cos\alpha = 2.5T$

$T = 56$ N

b)

$R_\rightarrow$:

$$T\sin\alpha = R$$

$$R = 56 \times \frac{3}{5} = 33.6 \text{ N}$$

$R_\uparrow$:

$$T\cos\alpha + \mu R = 70$$

$$56 \times \frac{4}{5} + \mu \times 33.6 = 70$$

$$33.6\mu = 25.2$$

$$\mu = 0.75$$

5.

$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{12}{13}$

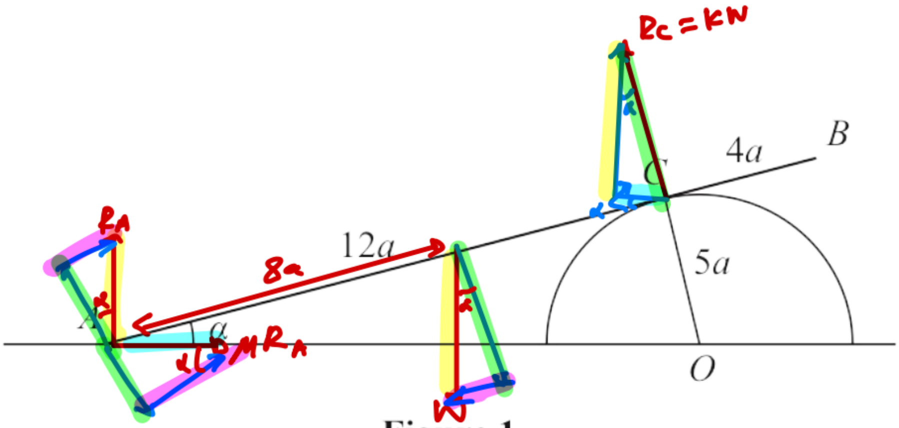


Figure 1

A smooth solid hemisphere is fixed with its flat surface in contact with rough horizontal ground. The hemisphere has centre O and radius $5a$.
A uniform rod AB, of length $16a$ and weight W, rests in equilibrium on the hemisphere with end A on the ground. The rod rests on the hemisphere at the point C, where $AC = 12a$ and angle $CAO = \alpha$, as shown in Figure 1.

Points A, C, B and O all lie in the same vertical plane.

(a) Explain why $AO = 13a$

(1)

The normal reaction on the rod at C has magnitude kW

(b) Show that $k = \dfrac{8}{13}$

(3)

The resultant force acting on the rod at A has magnitude R and acts upwards at $\theta°$ to the horizontal.

(c) Find

(i) an expression for R in terms of W

(ii) the value of θ

(8)

a) AC is a tangent ⇒ ACO is 90°.
⇒ ACO is a right angle triangle
⇒ By pythagoras theorem, AO is 13a

b) M(A):

$8a \times W\cos\alpha = 12a \times kW$

$8\cos\alpha = 12k \Rightarrow k = \dfrac{8\cos\alpha}{12} = \dfrac{8}{13}$

6.19

c)

$R_{\uparrow}$:

$$R_A + \frac{8}{13} w\cos\alpha = w$$

$$R_A + \frac{96}{169} w = w$$

$$R_A = \frac{73}{169} w$$

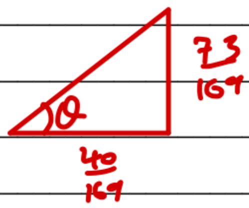
$R_{\rightarrow}$:

$$\frac{8}{13} w \sin\alpha = \mu R_A$$

$$\frac{40}{169} w = \mu R_A$$

$$\sqrt{(R_A)^2 + (\mu R_A)^2} = \sqrt{\left(\frac{73}{169}\right)^2 + \left(\frac{40}{169}\right)^2}\; w$$

$$= \frac{\sqrt{41}}{13} w$$



$$\tan\theta = \frac{\frac{73}{169}}{\frac{40}{169}} = \frac{73}{40}$$

$$\theta = 61.3$$

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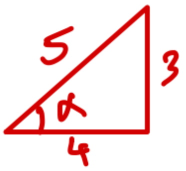

