January 2018

Q	Scheme	Marks	Notes
1.	Impulse- momentum equation	M1	Must be subtracting velocities (or equivalent). Dimensionally correct.
	$4\mathbf{i}+5\mathbf{j}=\frac{1}{2}(\mathbf{v}-(2\mathbf{i}-3\mathbf{j}))$	A1	Correct unsimplified equation.
	$\mathbf{v}=10\mathbf{i}+7\mathbf{j}$	A1	Seen or implied
	KE Gain	M1	Dimensionally correct. Condone ± Must be difference of two KE terms.
	$=\frac{1}{2}0.5(10^2+7^2-(2^2+(-3)^2))$	A1ft	Correct unsimplified expression Follow their **v** . Condone ±
	$=34$ J	A1	CSO
		(6)	
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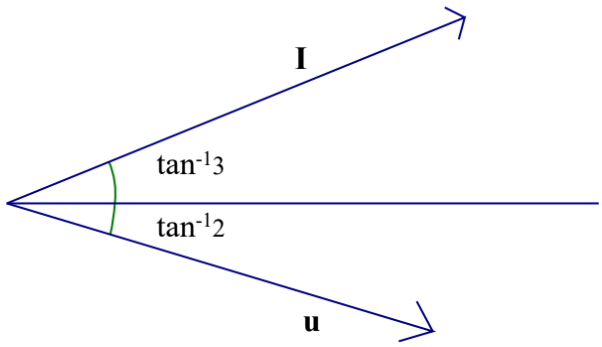
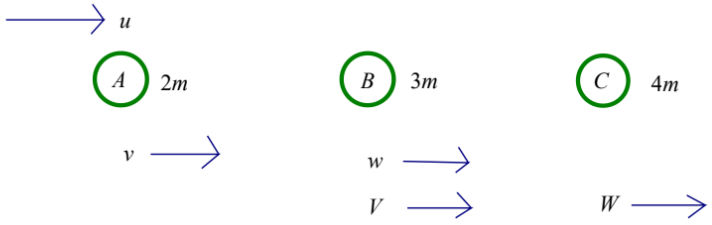
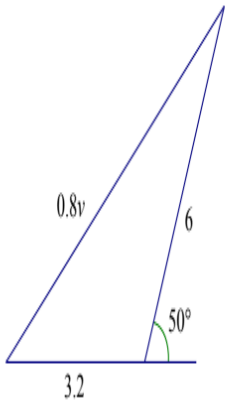
Q	Scheme	Marks	Notes
4(a)			NB three variants possible: both to right, both to the left or as diagram
	Conservation of momentum	M1	Dimensionally correct. All terms required. Condone sign errors
	$2mu-3mu=-2mv_P+mv_Q \quad \left(-u=-2v_P+v_Q\right)$	A1	Correct unsimplified equation
	Impact law	M1	Must be used the right way round. Condone sign errors.
	$4eu=v_P+v_Q$	A1	Correct unsimplified equation. Signs consistent with CLM equation
	Solve simultaneous equations for v_P or v_Q	DM1	Dependent on the 2 preceding M marks
	$v_P=\dfrac{u(1+4e)}{3}$	A1	One correct (must be positive) or equivalent
	$v_Q=\dfrac{u(8e-1)}{3}$	A1	Both correct (must be positive) or equivalent
		(7)	
4(b)	$\dfrac{u(1+4e)}{3}>0$	M1	Working from a correct inequality for their v_P
	Always true because $e\geq 0 \quad \left(\text{or } e>\dfrac{1}{8}\right)$	A1	Correct justification from correct work
		(2)	
4(c)	$e=\dfrac{3}{4}\Rightarrow v_Q=\dfrac{5u}{3}$	B1	Seen or implied
	Speed of Q after collision $=fv_Q$	M1	Impact law for collision with the wall
	To collide with P: $fv_2>v_1=\dfrac{4u}{3}$	M1	Correct inequality for second collision
	$1\geq f>\dfrac{4}{5}$	A1	Both limits required
		(4)	
		[13]	

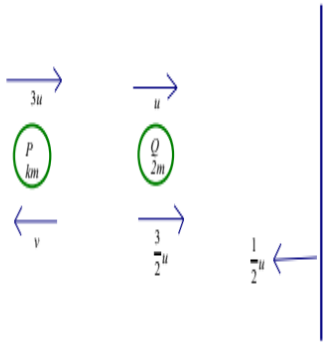
Question Number	Scheme	Marks
1a.	Use of $\mathbf{I}=m\mathbf{v}-m\mathbf{u}$	M1
	$=0.7\left(\left(3\mathbf{i}+4\mathbf{j}\right)-\left(\mathbf{i}-2\mathbf{j}\right)\right)$	A1
	$=0.7\left(2\mathbf{i}+6\mathbf{j}\right)=1.4\mathbf{i}+4.2\mathbf{j}$ (N s)	A1
		(3)
1b.		
	Required angle $=\tan^{-1}3+\tan^{-1}2$ Follow their impulse	M1 A1ft
	$=135°$ (Allow 225^0)	A1
		(3)
	Alternative for last 3 marks using scalar product:	
	Correct use of scalar product with relevant vectors M1	
	$\cos\theta=\dfrac{(\mathbf{i}-2\mathbf{j})\bullet(\mathbf{i}+3\mathbf{j})}{\sqrt{1+4}\sqrt{1+9}}=\dfrac{-5}{5\sqrt{2}}$ A1ft	
	$\theta=135°$ A1	
	Alternative for last 3 marks using cosine rule:	
	Correct use of cosine rule with relevant sides M1	
	$\cos\theta=\dfrac{(1.4^2+4.2^2)+(1^2+(-2)^2)-(0.4^2+6.2^2)}{2\sqrt{(1.4^2+4.2^2)}\sqrt{(1^2+(-2)^2)}}$ A1ft	
	$\theta=135°$ A1	
	1(a) M1 for Use of $\mathbf{I}=m\mathbf{v}-m\mathbf{u}$ (M0 if m omitted or g included) with **v** and **u** substituted but allow m and allow terms reversed First A1 for a correct equation Second A1 for a correct answer, **i**'s and **j**'s need to be collected Isw if they go on and give a magnitude.	
	1(b) M1 for a complete correct method to find required angle for their **I** (M0 if they are finding the wrong angle) First A1 ft for a correct expression from their **I** and **u** Second A1 for 135° (Accept 134.9.. or 225.1…)	

Question Number	Scheme	Marks
7a		
	CLM: $2mu = 2mv + 3mw$	M1A1
	Impact law: $eu = w - v$	M1A1
	Eliminate w: $5v = u(2-3e)$ oe	M1
	KE: $v = \frac{u}{3}$	M1
	$e = \frac{1}{9}$, 0.11 or better	A1
	OR: $v = \frac{u}{3}$ is used in CLM and NIL equations (See Alternative below)	
	ALTERNATIVE: $v = \frac{u}{3}$ M1 (KE) $2mu = 2mv + 3mw$ $\left(2mu = 2m\frac{u}{3} + 3mw\right)$ M1 A1 (CLM) $\Rightarrow w = \frac{4u}{9}$ M1 (eliminate w) $e = \frac{w-v}{u}$ $\left(e = \frac{\frac{4u}{9} - \frac{u}{3}}{u}\right)$ M1 A1 (Impact Law) $e = \frac{1}{9}$ A1	
7b(i)	Substitute $e\left(=\frac{1}{9}\right)$: $v = \frac{u}{3}$, $w = \frac{4}{9}u$	M1
	CLM & Impact equations: $3m \times w = 3mV + 4mW$ $\left(\frac{4}{3}u = 3V + 4W\right)$ $wf = W - V$ $\left(\frac{16u}{9}f = 4W - 4V\right)$ Follow their w	M1 A1
	Solve for V: $V = \frac{4u}{21} - \frac{16uf}{63}$ N.B. answer must be positive	M1 A1
7b(ii)	since $f \geq 0$, $V \leq \frac{4u}{21} < \frac{u}{3}$ and hence A collides with B	M1A1

Question Number	Scheme	Marks
	N.B. If $f = 0$ is considered, to give $V = \frac{4u}{21} < \frac{u}{3}$ M1A0	(7)
		[14]
	Notes for Qu7	
	7(a) First M1 for CLM, usual rules First A1 for a correct equation Second M1 for NIL, with e on the correct side Second A1 for a correct equation Third M1 for <u>eliminating</u> w Fourth M1 (this is really a B mark) for use of $v = \frac{u}{3}$ oe (M0 for use of $v = \frac{u}{9}$ oe) Third A1 for 1/9, 0.11 or better	
	7(b) (i) First M1 for using their e to find v and w (can be done in (a) but must be used in (b)) Second M1 for CLM and NIL equations for 2nd collision First A1 for two correct equations for **their** w, V and W. Third M1 for finding V in terms of f and u Second A1 for **any** correct expression (must be > 0) (ii) Fourth M1 (<u>only available if they have a correct expression for V</u>) for a complete strategy to show that there is a 2nd collision between A and B, using $f \geq 0$ (Allow M1A0 if they use $f = 0$, see above) Third A1 for a fully correct justification. N.B. GIVEN ANSWER **(ii) Alternative:** Fourth M1 (<u>only available if they have a correct expression for V</u>) for a complete strategy to show that there is a 2nd collision between A and B: 2nd collision if $V < \frac{u}{3}$ i.e. if $\frac{4u}{21} - \frac{16uf}{63} < \frac{u}{3}$ i.e. if $\frac{-9}{16} < f$. Since $f \geq 0$ (given), there will be a 2nd collision. Third A1 for a fully correct justification. N.B. GIVEN ANSWER.	

