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Chapter 1: Projectiles

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7. A particle is projected from a point O with speed U at an angle of elevation α to the horizontal and moves freely under gravity. When the particle has moved a horizontal distance x , its height above O is y .

(a) Show that

$$y = x \tan \alpha - \frac{gx^2(1 + \tan^2 \alpha)}{2U^2} \quad (7)$$

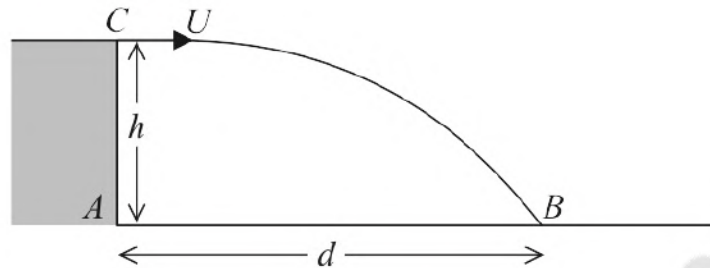


Figure 3

A small stone is projected horizontally with speed U from a point C at the top of a vertical cliff AC so as to hit a fixed target B on the horizontal ground. The point C is a height h above the ground, as shown in Figure 3. The time of flight of the stone from C to B is T , and the stone is modelled as a particle moving freely under gravity.

(b) Find, in terms of U , g and T , the speed of the stone as it hits the target at B .

(4)

It is found that, using the same initial speed U , the target can also be hit by projecting the stone from C at an angle α above the horizontal. The stone is again modelled as a particle moving freely under gravity and the distance $AB = d$.

(c) Using the result in part (a), or otherwise, show that

$$d = \frac{1}{2} g T^2 \tan \alpha \quad (6)$$

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6. A particle P is projected from a fixed point A with speed 12 m s^{-1} at an angle α above the horizontal and moves freely under gravity. As P passes through the point B on its path, P is moving with speed 8 m s^{-1} at an angle β below the horizontal.

(a) By considering energy, find the vertical distance between A and B .

(4)

Particle P takes 1.5 seconds to travel from A to B .

(b) Find the size of angle α .

(3)

(c) Find the size of angle β .

(3)

(d) Find the length of time for which the speed of P is less than 8 m s^{-1} .

(4)

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4. At time $t = 0$ a ball is projected from a fixed point A on horizontal ground to hit a target. The ball is projected from A with speed $u \text{ m s}^{-1}$ at an angle θ° to the horizontal. At time $t = 2 \text{ s}$ the ball hits the target. At the instant when it hits the target, the ball is travelling downwards at 30° below the horizontal with speed 12 m s^{-1} . The ball is modelled as a particle moving freely under gravity and the target is modelled as the point T .

- (a) Find
- (i) the value of θ ,
 - (ii) the value of u .
- (7)

The height of T above the ground is h metres.

- (b) Find the value of h .
- (3)
- (c) Find the length of time for which the ball is more than h metres above the ground during the flight from A to T .
- (3)

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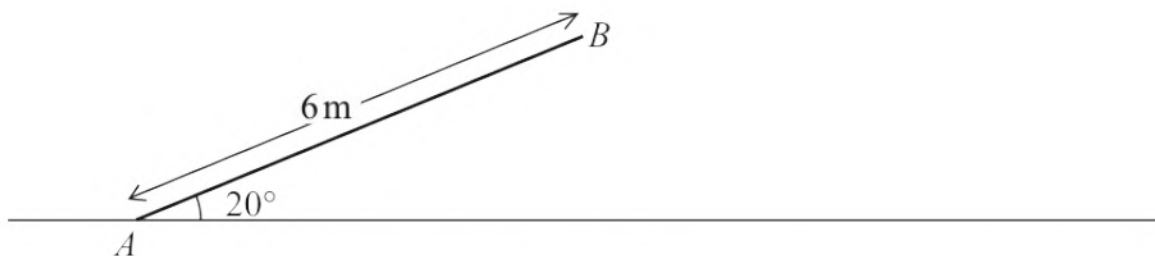


Figure 3

After leaving the ramp at B , the particle P moves freely under gravity until it hits the horizontal ground at the point C . The speed of P as it hits the ground at C is $w \text{ ms}^{-1}$.