$\cos\alpha = \frac{4}{5}$

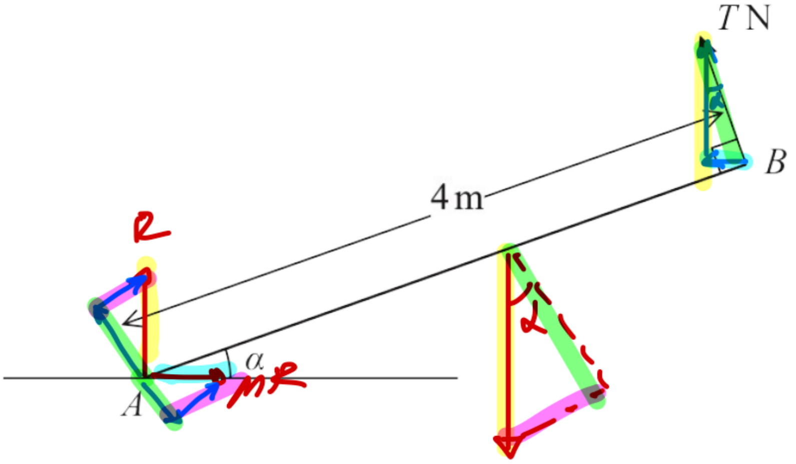

Figure 2

A uniform rod AB has length 4 m and weight 50 N.
The rod has its end A on rough horizontal ground. The rod is held in equilibrium at an angle α to the ground by a light inextensible cable attached to the rod at B, as shown in Figure 2. The cable and the rod lie in the same vertical plane and the cable is perpendicular to the rod. The tension in the cable is T newtons.

Given that $\sin\alpha = \dfrac{3}{5}$

(a) show that $T = 20$

(3)

Given also that the rod is in limiting equilibrium,

(b) find the value of the coefficient of friction between the rod and the ground.

(6)

a)

M(A):

$2 \times 50\cos\alpha = 4 \times T$

$80 = 4T$

$T = 20$ N

b) $R_{\uparrow}$:

$R + T\cos\alpha = 50$

$R = 50 - 20(\frac{4}{5}) = 34$

$R_{\rightarrow}$:

$\mu R = T\sin\alpha$

$34\mu = 12$

$\mu = \frac{6}{17} = 0.352$

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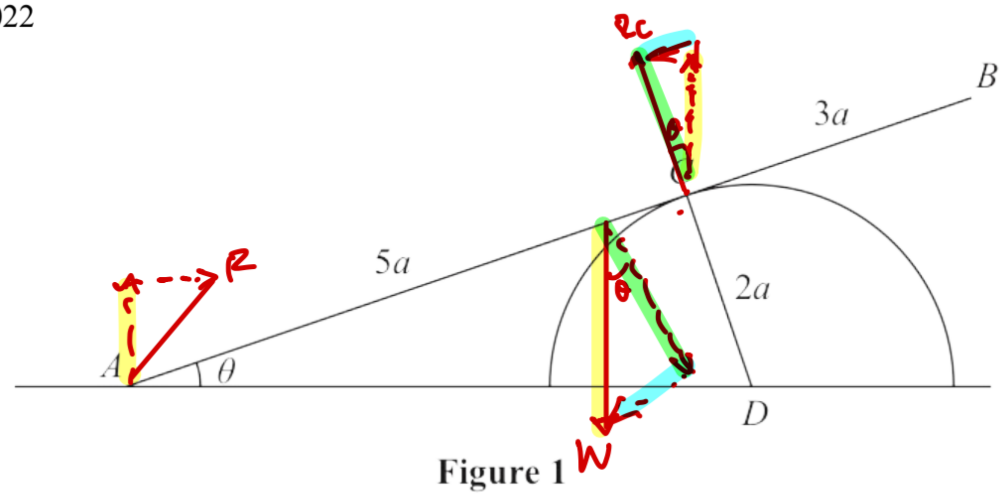
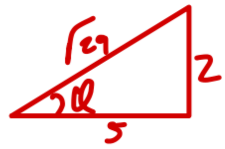


Figure 1



$\sin\theta = \frac{2}{\sqrt{29}}$

$\cos\theta = \frac{5}{\sqrt{29}}$

A uniform rod AB has length $8a$ and weight W.
The end A of the rod is freely hinged to horizontal ground.
The rod rests in equilibrium against a block which is also fixed to the ground.
The block is modelled as a smooth solid hemisphere with radius $2a$ and centre D.
The point of contact between the rod and the block is C, where $AC = 5a$
The rod is at an angle θ to the ground, as shown in Figure 1.
Points A, B, C and D all lie in the same vertical plane.