WME02 - Mechanics 2 - Mark Scheme

Q	Scheme	Marks	Notes
1.	Impulse-momentum equation:	M1	Must be working in 2 dimensions. Dimensionally correct. Condone subtraction in wrong order
	$\begin{pmatrix}6\cos 50°\\6\sin 50°\end{pmatrix}=0.8\mathbf{v}-0.8\begin{pmatrix}4\\0\end{pmatrix}$	A1	Correct unsimplified equation
	$\Rightarrow \mathbf{v}=\begin{pmatrix}4+7.5\cos 50°\\7.5\sin 50°\end{pmatrix}$	A1	$\begin{pmatrix}8.82\\5.74\end{pmatrix}$
	Pythagoras: $\lvert\mathbf{v}\rvert=\sqrt{\left(4+7.5\cos 50°\right)^2+\left(7.5\sin 50°\right)^2}$	M1	Must have 2 components
	$\lvert\mathbf{v}\rvert=\left(=\sqrt{8.82^2+5.75^2}\right)=10.5\,(\text{m s}^{-1})$	A1	Accept 10.52 and 10.53
		[5]	
Alt1	Impulse-momentum equation:	M1	Dimensionally correct. Condone subtraction in wrong order
	$\begin{pmatrix}6\\0\end{pmatrix}=0.8\begin{pmatrix}v\cos\theta-4\cos 50°\\v\sin\theta-4\sin 50°\end{pmatrix}$	A1	Working parallel and perpendicular to the impilse
	$\Rightarrow\begin{pmatrix}v\cos\theta\\v\sin\theta\end{pmatrix}=\begin{pmatrix}7.5+4\cos 50°\\4\sin 50°\end{pmatrix}$	A1	$\begin{pmatrix}10.071...\\3.064..\end{pmatrix}$
	Use of Pythagoras	M1	
	$\lvert\mathbf{v}\rvert=10.5\,(\text{m s}^{-1})$	A1	
		[5]	
Alt2			Momentum (or velocity) triangle

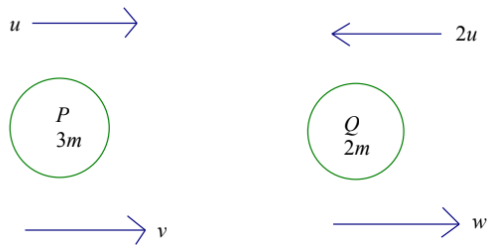
Q	Scheme	Marks	Notes
			
7a	Impact law: $\frac{3}{2}u+v=e(3u-u)\left(=2eu\right)$	M1	Used the right way round
	$\left(v=2eu-\frac{3}{2}u\right)$	A1	Correct unsimplified equation
	$v>0\Rightarrow 2e>\frac{3}{2}$	M1	Form and solve correct inequality for their v
	$\left(1\geq\right)e>\frac{3}{4}$	A1	Accept $1>e>\frac{3}{4}$ and $e>\frac{3}{4}$
		(4)	
7a alt	Impact law: $\frac{3}{2}u+v=e(3u-u)\left(=2eu\right)$	M1	Used the right way round
	$\left(v=2eu-\frac{3}{2}u\right)$	A1	Correct unsimplified equation
	CLM $\Rightarrow v=\frac{u}{k}(1-3k)>0\Rightarrow k<\frac{1}{3}$	M1	Use CLM to form inequality in k and substitute into impact equation
	$e=\frac{1}{2k}-\frac{3}{4}>\frac{3}{2}-\frac{3}{4}\Rightarrow\frac{3}{4}<e\left(\leq 1\right)$	A1	
		(4)	

Q	Scheme	Marks	Notes
7b	$e=\frac{7}{8} \Rightarrow v=\frac{7}{4}u-\frac{3}{2}u=\frac{1}{4}u$	B1	
	CLM : $3kmu+2mu=2m\times\frac{3}{2}u-kmv \qquad (3ku=u-kv)$	M1	Need all terms and dimensionally consistent. If only seen in (a) it must be used in (b) to score.
	$\left(3k+2=3-\frac{1}{4}k\right)$	A1	Correct unsimplified equation
	$k=\frac{4}{13}$	A1	
	KE lost: $\frac{1}{2}\times\frac{4}{13}m\left(9u^2-\frac{u^2}{16}\right)$	M1	Accept in terms of k e.g. $\frac{1}{2}km\left(9u^2-\frac{1}{16}u^2\right)$
	$=\frac{2}{13}m\times\frac{143}{16}u^2=\frac{11}{8}mu^2$ ***Given answer***	A1	Obtain given answer from correct working Fully correct substitution seen
		(6)	
7c	Time for Q to reach wall: $\frac{2d}{3u}$	B1	
	Speed of Q after collision with wall: $\frac{1}{3}\times\frac{3}{2}u=\frac{1}{2}u$	B1	
	P has moved $\frac{u}{4}\times\frac{2d}{3u}=\frac{d}{6}$	B1	
	Gap $d+\frac{d}{6}=\frac{7d}{6}$ closing at $\frac{1}{2}u-\frac{1}{4}u=\frac{1}{4}u$	M1	
	takes $\frac{7d}{6}\div\frac{u}{4}=\frac{14d}{3u}$	M1	Terms dimensionally correct
	Total time $\frac{14d}{3u}+\frac{2d}{3u}=\frac{16d}{3u}$	A1	
		(6)	

Q	Scheme	Marks	Notes
7c alt	Time for Q to reach wall: $\frac{2d}{3u}$	B1	
	Speed of Q after collision with wall: $\frac{1}{3}\times\frac{3}{2}u=\frac{1}{2}u$	B1	
	Total time for Q: $\frac{2d}{3u}+\frac{2x}{u}$	B1	
	Equal times: $\frac{2d}{3u}+\frac{2x}{u}=\frac{4(x-d)}{u}$	M1	Terms dimensionally correct. Condone a sign error
	Solve for x: $2d+6x=12x-12d,\quad x=\frac{7d}{3}$	M1	
	Time $=\frac{4}{u}\times\frac{4d}{3}=\frac{16d}{3u}$	A1 (6)	
		[16]	

January 2019

Q	Scheme	Marks	Notes
2a	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$:	M1	Accept $\pm m(\mathbf{v}-\mathbf{u})$
	$\frac{3}{4}\mathbf{v} = \frac{3}{4}(4\mathbf{i}+\mathbf{j}) + (-6\mathbf{i}+4\mathbf{j})\left(=\left(-3\mathbf{i}+\frac{19}{4}\mathbf{j}\right)\right)$	A1	Correct unsimplified equation
	$\mathbf{v} = \frac{4}{3}\left(-3\mathbf{i}+\frac{19}{4}\mathbf{j}\right) = -4\mathbf{i}+\frac{19}{3}\mathbf{j}$ (m s^{-1})	A1	$-4\mathbf{i}+6.3\mathbf{j}$ (m s^{-1}) or better. ISW Accept as a column vector.
		(3)	
2b	Change in direction	M1	Use **u** and their **v** to find a relevant angle between the two velocities (e.g.not just $\tan^{-1}\frac{1}{4}$)
	$=180° - \tan^{-1}\frac{1}{4} - \tan^{-1}\frac{19}{12}$ or $\tan^{-1}4 + \tan^{-1}\frac{12}{19}$	A1ft	Correct unsimplified. Follow their **v** $(180° - 14.036...° - 57.724...°)$
	$=108°$ $(108.2°)$ (1.89 radians)	A1	108.2° or better. Accept 252°
		(3)	
2b alt	Using scalar product:	M1	Using vectors **u** and their **v** or equivalent
	$\cos\theta = \dfrac{-16+\frac{19}{3}}{\sqrt{17}\sqrt{16+\left(\frac{19}{3}\right)^2}}$	A1ft	follow their **v**
	$\theta = 108°$ $(108.2°)$	A1	(1.89 radians) Accept 252°
		(3)	
2b alt	Use cosine rule in triangle with sides $\sqrt{17}$, $\frac{\sqrt{505}}{3}$, $\frac{8\sqrt{13}}{3}$	M1	Or equivalent
	$\cos\theta = \dfrac{17+\frac{505}{9}-\frac{64\times13}{9}}{2\sqrt{17}\frac{\sqrt{505}}{3}} (=-0.313)$	A1	
	$\theta = 108°$ $(108.2°)$	A1	(1.89 radians) Accept 252°
		(3)	
		[6]	