Find

- (b) (i) the value of w ,
- (ii) the direction of motion of P as it hits the ground at C ,
- (c) the greatest height of P above the ground as P moves from A to C .

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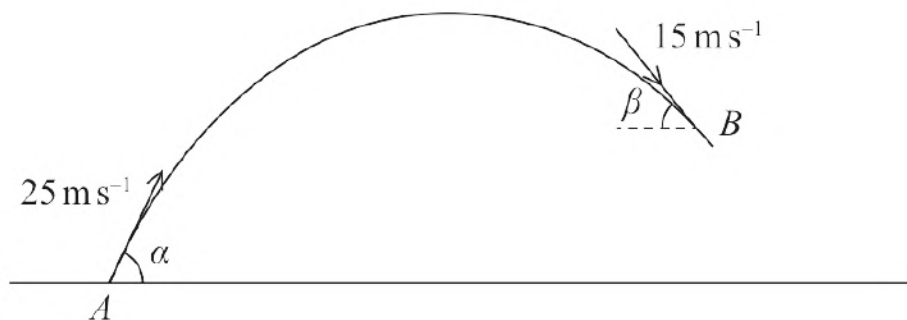


Figure 5

At time $t = 0$ a particle P is projected from a fixed point A on horizontal ground. The particle is projected with speed 25 m s^{-1} at an angle α to the ground. The particle moves freely under gravity. At time $t = 3$ seconds, P is passing through the point B with speed 15 m s^{-1} and is moving downwards at an angle β to the horizontal, as shown in Figure 5.

(a) By considering energy, find the height of B above the ground. (3)

(b) Find the size of angle α . (3)

(c) Find the size of angle β . (3)

(d) Find the least speed of P as P travels from A to B . (2)

As P travels from A to B , the speed, $v \text{ m s}^{-1}$, of P is such that $v \leq 15$ for an interval of T seconds.

(e) Find the value of T . (3)

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8. [In this question, the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in a vertical plane, with $\mathbf{i}$ being horizontal and $\mathbf{j}$ being vertically upwards.]

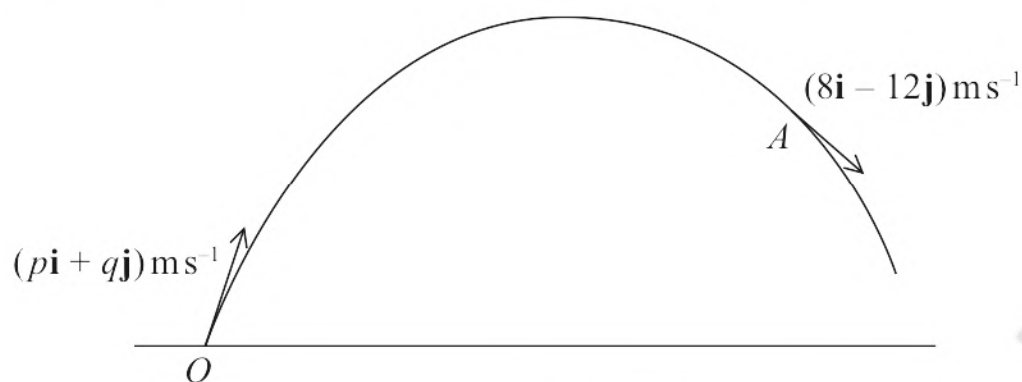


Figure 4

At time $t = 0$, a small ball is projected from a fixed point O on horizontal ground. The ball is projected from O with velocity $(p\mathbf{i} + q\mathbf{j})\text{ m s}^{-1}$, where p and q are positive constants. The ball moves freely under gravity.

At time $t = 3$ seconds, the ball passes through the point A with velocity $(8\mathbf{i} - 12\mathbf{j})\text{ m s}^{-1}$, as shown in Figure 4.