(a) Show that $AD = \sqrt{29}a$

(1)

(b) Show that the magnitude of the normal reaction at C between the rod and the block is $\frac{4}{\sqrt{29}}W$

(3)

The resultant force acting on the rod at A has magnitude kW and acts at an angle α to the ground.

(c) Find (i) the exact value of k
(ii) the exact value of $\tan\alpha$

(8)

a) $\sqrt{(5a)^2+(2a)^2} = \sqrt{29a^2} = \sqrt{29}\,a$

b) M(A):

$4a \times W\cos\theta = 5a \times R_C$

$4W\left(\frac{5}{\sqrt{29}}\right) = 5R_C$

$R_C = \frac{4}{\sqrt{29}}W$

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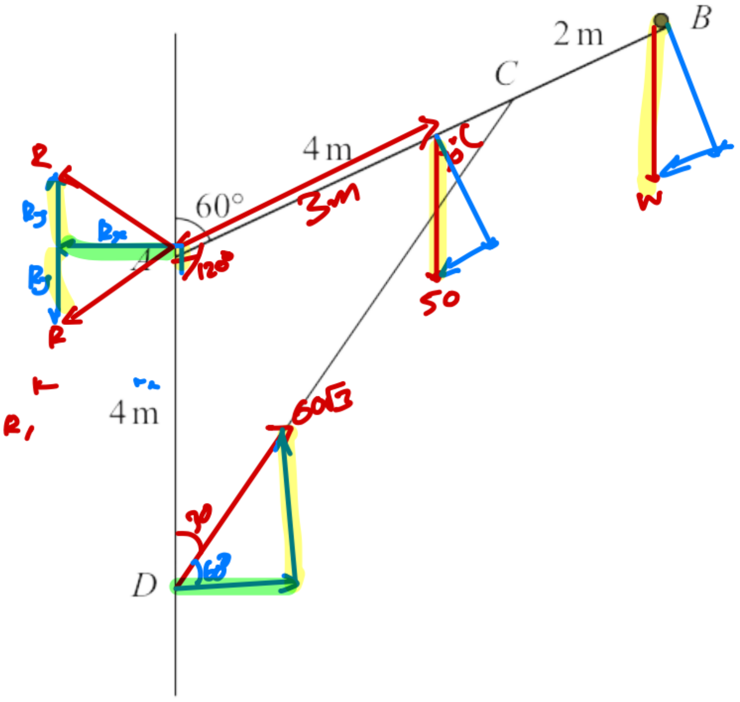

Figure 3

A uniform pole AB, of weight 50 N and length 6 m, has a particle of weight W newtons attached at its end B. The pole has its end A freely hinged to a vertical wall.
A light rod holds the particle and pole in equilibrium with the pole at 60° to the wall.
One end of the light rod is attached to the pole at C, where $AC = 4$ m.
The other end of the light rod is attached to the wall at the point D.
The point D is vertically below A with $AD = 4$ m, as shown in Figure 3.

The pole and the light rod lie in a vertical plane which is perpendicular to the wall.

The pole is modelled as a rod.

Given that the thrust in the light rod is $60\sqrt{3}$ N,

(a) show that $W = 15$

(4)

(b) find the magnitude of the resultant force acting on the pole at A.

(6)

a) M(A)

$3 \times 50\cos 30 + 6 \times W\cos 30 = 4 \times 60\sqrt{3}\cos 60$

$75\sqrt{3} + 3\sqrt{3}W = 120\sqrt{3}$

$3\sqrt{3}W = 45\sqrt{3}$

$3W = 45$

$W = 15$

b) R_1:

$$R_y + 50 + 15 = 60\sqrt{3}\sin 60$$
$$R_y = 25$$

$$R_x = 60\sqrt{3}\cos 60$$
$$= 30\sqrt{3}$$

$$R = \sqrt{R_x^2 + R_y^2} = 5\sqrt{133} = 57.7$$

May 2023

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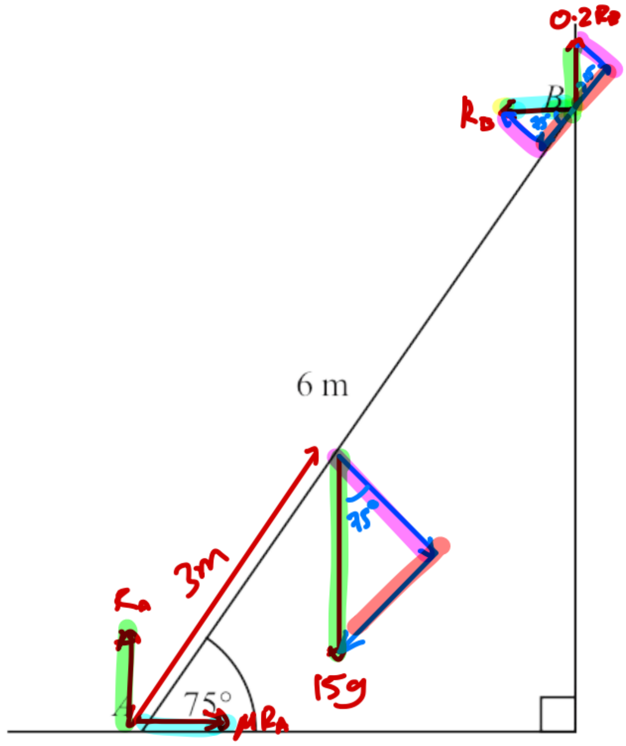


Figure 2

A uniform beam AB, of mass 15 kg and length 6 m, rests with end A on rough horizontal ground. The end B of the beam rests against a rough vertical wall.

The beam is inclined at 75° to the ground, as shown in Figure 2.

The coefficient of friction between the beam and the wall is 0.2

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod which lies in a vertical plane perpendicular to the wall.

The beam rests in limiting equilibrium.

(a) Find the magnitude of the normal reaction between the beam and the wall at B.

(5)

(b) Find the value of μ

(6)

M(A):

$3 \times 15g\cos 75 = 6 \times R_B \sin 75 + 6 \times 0.2R_B \sin 15$

$45g\cos 75 = R_B(6\sin 75 + 1.2\sin 15)$

$R_B = \dfrac{45g\cos 75}{6\sin 75 + 1.2\sin 15} = 18.7\text{N}$

6.27

$R_\uparrow$:

$$R_A + 0.2R_B = 15g$$

$$R_A = 15g - 0.2R_B$$

$R_\rightarrow$

$$\mu R_A = R_B$$

$$\mu(15g - 0.2R_B) = R_B$$

$$\mu = \frac{R_B}{15g - 0.2R_B} = \frac{18.7}{15(9.8) - 0.2(18.7)}$$

$$= 0.13$$

6.

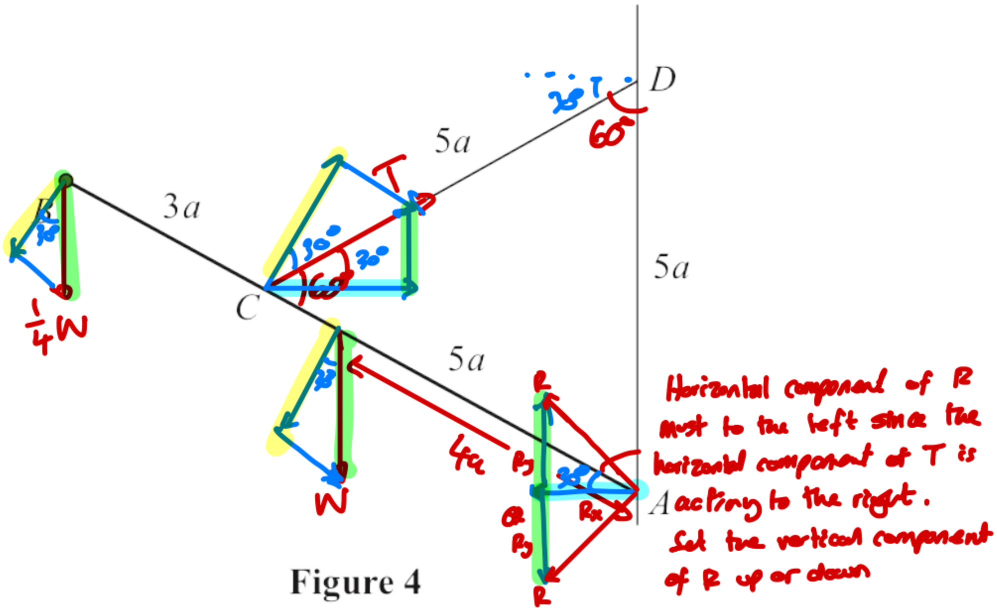


Figure 4

A uniform rod AB has length $8a$ and weight W.