Q	Scheme	Marks	Notes
7a			
	CLM (taking the initial direction of P to be +ve)	M1	All terms needed. Dimensionally correct. Condone sign error(s)
	$3m\times u-2m\times 2u=3mv+2mw\left(=-mu\right)$	A1	Correct unsimplified equation
	Negative total $\Rightarrow v<0$ and $w<0$ or $v<0$ and $w>0$ because $w<0$ and $v>0$ is impossible therefore $v<0$ - P has changed direction ***Given answer***	A1	Alt: solve CLM & impact equation for $v\left(=-\frac{u}{5}(1+6e)\right)$ and use the range of possible values for e to justify **given answer**
		(3)	
7b	Impact law:	M1	Used correctly
	$w-v=e\left(u+2u\right)=3eu$	A1	Correct unsimplified equation Signs consistent with CLM equation Allow M1A1 if seen in (a) and used in (b)
	Solve for kw: $v=w-3eu\ \Rightarrow 3\left(w-3eu\right)+2w=-u$ $5w=9eu-u$	DM1	Dependent on the preceding M1
	Correct inequality for their w: $w>0$ $\Rightarrow 9eu-u>0$	DM1	Form inequality in e Dependent on the preceding M1
	$\left(1\geq\right)e>\frac{1}{9}$	A1	Allow if upper limit not stated. A0 if upper limit incorrect. Condone $1>e$
		(5)	
7c	v and w in terms of u: $5w=\frac{9}{2}u-u=\frac{7}{2}u$, $w=\frac{7}{10}u$	M1	Solve for v or w
	$v=w-\frac{3}{2}u=-\frac{8}{10}u$	A1	Both values correct
	Loss in KE	M1	Accept change in KE Must be using two different masses ($3m$ and $2m$)
	$=\frac{1}{2}\times 3m\times\left(u^2-v^2\right)+\frac{1}{2}\times 2m\times\left(4u^2-w^2\right)$	A1ft	Follow their v, w. Correct unsimplified equation
	$=\frac{3}{2}mu^2\left(1-0.64\right)+mu^2\left(4-0.49\right)=4.05mu^2$	A1	$\left(\frac{81}{20}mu^2\text{ or equivalent}\right)$
		(5)	
		[13]	

Question Number	Scheme	Marks
3	Use of $m\mathbf{v} = \mathbf{I} + m\mathbf{u}$	M1
	Component of momentum parallel to original direction $= 6\times 0.75 + \sqrt{24}\cos 60 \ \left(= 4.5 + \sqrt{6}\right)$	A1 A1
	Use of Pythagoras: $\left(\frac{3}{4}v =\right)\sqrt{\left(4.5+\sqrt{6}\right)^2 + 18}$	M1
	$v = 10.9 \ \left(\text{m s}^{-1}\right), 11\left(\text{m s}^{-1}\right)$	A1
	Alternative for the 1st 5 marks:	
	Vector triangle for impulses or velocities	M1
	Use of cosine rule	M1
	$\left(\frac{3}{4}v\right)^2 = 4.5^2 + 24 - 2\times 4.5\times\sqrt{24}\times\cos 120°$	A1 A1
	$v = 10.9 \ \left(\text{m s}^{-1}\right), 11\left(\text{m s}^{-1}\right)$	A1
	Change in direction $= \tan^{-1}\dfrac{3\sqrt{2}}{4.5+\sqrt{6}}$	M1
	$= 31.4° \ (31°)$	A1 (7)
	Notes	**[7]**
M1	Need to consider both components.	
A1	Or equivalent	
M1	Or equivalent.	
A1	Correct LHS	
A1	Or better	
M1	Must be using correct triangle - need $120°$ seen or implied	
A1	Correct unsimplified	
A1	Or better	
M1	Or equivalent use of trig. With their components to find **the required angle** Eg angle $= \cos^{-1}\left(\dfrac{4.5^2 + (mv)^2 - 24}{2\times 4.5\times(mv)}\right)$ Or from scalar product, $\cos^{-1}\left(\dfrac{6\times 9.27...}{6\times 10.9...}\right)$	
A1	0.548 radians (0.55 radians) or better. Do not ISW	

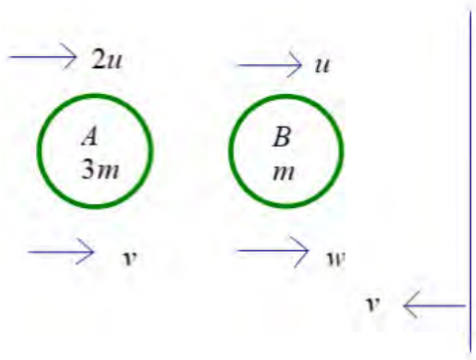
Question Number	Scheme	Marks
7a	NB: sine/cosine confusion is not condoned in projectile questions	M1
	Use of conservation of energy $\frac{1}{2}m\times 25^2=\frac{1}{2}m\times 15^2+mgh$	A1
	$\Rightarrow h=20$ or 20.4 (m)	A1 (3)
7b	Vertical distance $20.4=25\sin\alpha\times 3-4.5\times 9.8$	M1 A1ft
	$\alpha=59°$ or $59.3°$	A1 (3)
7c	Horizontal component of speed is constant $\Rightarrow 25\cos\alpha=15\cos\beta$	M1 A1ft
	$\beta=32°$ or $31.8°$	A1 (3)
7c alt	Vertical distance $20.4=-15\sin\beta\times 3+4.5\times 9.8$	M1 A1ft
	$\beta=32°$ or $31.8°$	A1 (3)

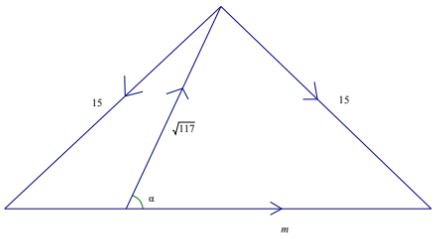
Notes

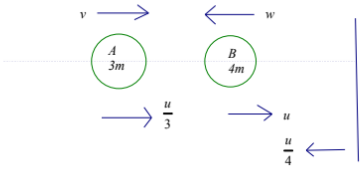
7a M1	Need energy equation with all 3 terms. Must be dimensionally correct. Condone sign errors.
A1	Correct unsimplified equation
A1	Max 3 sf Not $\frac{1000}{49}$ nor $\frac{200}{g}$
7b M1	Use of *suvat* to find α
A1ft	Correct unsimplified equation in their h
A1	0.554 rads. Max 3 sf From CWO
7c M1	Or horizontal distance travelled
A1ft	Correct unsimplified in α or their α
A1	0.554 rads. Max 3 sf From CWO
7c alt	
M1	Use of *suvat* to find β
A1ft	e.g. using $s=vt-\frac{1}{2}gt^2$. Correct unsimplified equation in their h
A1	0.554 rads. Max 3 sf From CWO

Question Number	Scheme	Marks
7d	Min speed = horizontal component $= 25\cos\alpha \left(= 15\cos\beta\right)$ $= 13$ or 12.8 (m s^{-1})	M1 A1 (2)
7e	By considering vertical component of speed at B: $15\sin 31.8° - gT = -15\sin 31.8°$ $T = 1.6$ or 1.61 (s)	M1 A1ft A1 (3)
	Notes	**[14]**
7d M1 **A1**	Follow their angle. Must show working if using incorrect angle. Max 3 sf From CWO	
7e M1 **A1ft** **A1**	Complete method using *suvat* to find T Correct unsimplified equation in T - follow their angles. Max 3 sf From CWO	