- (a) Find the speed of the ball at the instant it is projected from O . (5)

For an interval of T seconds the speed, $v\text{ m s}^{-1}$, of the ball is such that $v \leq 10$

- (b) Find the value of T . (4)

At the point B on the path of the ball, the direction of motion of the ball is perpendicular to the direction of motion of the ball at A .

- (c) Find the vertical height of B above A . (4)

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Figure 4

- (a) By considering energy, find the speed of P as it hits the ground at B . (4)
- (b) Find the least speed of P as it moves from A to B . (2)
- (c) Find the length of time for which the speed of P is more than 10 ms^{-1} . (6)

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7. [In this question, the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in a vertical plane, $\mathbf{i}$ being horizontal and $\mathbf{j}$ being vertically upwards.]

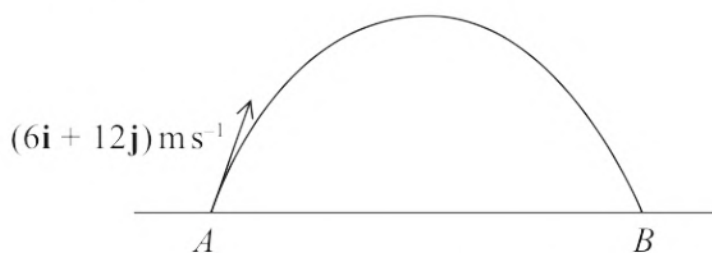


Figure 5

A small ball is projected with velocity $(6\mathbf{i} + 12\mathbf{j})\text{ms}^{-1}$ from a fixed point A on horizontal ground. The ball hits the ground at the point B , as shown in Figure 5. The motion of the ball is modelled as a particle moving freely under gravity.

- (a) Find the distance AB . (4)

When the height of the ball above the ground is more than h metres, the speed of the ball is less than 10ms^{-1}

- (b) Find the smallest possible value of h . (4)

When the ball is at the point C on its path, the direction of motion of the ball is perpendicular to the direction of motion of the ball at the instant before it hits the ground at B .

- (c) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the velocity of the ball when it is at C . (3)

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8.

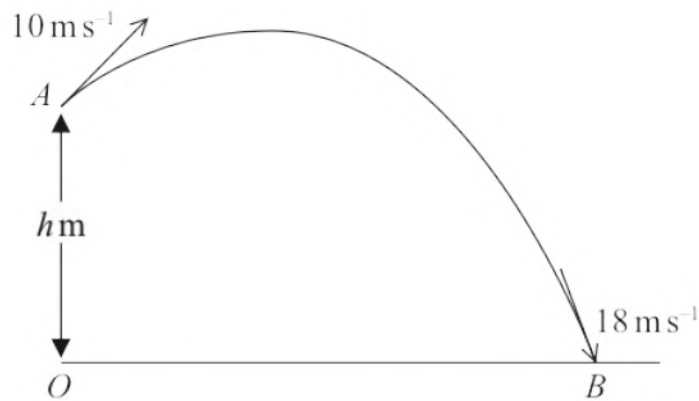


Figure 4

The fixed point A is h metres vertically above the point O that is on horizontal ground. At time $t = 0$, a particle P is projected from A with speed 10 ms^{-1} . The particle moves freely under gravity. At time $t = 2.5$ seconds, P strikes the ground at the point B . At the instant when P strikes the ground, the speed of P is 18 ms^{-1} , as shown in Figure 4.

- (a) By considering energy, find the value of h . (3)
- (b) Find the distance OB . (5)

As P moves from A to B , the speed of P is less than or equal to 8 m s^{-1} for T seconds.

- (c) Find the value of T (6)

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7. A particle P is projected from a fixed point O on horizontal ground. The particle is projected with speed u at an angle α above the horizontal. At the instant when the horizontal distance of P from O is x , the vertical distance of P above the ground is y . The motion of P is modelled as that of a particle moving freely under gravity.

(a) Show that $y = x \tan \alpha - \frac{gx^2}{2u^2} (1 + \tan^2 \alpha)$

(6)

A small ball is projected from the fixed point O on horizontal ground. The ball is projected with speed 20 m s^{-1} at angle θ° above the horizontal. A vertical pole AB , of height 2 m , stands on the ground with $OA = 10 \text{ m}$, as shown in Figure 3.