The end A of the rod is freely hinged to a fixed point on a vertical wall.

A particle of weight $\frac{1}{4}W$ is attached to the rod at B.

A light inelastic string of length $5a$ has one end attached to the rod at the point C, where $AC = 5a$.

The other end of the string is attached to the wall at the point D, where D is above A and $AD = 5a$, as shown in Figure 4.

The rod rests in equilibrium.

The tension in the string is T.

(a) Show that $T = \frac{6}{5}W$ **(3)**

(b) Find, in terms of W, the magnitude of the force exerted on the rod by the hinge at A. **(6)**

a) M(A)

$4a \times W\cos 30 + 8a \times \frac{1}{4}W\cos 30 = 5a \times T\cos 30$

$3\sqrt{3}W = \frac{5}{2}\sqrt{3}T$

$3W = \frac{5}{2}T$

$T = \frac{6}{5}W$

R_x :

$$R_x = T\cos 30$$

$$= \frac{6}{5}W\cos 30 = \frac{3}{5}\sqrt{3}$$

R_y :

$$\frac{1}{4}W + W = T\sin 30 + R_y$$

$$\frac{5}{4}W = \frac{6}{5}W\sin 30 + R_y$$

$$R_y = \frac{13}{20}W$$

$$\sqrt{R_x^2 + R_y^2} = \frac{\sqrt{601}}{20}W = 1.23W$$

6.

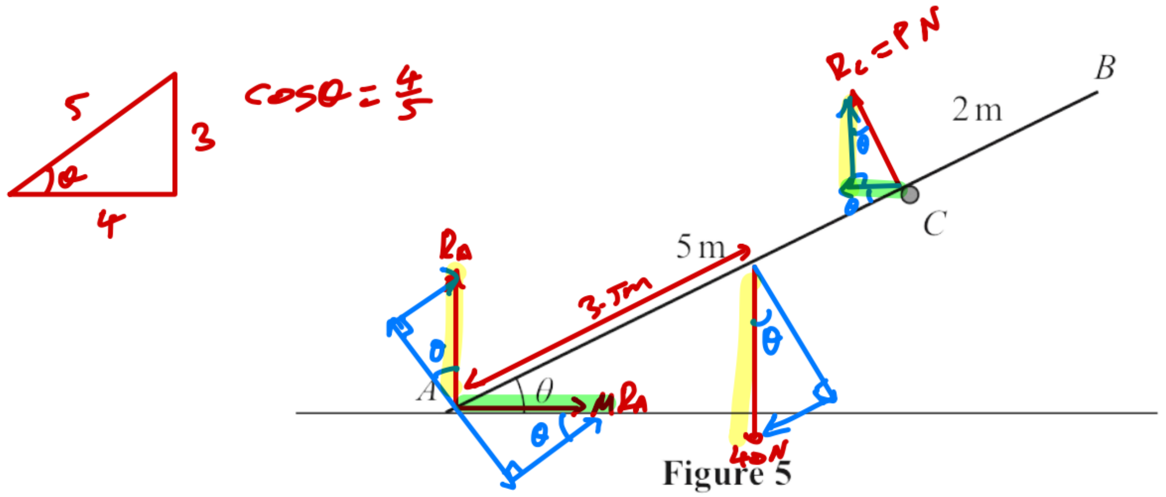


Figure 5

A uniform beam AB, of weight 40 N and length 7 m, rests with end A on rough horizontal ground.

The beam rests on a smooth horizontal peg at C, with $AC = 5$ m, as shown in Figure 5.

The beam is inclined at an angle θ to the ground, where $\sin\theta = \frac{3}{5}$

The beam is modelled as a rod that lies in a vertical plane perpendicular to the peg.

The normal reaction between the beam and the peg at C has magnitude P newtons.

Using the model,

(a) show that $P = 22.4$

(3)

(b) find the magnitude of the resultant force acting on the beam at A.