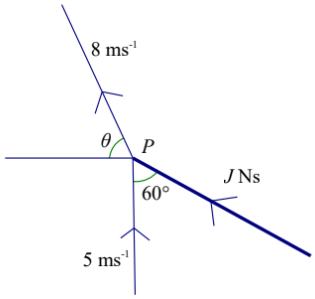
Question Number	Solution	Marks	Notes
1.	$\mathbf{I} = 2\left[\lambda\mathbf{i} + \lambda\mathbf{j} - 5\mathbf{i} - 3\mathbf{j}\right]$	M1	Use of $\mathbf{I} = m\left(\mathbf{v} - \mathbf{u}\right)$
	$= 2\left(\lambda - 5\right)\mathbf{i} + 2\left(\lambda - 3\right)\mathbf{j}$	A1	Any equivalent form
	$\left\|I\right\| = \sqrt{40} \Rightarrow \left(\lambda - 5\right)^2 + \left(\lambda - 3\right)^2 = 10$	M1	Correct use of Pythagoras and their impulse to form an equation in λ
	$\lambda^2 - 8\lambda + 12 = 0 \Rightarrow \lambda = 2$ or $\lambda = 6$	DM1	Solve to find both values for λ. Dependent on the 2 preceding M marks
	$\mathbf{I} = -6\mathbf{i} - 2\mathbf{j}$ or $\mathbf{I} = 2\mathbf{i} + 6\mathbf{j}$ $(a = -6, b = -2$ or $a = 2, b = 6)$	A1	And no others
		(5)	
	Alternative working:		
	$\mathbf{I}\left(= a\mathbf{i} + b\mathbf{j}\right) = 2\left(\mathbf{v} - \left(5\mathbf{i} + 3\mathbf{j}\right)\right)$	M1A1	
	$\mathbf{v} = \frac{a+10}{2}\mathbf{i} + \frac{b+6}{2}\mathbf{j} \quad \Rightarrow \quad \left(\Rightarrow a + 10 = b + 6\right)$		
	$a^2 + b^2 = 40 \Rightarrow b^2 - 4b - 12 = 0$ or $a^2 + 4a - 12 = 0$	M1	Correct use of Pythagoras and impulse to form an equation in a or b Any equivalent form
	$b^2 - 4b - 12 = 0 \quad \Rightarrow b = 6$ or $b = -2$	DM1	
	$\mathbf{I} = -6\mathbf{i} - 2\mathbf{j}$ or $\mathbf{I} = 2\mathbf{i} + 6\mathbf{j}$	A1	Or simplified equivalent
		[5]	

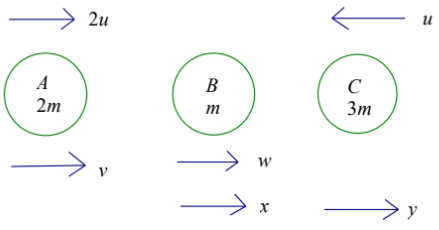
Question Number	Solution	Marks	Notes
7			
7a	KE gain = final KE – initial KE	M1	KE equation for *B*. Allow for change in KE
	$\frac{48}{25}mu^2 = \frac{1}{2}mw^2 - \frac{1}{2}mu^2$	A1	Correct unsimplified equation to find *w*
	$\left(w^2 = \frac{121}{25}u^2, \quad w = \frac{11}{5}u\right)$		
	CLM: $3m \times 2u + mu = 3mv + mw$	M1	All terms required. Condone sign errors
	$\left(7mu = 3mv + \frac{11}{5}mu\right) \left(v = \frac{8}{5}u\right)$	A1	Correct unsimplified equation in *v* and *w* or their *w*
	Impact law:	M1	Used correctly
	$w - v = e(2u - u)$	A1	Correct unsimplified equation in *v* and *w* or their *v* and *w*
	Solve for *e*	DM1	Dependent on the preceding M marks
	$\frac{3}{5}u = eu, \quad e = \frac{3}{5}$	A1	
		(8)	
7b	Impact law: $fw = v$	M1	Condone sign error
	$f = \frac{8}{11}$	A1	0.73 or better Final answer must be positive
		(2)	
		[10]	

Question Number	Scheme	Marks	
1	$(\mathbf{I}=)1.5\{v\mathbf{i}-(4\mathbf{i}+6\mathbf{j})\}$	M1	Use of $\mathbf{I}=m\mathbf{v}-m\mathbf{u}$. Must be using $v\mathbf{i}$. Condone **u**, **v** confusion. Ignore the left hand side
	$=1.5\{(v-4)\mathbf{i}-6\mathbf{j}\}$	A1	Or equivalent seen or implied Condone subtraction the wrong way round. Ignore the left hand side
	$\Rightarrow 15^2=1.5^2\{(v-4)^2+6^2\}$	M1	Use of modulus. Allow for $p^2+q^2=100$
	$(100=(v-4)^2+36)$	A1	Correct unsimplified equation in v
	$(v^2-8v-48=0)$	A1	Correct simplified equation in v seen or implied.
	$\Rightarrow v=12$	A1	One correct value
	or $v=-4$	A1	Both correct values
		[7]	
1 alt1			
	Initial momentum $=(6\mathbf{i}+9\mathbf{j})$ Ns	M1	Impulse momentum triangle. Accept $\sqrt{117}$ Ns
	$\cos\alpha=\frac{6}{\sqrt{117}}\left(=\frac{2}{\sqrt{13}}\right)$	A1	Or equivalent
	$m^2+117-2m\sqrt{117}\cos\alpha=225$	M1	Use of cosine formula (final momentum m)
	$m^2-12m-108=0$	A1	Or equivalent
	$\Rightarrow m=-6$ or $m=18$	A1	
	$\Rightarrow v=12$	A1	One correct value
	or $v=-4$	A1	Both correct values
		[7]	
1alt2	Initial momentum $=(6\mathbf{i}+9\mathbf{j})$ Ns	M1	Impulse momentum triangle. Accept $\sqrt{117}$ Ns
	$\sin\alpha=\frac{3}{\sqrt{13}}$	A1	Or equivalent
	$\frac{15}{\sin\alpha}=\frac{\sqrt{117}}{\sin\theta}$	M1	Use of sine formula
	$\Rightarrow \sin\theta=\frac{3}{5}$, $\theta=36.9°$ or $\theta=143.1°$	A1	
	$\frac{m}{\sin 86.8}=\frac{15}{\sin\alpha}$ or $\frac{m}{\sin 19.4}=\frac{15}{\sin(180-\alpha)}$	A1	Correct equation in m
	$\Rightarrow v=12$	A1	One correct value
	or $v=-4$	A1	Both correct values
		[7]	

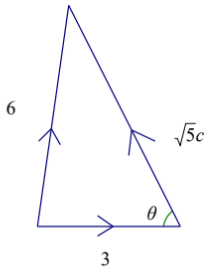
Question Number	Scheme	Marks	Notes
8a			
	Impulse on A: $8mu = 3mv - 3m \times \frac{u}{3}$	M1	Terms dimensionally correct. Must be subtracting. Condone sign errors. Must be combining correct mass and speed
	$v = 3u$	A1	
	Impulse on B: $8mu = 4mu + 4mw$	M1	Terms dimensionally correct. Condone sign errors Or use CLM: $9mu - 4mw = 3m\frac{u}{3} + 4mu$ Must be combining correct mass and speed
	$w = u$	A1	
	Impact law: $u - \frac{u}{3} = e(3u + u)$	M1	Used the right way round. Condone sign errors
	$e = \frac{1}{6}$	A1	
	Award first 4 marks in order on the scheme. Marks for CLM equation, if used, should be given in place of whichever impulse equation is not used.		
	Watch out for sign errors in the equations If they have $3mv + 4mw$ in the equation for CLM they might combine this with $w = -u$ to obtain a "correct" answer. The sign error in the CLM is due to a misread so the maximum score for this double sign error is 4/6		
		[6]	
8b	Gap when B hits wall $= \frac{2d}{3}$	B1	Or find distances from the first impact: $s_A = \frac{d}{3} + \frac{u}{3}t$ and $s_B = d - \frac{u}{4}t$
	Speed of rebound from wall $= \frac{u}{4}$	B1	Allow + / -
	Time to close gap $= \frac{\frac{2d}{3}}{\frac{u}{3} + \frac{u}{4}}$	M1	
	$= \frac{8d}{7u}$	A1	
	Distance from wall $= \frac{8d}{7u} \times \frac{u}{4}$	DM1	Dependent on the preceding M1
	$= \frac{2d}{7}$	A1 [6]	
8balt	Time for A $\frac{d-x}{u/3}$ $\left(= \frac{3d-3x}{u}\right)$	B1	
	Speed of rebound from wall $= \frac{u}{4}$	B1	

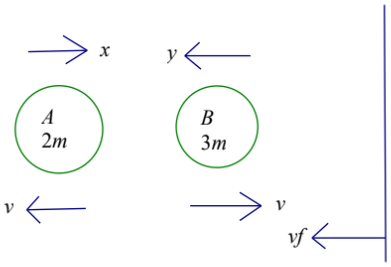
	Time for $B = \frac{d}{u} + \frac{x}{u/4}$	M1	
	$\left(= \frac{d+4x}{u}\right)$	A1	
	$3d - 3x = d + 4x$	DM1	Solve for x Dependent on the preceding M1
	$x = \frac{2d}{7}$	A1 [6]	
		[12]	