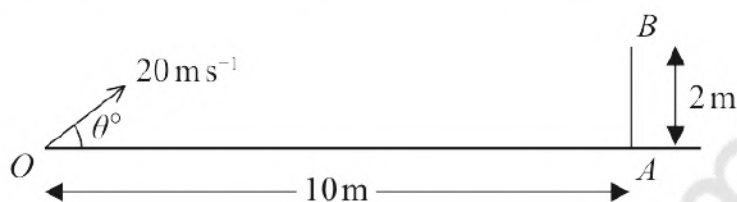


Figure 3

The ball is modelled as a particle moving freely under gravity and the pole is modelled as a rod.

The path of the ball lies in the vertical plane containing O , A and B .

Using the model,

- (b) find the range of values of θ for which the ball will pass over the pole.

(3)

Given that $\theta = 40^\circ$ and that the ball first hits the ground at the point C

- (c) find the speed of the ball at the instant it passes over the pole,

(5)

- (d) find the distance OC .

(2)

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8. [In this question, the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in a vertical plane, with $\mathbf{i}$ being horizontal and $\mathbf{j}$ being vertically upwards.]

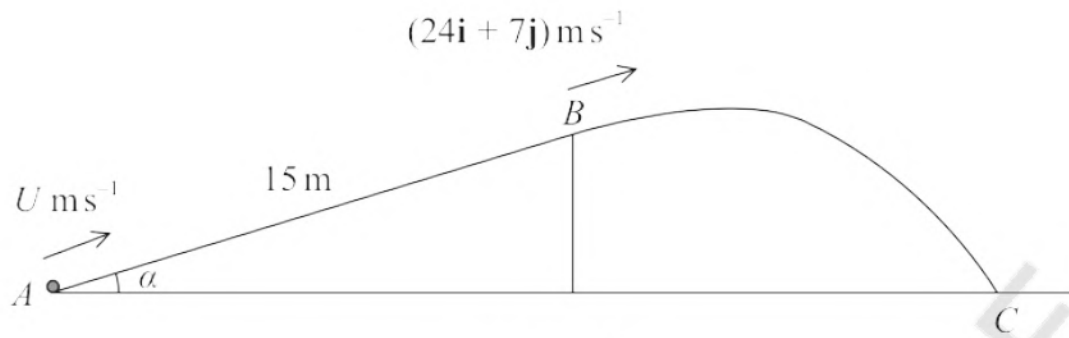


Figure 3

A rough ramp is fixed to horizontal ground.

The ramp is inclined to the ground at an angle α , where $\tan \alpha = \frac{7}{24}$.

The point A is at the bottom of the ramp and the point B is at the top of the ramp. The line AB is a line of greatest slope of the ramp and $AB = 15 \text{ m}$, as shown in Figure 3.

A particle P of mass 0.3 kg is projected with speed $U \text{ m s}^{-1}$ from A directly towards B . At the instant P reaches the point B , the velocity of P is $(24\mathbf{i} + 7\mathbf{j}) \text{ m s}^{-1}$. The particle leaves the ramp at B , and moves freely under gravity until it hits the horizontal ground at the point C .

The coefficient of friction between P and the ramp is $\frac{1}{5}$.

- (c) Find the time taken by P to move from B to C . (3)

At the instant immediately before P hits the ground at C , the particle is moving downwards at θ° to the horizontal.

- (d) Find the value of θ . (4)

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8. A particle P is projected from a fixed point O . The particle is projected with speed $u \text{ m s}^{-1}$ at angle α above the horizontal. The particle moves freely under gravity. At the instant when the horizontal distance of P from O is x metres, P is y metres vertically above the level of O .

(a) Show that $y = x \tan \alpha - \frac{gx^2}{2u^2} (1 + \tan^2 \alpha)$

(6)

A small ball is projected from a fixed point A with speed $U \text{ m s}^{-1}$ at θ° above the horizontal.