(6)

a)

M(A):

$$3.5 \times 40\cos\theta = 5 \times P$$
$$112 = 5P$$
$$P = 22.4$$

b) $R_{\rightarrow}$:

$$22.4\sin\theta = \mu R_A$$
$$\mu R_A = 13.44 \text{ N}$$

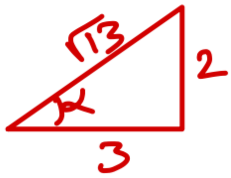
$R_{\uparrow}$:

$$R_A + 22.4\cos\theta = 40$$
$$R_A = 40 - 22.4\left(\tfrac{4}{5}\right)$$
$$= 22.08 \text{ N}$$

$R = \sqrt{(13.44)^2 + (22.08)^2} \quad = \frac{24}{5}\sqrt{29}$

$= 25.9 \text{ N}$

6.



$\sin\alpha = \frac{2}{\sqrt{13}}$

$\cos\alpha = \frac{3}{\sqrt{13}}$

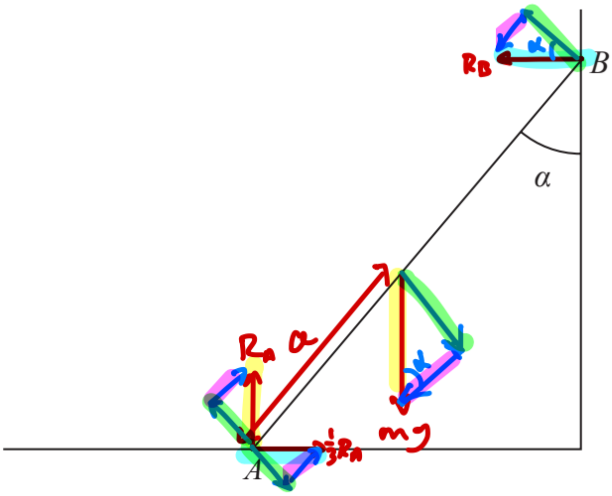


Figure 2

A uniform rod, AB, of mass m and length $2a$, rests in limiting equilibrium with its end A on rough horizontal ground and its end B against a smooth vertical wall.
The vertical plane containing the rod is at right angles to the wall.
The rod is inclined to the wall at an angle α, as shown in Figure 2.

The coefficient of friction between the rod and the ground is $\frac{1}{3}$

(a) Show that $\tan\alpha = \frac{2}{3}$

(6)

With the rod in the same position, a horizontal force of magnitude kmg is applied to the rod at A, towards the wall. The line of action of this force is at right angles to the wall.

The rod remains in equilibrium.

(b) Find the largest possible value of k.

(4)

M(A):

$a \times mg\sin\alpha = 2a \times R_B\cos\alpha$

$mg\sin\alpha = 2R_B\cos\alpha$

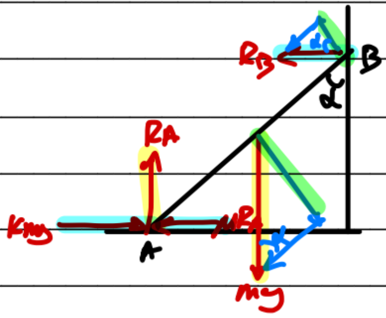
$R\uparrow$: $R_A = mg$

$R\rightarrow$: $R_B = \frac{1}{3}R_A = \frac{1}{3}mg$

$\Rightarrow mg\sin\alpha = 2\left(\frac{1}{3}mg\right)\cos\alpha$

$\Rightarrow \sin\alpha = \frac{2}{3}\cos\alpha$

$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{2}{3} \Rightarrow \tan\alpha = \frac{2}{3}$

b) 

For greatest value of k, friction is acting to the left.

M(A):

$2a \times R_B\cos\alpha = a \times mg\sin\alpha$

$2R_B = mg\sin\alpha$

$R_B = \frac{1}{2}mg\tan\alpha$

$= \frac{1}{2}mg \times \frac{2}{3} = \frac{1}{3}mg$

$R_\uparrow$:

$R_A = mg$

$R_\rightarrow$:

$kmg = \mu R_A + R_B$

$kmg = \frac{1}{3}mg + \frac{1}{3}mg$

$k = \frac{2}{3}$