4			Resolving parallel and perpendicular to the original direction of motion
	Use of $J = m(v-u)$	M1	Use of $J = m(v-u)$ parallel or perpendicular to original direction
	$J\cos 30° = 2.4\cos\theta$ or $J\cos 60° = 2.4\sin\theta - 1.5$	A1	One correct unsimplified equation
	Use of $J = m(v-u)$	M1	Use of $J = m(v-u)$ to form second equation
		A1	2nd correct unsimplified equation
	The first 4 marks are available for a correct equation in vector form.		$\begin{pmatrix} -2.4\cos\theta \\ 2.4\sin\theta \end{pmatrix} = \begin{pmatrix} -J\cos 30° \\ J\cos 60° + 1.5 \end{pmatrix}$
	$2.4^2 = \frac{3J^2}{4} + \frac{J^2}{4} + 1.5J + 1.5^2$ $\left(J^2 + 1.5J - 3.51 = 0\right)$	DM1	Form an equation in J only Dependent on previous two M1 marks
	$J = 1.3$	A1	1.3 or better (1.268….)
		(6)	
			See over for alternatives

8a			
	Use CLM: $4mu = 2mv + mw$	M1	Need all terms. Condone sign errors. Dimensionally correct but allow with m cancelled
	$(4u = 2v + w)$	A1	Correct unsimplified. Signs correct for their v, w
	Use Impact law	M1	Used the right way round. Condone sign errors.
	$w - v = 2ue$	A1	Correct unsimplified. Signs consistent with CLM equation.
	$\Rightarrow 4u = 2(w - 2ue) + w$	DM1	Solve for v or w. Dependent on previous 2 M marks
	$3w = 4u + 4ue, \quad w = \frac{4}{3}u(1+e)$ *	A1*	Obtain **given result** from correct working
	$v = \frac{2}{3}u(2-e)$	A1	Or equivalent. Must be positive
		(7)	
8b	$2 > e$ so A moving towards centre	B1	Correct statement about direction of travel for A or B
	$mw - 3mu = mx + 3my$ $y - x = e\left(u + \frac{4u}{3} + \frac{4eu}{3}\right)$	M1	Use CLM and impact law correctly to form simultaneous equations in x and y.
	$\frac{4}{3}eu - \frac{5}{3}u = x + 3y$ $3y - 3x = e(7u + 4ue)$	A1	Both equations correct unsimplified
	$4x = \frac{4}{3}ue - \frac{5}{3}u - 7ue - 4ue^2$	DM1	Solve for x
	$x = -\frac{5}{12}u - \frac{17}{12}ue - ue^2$	A1	
	$e > 0, u > 0$ so B moving towards centre from opposite direction, hence they collide.*	A1*	Obtain **given answer** from correct working
		(6)	
	Alternative for last 3 marks;		
	C moving towards centre implies B moving towards centre, so collision. C moving away from centre , so $y > 0$,	DM1	Consider direction of C

	$x = w - 3u - 3y = -\frac{8u}{3} + \frac{4eu}{3} - 3y$		
	$= -\frac{u}{3}(8 - 4e) - 3y$	A1	
	< 0 because $e \leq 1$ and $y > 0$ hence B moving towards centre from opposite direction, and they will collide.*	A1*	Obtain **given answer** from correct working
		[13]	

4	Use of $\mathbf{I} = m(\mathbf{v} - \mathbf{u})$	M1	As a single vector equation or two separate equations.
	$c\begin{pmatrix}-1\\2\end{pmatrix} = \frac{3}{4}\mathbf{v} - \begin{pmatrix}3\\0\end{pmatrix}$ $\left(\mathbf{v} = \frac{4}{3}\begin{pmatrix}3-c\\2c\end{pmatrix}\right)$	A1	Any equivalent substituted form
	Use of Pythagoras	M1	
	$64 = \frac{16}{9}\left((3-c)^2 + 4c^2\right)$	A1	Correct unsimplified equation in c or a component of $\mathbf{v}$. $\left(5a^2 - 32a = 0 \text{ or } 5b^2 - 16b - 192 = 0\right)$
	Simplify to 3 term quadratic and solve for c	M1	$5c^2 - 6c - 27 = 0$
	$c = 3 \quad \text{or} \quad c = -\frac{9}{5}(-1.8)$	A1	Correct only
		(6)	
4 Alt		M1 A1	Form vector triangle. Dimensionally correct Three correct lengths and $\lvert\cos\theta\rvert = \frac{1}{\sqrt{5}}$ seen or implied
	Use of cosine rule	M1	
	$36 = 9 + 5c^2 - 2\times 3\sqrt{5}c\cos\theta$	A1	Correct unsimplified equation in c with $\cos\theta$ or their $\cos\theta$
	Rearrange as 3 term quadratic and solve for c.	M1	$5c^2 - 6c - 27 = 0$
	$c = 3 \quad \text{or} \quad c = -\frac{9}{5}(-1.8)$	A1	Correct only
		(6)	
		[6]	

6a			
	$5mv = 2m\left(v-\left(-x\right)\right)$	M1	Use of $I = mv - mu$
	$x = \frac{3v}{2}$	A1	Seen or implied
	$5mv = 3m\left(v-\left(-y\right)\right)$ or $2mx - 3my = 3mv - 2mv$	M1	Use of $I = mv - mu$ or use of CLM
	$y = \frac{2v}{3}$	A1	Seen or implied
	$2v = e\left(\frac{3v}{2} + \frac{2v}{3}\right)$	M1	Correct use of impact law (not necessarily with values in terms of v) **Allow $v - v$ on LHS**
	$e = \frac{12}{13}$	A1	0.92 or better
		(6)	
6b	Speed of B after collision with wall $= vf$	B1	Seen or implied
	$2\times\frac{1}{2}\times 3m\left(y^2-\left(vf\right)^2\right) = \frac{1}{2}\times 2m\left(x^2-v^2\right)$	M1	Use KE to form an equation in f. Condone use of change in KE rather than loss **Condone 2 on wrong side**
	$3\left(\frac{4}{9}-f^2\right) = \left(\frac{9}{4}-1\right)$	A1	Correct unsimplified equation for f
	$\left(f^2 = \frac{1}{36}\right)\quad f = \frac{1}{6}$	A1	cao NB: $\frac{\sqrt{31}}{6}$ comes from inconsistent subtraction.
		(4)	
		[10]	

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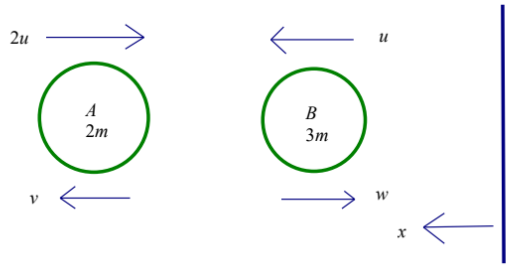
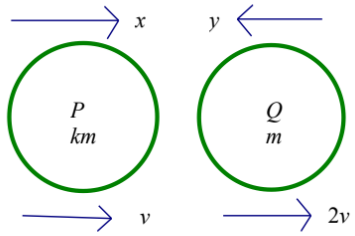
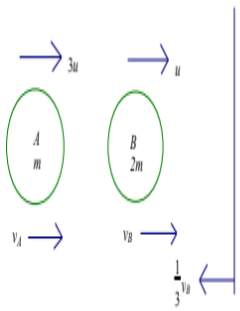
1a	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	M1	Condone subtraction in the wrong order.
	$\begin{pmatrix} -4 \\ 6 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} x-2 \\ y-4 \end{pmatrix}$	A1	Correct unsimplified equation Any equivalent form. Allow with **v**
	$\mathbf{v} = -6\mathbf{i} + 16\mathbf{j}\left(\text{m s}^{-1}\right)$	A1	Correct only. Seen or implied SR: Allow 3/3 if stop at $\mathbf{v} = 6\mathbf{i} - 16\mathbf{j}\left(\text{m s}^{-1}\right)$
	$\lvert\mathbf{v}\rvert = \sqrt{(-6)^2 + 16^2}$	M1	Correct use of Pythagoras with their **v**
	$= \sqrt{292}\left(= 2\sqrt{73}\right)\left(\text{m s}^{-1}\right)$	A1	Correct simplified value. 17 or better (17.088……)
			Allow 5/5 if working from the negative of the velocity.
		[5]	
1b	Correct use of trigonometry to find 2 relevant angles - as values or in inverse tangent form	M1	For their **v** e.g. $\pm 69.44..°, 63.43..°$ or $\pm 1.212..., 0.4636..$
	$\theta = \left(180° - \tan^{-1}\frac{16}{6}\right) - \tan^{-1}\frac{4}{2}$	A1ft	Correct unsimplified expression for θ Any equivalent form
	$= 47°$	A1	$47°$ or better (47.121..) $312.9°$ Accept radians (0.8224..)
		[3]	
1b alt	Use of scalar product with two relevant vectors	M1	For their **v**
	$\theta = \cos^{-1}\left(\frac{-12+64}{\sqrt{20}\sqrt{292}}\right)$	A1ft	Correct unsimplified expression for $\cos\theta$ or equivalent
	$= 47°$	A1	$47°$ or better (47.121..) $312.9°$ Accept radians (0.8224..)
		[3]	
		(8)	