The point B is on horizontal ground and is vertically below the point A , with $AB = 20 \text{ m}$.

The ball hits the ground at the point C , where $BC = 30 \text{ m}$, as shown in Figure 4.

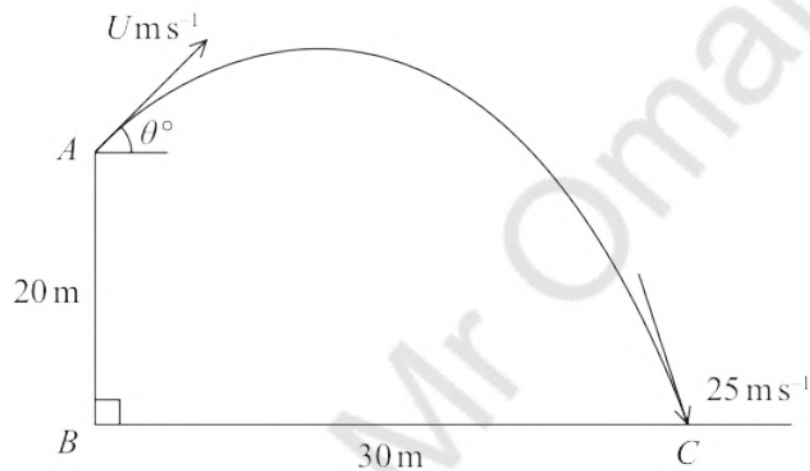


Figure 4

The speed of the ball immediately before it hits the ground is 25 m s^{-1}

The motion of the ball is modelled as that of a particle moving freely under gravity.

- (b) Use the principle of conservation of mechanical energy to find the value of U .

(3)

- (c) Find the value of θ

(3)

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7. [In this question, the perpendicular unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in a vertical plane with $\mathbf{i}$ being horizontal and $\mathbf{j}$ being vertically upwards.]

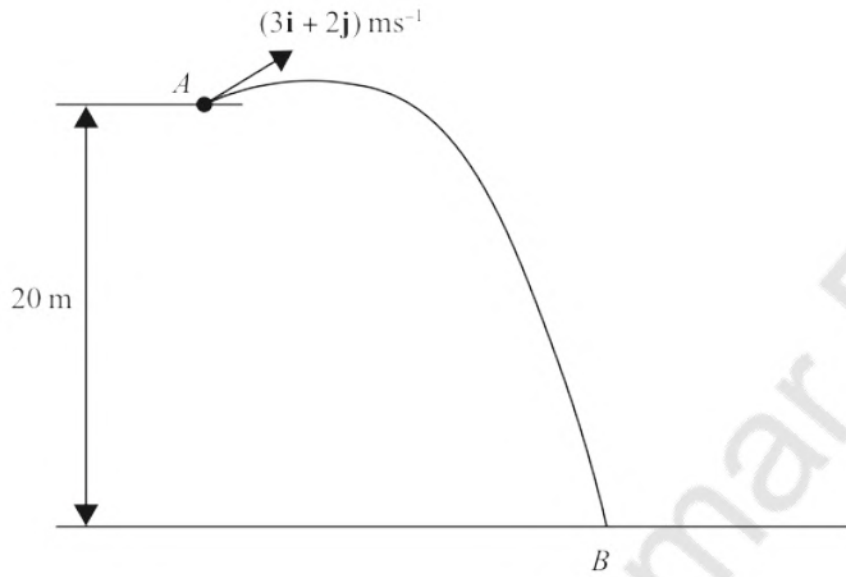


Figure 4

A small ball is projected with velocity $(3\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}$ from the fixed point A .

The point A is 20 m above horizontal ground.

The ball hits the ground at the point B , as shown in Figure 4.

The ball is modelled as a particle moving freely under gravity.

- By considering energy, find the speed of the ball at the instant immediately before it hits the ground. (3)
- Find the direction of motion of the ball at the instant immediately before it hits the ground. (3)
- Find the time taken for the ball to travel from A to B . (3)

At the instant when the direction of motion of the ball is perpendicular to $(3\mathbf{i} + 2\mathbf{j})$ the ball is h metres above the ground.

- Find the value of h . (6)

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4. [In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors, with $\mathbf{i}$ horizontal and $\mathbf{j}$ vertical.]

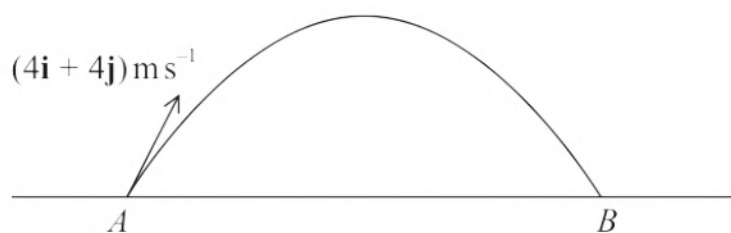


Figure 3

The fixed points A and B lie on horizontal ground.

At time $t = 0$, a particle P is projected from A with velocity $(4\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$.

Particle P moves freely under gravity and hits the ground at B , as shown in Figure 3.

At time T_1 seconds, P is at its highest point above the ground.

- (a) Find the value of T_1 (2)

At time $t = 0$, a particle Q is also projected from A but with velocity $(5\mathbf{i} + 7\mathbf{j}) \text{ m s}^{-1}$.

Particle Q moves freely under gravity.

- (b) Find the vertical distance between Q and P at time T_1 seconds, giving your answer to 2 significant figures. (3)

At the instant when particle P reaches B , particle Q is moving at α° below the horizontal.

- (c) Find the value of α . (4)

At time T_2 seconds, the direction of motion of Q is perpendicular to the initial direction of motion of Q .

- (d) Find the value of T_2 (3)

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A diagram showing a parabolic trajectory of a projectile. The path starts at point A on the left and ends at point B on the right, both on a horizontal ground line. An arrow at point A indicates the initial velocity vector, labeled $(4\mathbf{i} + 7\mathbf{j}) \text{ m s}^{-1}$.

The fixed points A and B lie on horizontal ground.

Particle P moves freely under gravity and hits the ground at B , as shown in Figure 6.

- The speed of P is less than 5 ms^{-1} for an interval of length T seconds.

- At the instant when the direction of motion of P is perpendicular to the initial direction of motion of P , the particle is h metres above the ground.

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7. [In this question, the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in a vertical plane, $\mathbf{i}$ being horizontal and $\mathbf{j}$ being vertically upwards.]

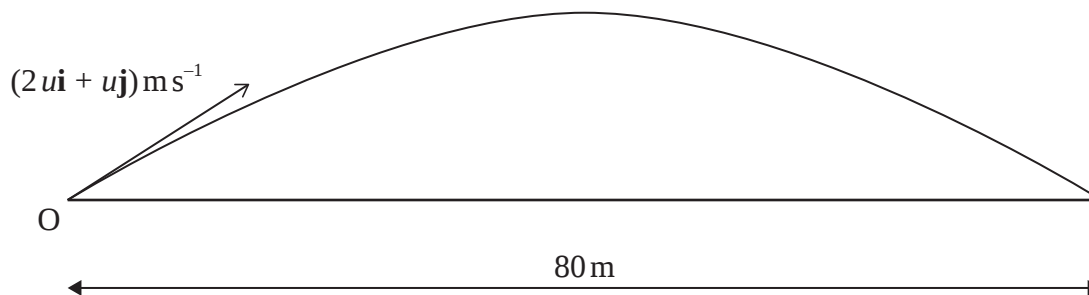


Figure 3

A golf ball is hit from a point O on horizontal ground and is modelled as a particle moving freely under gravity. The initial velocity of the ball is $(2u\mathbf{i} + u\mathbf{j})\text{ms}^{-1}$. The ball first hits the horizontal ground at a point which is 80m from O , as shown in Figure 3.

Use the model to

- (a) show that $u = 14$ (6)
- (b) find the total time, while the ball is in the air, for which the speed of the ball is greater than $7\sqrt{17} \text{ m s}^{-1}$ (5)

[illegible]

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