4a			
	Use of CLM	M1	Need all terms, dimensionally correct. Condone sign errors.
	$4mu-3mu=3mw-2mv$ $(u=3w-2v)$	A1	Correct unsimplified equation
	Use of impact law	M1	Used correctly. Condone sign errors
	$v+w=3eu$	A1	Correct unsimplified equation. Signs consistent with their CLM equation
	$\begin{cases} u=3w-2v \\ 6eu=2w+2v \end{cases}$	DM1	Dependent on both preceding M marks. Solve to find speed of B.
	$\Rightarrow 5w=u+6eu, \quad w=\frac{1}{5}u(1+6e)$ *	A1*	Obtain **given answer** from correct working
		[6]	
4b	$v=3eu-w=\frac{u}{5}(9e-1)$	B1	Check their diagram / directions and allow $v=\frac{u}{5}(1-9e)$ if correct for their working. Any equivalent form. Must be seen or used in (b)
	$x=\frac{u}{7}(1+6e)$	B1	Seen or implied. Accept $\pm$
	Second collision if $\frac{u}{7}(1+6e)>\frac{u}{5}(9e-1)$	M1	Correct inequality to find the upper limit for e, using their v and x
	$(0<)e<\frac{4}{11}$	A1	Final answer. Or equivalent Do not need to mention the lower limit, but if they do it must be stated correctly (strict inequality).
		[4]	

3			**Allow column vectors**
	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	M1	Must be subtracting
	$(\mathbf{I} =) \pm 0.5\left((4-\lambda)\mathbf{i} + (-\lambda)\mathbf{j}\right)$	A1	Accept $\pm$ correct unsimplified expression on right hand side. (Ignore the left hand side) Allow $2\mathbf{i} - \frac{\lambda}{2}(\mathbf{i}+\mathbf{j})$ or equivalent
	Use of magnitude to form an equation in one variable	M1	Correct use of Pythagoras
	$\frac{5}{2} = \frac{1}{4}\left((4-\lambda)^2 + (-\lambda)^2\right)$	A1ft	Follow their **I**
	$0 = 2\lambda^2 - 8\lambda + 6 \quad \left(= (2\lambda - 6)(\lambda - 1)\right)$	DM1	Form a 3 term quadratic (seen or implied). Not necessarily stated "= 0" From $\mathbf{I} = a\mathbf{i} + b\mathbf{j}$ can obtain $4a^2 - 8a + 3 = 0$ or $4b^2 + 8b + 3 = 0$ Dependent on the preceding M1 **Solving not required for the M1.**
	$\lambda = 3$ and $\lambda = 1$	A1cso	**From correct solution only.** Do not need to see method of solution.
		[6]	
3alt	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ to form a vector triangle	M1	
	Triangle with sides of length $\sqrt{\frac{5}{2}}$, $\lvert 2\mathbf{i}\rvert$ and $\left\lvert \frac{\lambda}{2}(\mathbf{i}+\mathbf{j}) \right\rvert$	A1	
	Use of cosine rule with $45°\left(\frac{\pi}{4}\right)$	M1	
	$\frac{5}{2} = 2^2 + \left(\frac{\lambda}{2}\right)^2 \times 2 - 2 \times 2 \times \frac{\lambda}{2}\sqrt{2}\cos 45°$	A1ft	Correct unsimplified equation Follow their magnitudes
	$0 = \lambda^2 - 4\lambda + 3 \quad \left(= (\lambda - 3)(\lambda - 1)\right)$	DM1	Form a 3 term quadratic (seen or implied) Dependent on the preceding M1
	$\lambda = 3$ and $\lambda = 1$	A1	Correct solution only
		[6]	
		(6)	

6a			
	They need to form three equations, one of which must be the impact law. Mark them as you see them, so the first M1A1 on epen is available for the first equation seen, the second M1A1is for the second equation seen etc. If there are more than 3 equations, mark this as multiple attempts and all the marks for the equations actually used in the solution. **Treat the second and third A marks as follow through marks if they are substituting values they have already found.**		
	Use of $I = mv - mu$ for P or Q	M1	Dimensionally correct. Need all terms. M0 if m is missing on RHS
	$5mv = m\left(2v - \left(-y\right)\right)$ or $-5mv = km\left(v - x\right)$	A1	Correct unsimplified equation
	Use of CLM or second use of $I = mv - mu$	M1	Dimensionally correct. Need all terms. In CLM allow cancelled m and extra common factor (eg g) throughout
	$kmx - my = kmv + 2mv$ $(kx - y = kv + 2v)$ or $-5mv = km\left(v - x\right)$	A1	Correct unsimplified equation
	Use of impact law	M1	Must be used with e on the correct side. Condone sign errors
	$2v - v = \frac{1}{5}\left(x + y\right)$	A1	Correct unsimplified equation
	$y = 3v$	A1	cao
	$x = 2v$	A1	cao
	$k = 5$	A1	cao
		[9]	
6b	KE lost	M1	Dimensionally correct. Accept change in KE. Not scored until they form the complete substituted equation.
	$= \frac{1}{2} \times km\left(x^2 - v^2\right) + \frac{1}{2} \times m\left(y^2 - 4v^2\right)$ $\left(= \frac{15}{2}mv^2 + \frac{5}{2}mv^2\right)$	A1ft	Correct unsimplified expression. Follow their x, y, k Condone sign change without explanation. $\left(\text{KE before} = 14.5mv^2\right.$ $\left.\text{KE after} = 4.5mv^2\right)$
	$= 10mv^2$	A1	Only
		[3]	
		(12)	

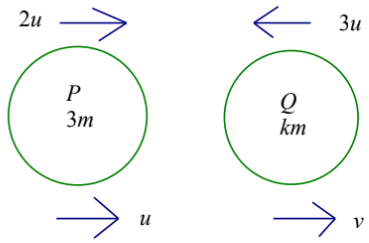
Q	Mark Scheme	Marks	Marking guidance
3.	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	M1	Accept equivalent e.g. $\mathbf{I} + m\mathbf{u} = m\mathbf{v}$. Dimensionally correct and must be using subtraction (but could be the wrong way round). The use of 7 in place of the velocity in the impulsemomentum equation is M0 unless they recover. **See below**
	$0.2(\mathbf{v} - + = 4\mathbf{i}3\mathbf{j}) \quad \lambda(\mathbf{i} + \mathbf{j})$ $((x - + +4)\mathbf{i} \; (y3)\mathbf{j} = 5\lambda \; \lambda\mathbf{i} + 5\,\mathbf{j})$	A1	Correct unsimplified vector equation or pair of separate equations for the **i** and **j** components. Condone column vectors with **i** and **j** included in the components.
	Use of Pythagoras for the speed	M1	Correct use of Pythagoras and 49 for their speed
	$x^2 + = y^2 \; 49$	A1ft	Correct unsimplified equation for their x, y
	Form quadratic in x y, orλ and reach $\lambda = \ldots$	DM1	Dependent on both previous M marks. $x^2 + -(x \; 7)^2 = 49$ or $(y + 7)^2 + = y^2 \; 49$ or $(5\lambda + 4)^2 + (5\lambda - = 3)^2 \; 49$
	$\lambda = {}_5{}^3$—or $\lambda = - \; {}_5{}^4$	A1	Or equivalent
**	Special case: Candidates who use 7 as a vector can score a **maximum** of		This maximum of 3 marks is only available for those candidates who "recover".
**			So, **if all you see** is $\lambda \; \lambda\mathbf{i} + = -\mathbf{j}$ 1.4 0.2 4(
**			$\mathbf{i} - 3\mathbf{j})$ they score
**	M1A0M1A0 for $1.4^2 = +(\lambda \; 0.8)^2 + -(\lambda \; 0.6)^2$ or equivalent		M0M0M0
**	DM1A0 for forming and solving a quadratic in λ .		

7 (a)			Check their diagram but remember that the directions used in their equations might not be consistent with the diagram. In this case, ignore their diagram.
	Conservation of momentum	M1	Need all terms. Dimensionally correct. Condone sign errors Condone m missing throughout or g present throughout
	$3mu + 2mu = mv_A + 2mv_B$ $\left(5u = +v_A 2v_B\right)$	A1	Correct unsimplified equation. Allow with v_A negative
	Use of NEL	M1	Used the right way round. Condone sign errors
	$v_B - = v_A\ e\left(3u - u\right)$ $\left(2ue = -v_B v_A\right)$	A1	Correct unsimplified equation. Allow with v_A negative. Signs consistent between the two equations.
	Solve for v_A or v_B	DM1	Dependent on two previous M marks
	Obtain $v_B = \frac{5+2e}{3}u$ *	A1*	Obtain **given answer** from correct working
	Obtain $v_A = \frac{5-4e}{3}u$	A1	Or equivalent. v_A must be positive
		7	
7 (b)	Time for B to reach the wall $t_B = \frac{d}{2u}$	B1	Or equivalent seen or implied. Allow $\frac{d \times 3}{\left(5\,2+ e\,u\right)}$

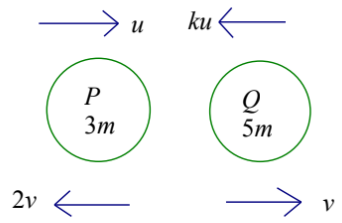
	Speed of B after impact with wall $= \frac{2}{3}u$	B1	Seen or implied. Allow $\left\|-\frac{1}{3}(5u + 2eu)\right\|$
	Distance travelled by A before B hits the wall d $= u \times \left(\frac{d}{2u}\right) = \left(\frac{2u}{2}\right)$	M1	Substitute $e = \frac{1}{2}$ and use their v_A and their t_b to find distance
	Time to close the gap	M1	Correct formula with their relevant speeds
	$d - 2ut = \left(5ut + \frac{3d}{2}\right)$ $\left(\frac{d}{2}\right) = ut + \frac{3}{2}t$ $t = \left(\frac{d}{2}\right) \div \left(\frac{3}{2}u + 10u\right)$	A1	Correct unsimplified equation (t x speed of approach)
	Total time $= \frac{d}{2u} + \frac{3d}{10u} = \frac{8d}{10u} = \frac{4d}{5u}$	A1	ISW Any equivalent form
		6	

7(b) alt	In time T, A travels x metres $x = uT$ B travels d metres in t sec $d = 2ut$ First B1 *B* travels $d - x$ metres in t' sec $d - x = 2ut'/3$ Second B1 and first M1 $t + t' = T$ $(d + 3d - 3uT)/2u = T$ Second M1 And first A1 $T = 4d/5u$ Second A1		Equivalent statement Correct value implied Distance travelled by *B* after it hits the wall Correct formula for time Correct unsimplified equation Correct answer
		(13)	

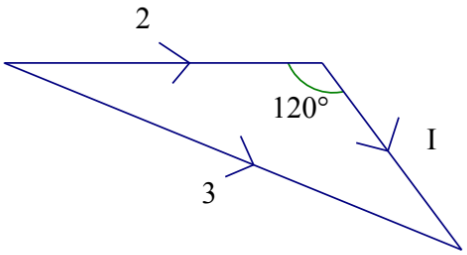
	———		
	——— –		
2	1st equation e.g. Equation for change in KE	M1	Dimensionally correct. Must be subtracting but condone sign error.
	$\frac{1}{2}\times 0.5\left(x^2+y^2-\left(5^2+3^2\right)\right)=22$ $\left(x^2+y^2=122\right)\ \left(1^2+\left(2\lambda+3\right)^2=122\right)$	A1	Correct unsimplified equation seen or implied (They might have used impulse-momentum first and done some work before substituting x and y.)
	2nd equation e.g. Impulse-momentum equation	M1	Dimensionally correct. Must be subtracting but condone sign error.
	$0.5\left(x\mathbf{i}+y\mathbf{j}\right)-0.5\left(5\mathbf{i}+3\mathbf{j}\right)=\left(-2\mathbf{i}+\lambda\mathbf{j}\right)$ $\left(\left(x-5\right)\mathbf{i}+\left(y-3\right)\mathbf{j}=-4\mathbf{i}+2\lambda\mathbf{j}\right)$	A1	Correct unsimplified equation
	NB: epen has M1A1A1 for the final 3 marks but this should be marked DM1DM1A1		
	Form a quadratic equation in λ	DM1	e.g. $1^2+\left(3+2\lambda\right)^2=122$ Dependent on the 2 preceding M marks
	Solve for 2 values of λ	DM1	e.g. solve $4\lambda^2+12\lambda-112=0$ or $\left(3+2\lambda\right)^2=121$ Dependent on the preceding M1
	$\Rightarrow \lambda=4$ or $\lambda=-7$	A1	Correct only and no errors seen (watch out for $x=-1$ used)
alt	Form a quadratic in y	DM1	e.g. $1+y^2=122$ $\left(y^2=121\right)$ Dependent on the 2 preceding M marks
	Solve for 2 values of y and use these to obtain 2 values of λ	DM1	Dependent on the preceding M1
	$\Rightarrow \lambda=4$ or $\lambda=-7$	A1	
		7	

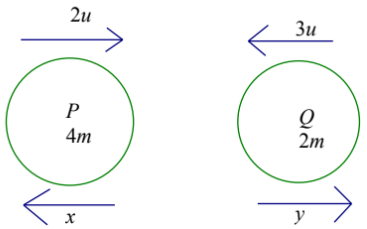
7a			
	Use CLM	M1	Need all terms and dimensionally correct. Condone sign errors. Might see them using equal (and opposite) impulses.
	$6mu-3kmu=3mu+kmv \quad \left((3-3k)u=kv\right)$	A1	Correct unsimplified equation
	$\Rightarrow v=\dfrac{(3-3k)}{k}u$ *	A1*	Obtain **given answer** from full and correct working
		3	
7b	Use of Impulse = change in momentum	M1	Must be subtracting. Can be for either particle.
	$\left\|I_Q\right\|=\left\|I_P\right\|=\left\|3mu-3m.2u\right\|=3mu$ or $\left\|kmv-(-3mku)\right\|=\left\|km.\dfrac{3-3k}{k}u+3mku\right\|=3mu$	A1	Correct only (Do not need to state that $\left\|I_Q\right\|=\left\|I_P\right\|$ if find $\left\|I_P\right\|$)
		2	
7c	Use impact law:	M1	Seen or implied. If stated in (a) must be used here. Must be used correctly but condone sign errors
	$\dfrac{v-u}{5u}=e$ or $\dfrac{3-3k}{k}u-u=5ue$	A1	Correct unsimplified equation
	NB: the second and third M mark are not dependent on the first M mark		
	Use $v>u$ or $e>0$ to form an inequality in k	M1	Could use $e\ldots0$ followed by $v\neq u$
	Use $e\geqslant 1$ to form an inequality in k	M1	
	$\dfrac{3-3k}{k}>1$ and $3-3k\geqslant 6k \quad \Rightarrow \dfrac{1}{3}\geqslant k<\dfrac{3}{4}$	A1	Correct answer only.
		5	
		(10)	

Question	Scheme	Marks	Notes
1a	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	M1	Must be subtracting but condone subtraction in wrong order
	$= 0.3\left(\left(7\mathbf{i} + 7\mathbf{j}\right) - 5\mathbf{i}\right) \quad \left(= 0.6\mathbf{i} + 2.1\mathbf{j}\right)$	A1	correct unsimplified equation Allow $\pm$
	$\left\|\mathbf{I}\right\| = \sqrt{0.6^2 + 2.1^2}$	M1	Use of Pythagoras
	$= \frac{3\sqrt{53}}{10}$	A1	2.2 or better (2.18403…)
		(4)	
1b	Correct method for a relevant angle	M1	e.g. use of trigonometry or scalar product for their **I** θ or 90 - θ
	Correct trig ratio for the required angle and no other angle involved.	A1	From correct **I** e.g. $\tan\theta = \frac{7}{2}$ or $\cos\theta = \frac{10}{\sqrt{53} \times 5}$
	$\theta = 74.1°$	A1	$74°$ or better $\left(74.0546..°\right)$ or 360 – 74 (286) (1.29… radians)
		(3)	

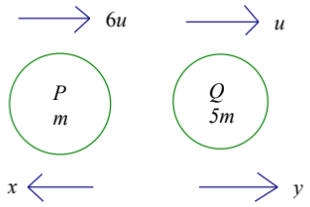
4a			
	Impulse-momentum equation for P:	M1	Correct use of $I = mv - mu$: Evidence of subtraction (can go straight to + you do not need to see $-(-)$) and dimensionally correct. Use of $3m$
	$15mv = 3m(2v-(-u))$	A1	Correct unsimplified equation
	$9mv = 3mu \Rightarrow u = 3v$ *	A1*	Obtain **given answer** from correct working
4a alt	Impulse-momentum equation for Q and CLM:	M1	CLM dimensionally consistent, all 4 terms, condone sign error(s). Correct use of $I = mv - mu$: Evidence of subtraction and dimensionally correct. Use of $5m$
	$15mv = 5m(v+ku)$, $k = 2\frac{v}{u}$ and substitute into CLM: $3mu - 5m\frac{2v}{u}u = 5mv - 6mv$	A1	Correct unsimplified equation in u and v
	$\Rightarrow u = 3v$ *	A1*	Obtain **given answer** from correct working
		(3)	
4b	Impulse-momentum equation for Q or use of CLM:	M1	Dimensionally consistent. All relevant terms.
	$15mv = 5m(v-(-ku))$ or $3mu - 5mku = 5mv - 6mv$	A1	Correct unsimplified equation
	$10v = 5ku = 15kv \Rightarrow k = \frac{2}{3}$	A1	Correct only. Accept 0.67 or better
		(3)	
4c	Use of impact law:	M1	Must be used the right way round. Condone sign error(s)
	$2v + v = e(u+ku)$ $\left(= e \times 3v \times \frac{5}{3}\right)$	A1ft	Correct unsimplified equation. Follow their k.
	$\Rightarrow e = \frac{3}{5}$	A1	Correct only
		(3)	

4d	Change in KE	M1	Allow for gain rather than loss. Dimensionally correct. Need to use all 4 terms and to be using the correct values for mass.
	$\frac{1}{2}\times 3m\left(u^2-(2v)^2\right)+\frac{1}{2}\times 5m\left((ku)^2-v^2\right)$	A1	Correct unsimplified equation. Allow for gain rather than loss. A0 if an error occurs before they form a single expression
	$\left(\frac{1}{2}\times 3m\left(5v^2\right)+\frac{1}{2}\times 5m\left(3v^2\right)=15mv^2\right)$		NB: $15mv^2=\frac{5}{3}mu^2$
	$\lambda=15$	A1	Correct only. Accept $15mv^2$
		(3)	

Question	Scheme	Mark	Notes
3	Form impulse-momentum equation	M1	Dimensionally correct. Accept answers in "vector" form, or as separate components. Condone sine / cosine confusion.
	One correct equation	A1	e.g. one correct component of $\begin{pmatrix} I\cos 60° \\ I\sin 60° \end{pmatrix} = \frac{1}{4}\left[\begin{pmatrix} 12\cos\alpha \\ 12\sin\alpha \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \end{pmatrix}\right]$ or $\left(= \begin{pmatrix} 3\cos\alpha - 2 \\ 3\sin\alpha \end{pmatrix}\right)$ $\begin{pmatrix} I\cos 60° \\ I\sin 60° \end{pmatrix} = \frac{1}{4}\left[\begin{pmatrix} v_x \\ v_y \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \end{pmatrix}\right]$ $\left(= \begin{pmatrix} 3\cos\alpha - 2 \\ 3\sin\alpha \end{pmatrix}\right)$ if working parallel and perpendicular to the initial direction or one of $8\sin 60° = 12\cos(30° + \alpha)$ or $I = 0.25(12\sin(30° + \alpha) - 8\cos 60°)$ if working parallel and perpendicular to the impulse
	Form a second impulse-momentum equation	M1	
	correct second equation	A1	
	Complete method to solve for I	DM1	Dependent on the two preceding M marks. e.g. from $36 = (I+4)^2 + 3I^2 \quad (4I^2 + 8I - 20 = 0)$
	$I = \sqrt{6} - 1$ (or 1.45 or 1.4)	A1	
		[6]	
3 alt		**M1**	Use of $I = mv - mu$ to draw a vector triangle. Dimensionally consistent.
		A1	Correct diagram
	Form an equation in I	**M1**	e.g. by using cosine rule
	$4 + I^2 - 4I\cos 120° = 9$	**A1**	Correct unsimplified equation A correct cosine rule equation can imply the first M1A1 if no diagram seen
	Solve for I	**DM1**	Dependent on the 2 preceding M marks $I^2 + 2I - 5 = 0$
	$I = \sqrt{6} - 1$ (or 1.45 or 1.4)	**A1**	
		(6)	

Question	Scheme	Mark	Notes
7a			
	Equation for CLM	M1	Dimensionally correct. All terms required. Condone sign errors.
	$8mu-6mu=2my-4mx$ $(u=y-2x)$	A1	Correct unsimplified equation
	Equation for kinetic energy ($\frac{1}{2}$ or 2 must be used)	M1	Dimensionally correct. Correct masses paired with correct velocities. All terms required. No sign errors. Condone 2 on the wrong side.
	$2mx^2+my^2=\frac{1}{2}(2m\times4u^2+m\times9u^2)$ $(17u^2=4x^2+2y^2)$	A1	Correct unsimplified equation
	Solve for y: $17u^2=2y^2+(y-u)^2$ $\Rightarrow 3y^2-2yu-16u^2=0$	DM1	Some working must be shown to obtain the quadratic in y (and u). Dependent on the preceding M marks $((3y-8u)(y+2u)=0)$
	$\Rightarrow y=\frac{8}{3}u$ *	A1*	Obtain given answer from correct working
		[6]	
7b	Use of Impact Law: $x+y=e\times5u$	M1	Condone sign errors but must be used the right way round.
	$e=\dfrac{\frac{1}{2}\left(\frac{8}{3}u-u\right)+\frac{8}{3}u}{5u}$	A1	Correct unsimplified equation. $\left(x=\dfrac{5u}{6}\right)$
	$=\dfrac{7}{10}$	A1	Correct only
		[3]	
7c	Velocity of Q after impact $=f\times\dfrac{8}{3}u$	B1	Allow $\pm$
	No collision if $f\times\frac{8}{3}u\le\frac{5}{6}u$ i.e. speed of $P\ge$ speed of Q	M1	Correct inequality with their values Accept strict inequality. Dimensionally correct.
	$\Rightarrow 0<f\le\frac{5}{16}$	A1	Both ends required. $(0<f\le0.3125)$
		[3]	
7d	Use of $I=\pm2m\left(y-\left(-\dfrac{1}{4}y\right)\right)$	M1	Subtraction seen or implied with *their* $\frac{1}{4}y$ Requires correct mass Requires correct impact law
	$\lvert I\rvert=\dfrac{20}{3}mu$	A1	Or equivalent. Must be positive $6.7mu$ or better Condone $-\dfrac{20}{3}mu\rightarrow\dfrac{20}{3}mu$ with no explanation
		[2]	
		(14)	

2a	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	M1	NB: Column vectors are acceptable. Condone wrong order but must be subtracting. Condone 5 in place of 0.5.
	$2\mathbf{i} + 5\mathbf{j} = 0.5\left(\mathbf{v} - \left(3\mathbf{i} + \mathbf{j}\right)\right)$ $\left(\mathbf{v} = 7\mathbf{i} + 11\mathbf{j}\right)$	A1	Correct unsimplified equation Accept as a vector equation or as a pair of equations, one for each component. Accept alternative notations provided the meaning is clear.
	Use of Pythagoras	M1	For their **v** Independent M1 but they must have a **v**
	$\lvert v\rvert = \sqrt{121+49} = \sqrt{170}\ \left(\text{m s}^{-1}\right)$	A1	$13\left(\text{m s}^{-1}\right)$ or better. (13.038…….)
		[4]	
2b	Correct use of trigonometry e.g. $\theta = \tan^{-1}\frac{11}{7} - \tan^{-1}\frac{1}{3}$ $\left(= 57.5 - 18.4\right)$	M1	Condone subtraction in either order. Allow if **both** fractions are the other way up. Alternatives: scalar product $\theta = \cos^{-1}\left(\frac{21+11}{\sqrt{10}\sqrt{170}}\right)$ cosine rule $4\times 29 = 10 + 170 - 2\sqrt{10}\sqrt{170}\cos\theta$
	$\theta = 39.1$	A1	Accept ± 39 or better (39.0938…) 0.68(2) radians is M1A0 Accept $\pm\left(360 - 39\right) = \pm 321$ or better
		[2]	
		(6)	

7a			If $6u$ and u are in opposite directions, mark as a sign error.
	Use of CLM	M1	Need all 4 terms. Dimensionally consistent. Condone sign errors Condone x in the wrong direction
	$6mu+5mu=5my-mx$ $(11u=5y-x)$	A1	Correct unsimplified equation
	Use of impact law	M1	Used correctly. Dimensionally correct. Condone sign errors
	$x+y=5eu$	A1	Correct unsimplified equation. Signs consistent with their CLM equation
	Solve for x in terms of e and u: $6x=25eu-11u$ or solve for e in terms of y and u: $e=\frac{6y-11u}{5u}$	DM1	Dependent on the first 2 M marks. As far as $kx=..$ Dependent on the previous 2 M marks
	Use $x>0$ $(\Rightarrow y>\frac{11}{5}u)$: $25e>11$	DM1	Use correct inequality for their x
	$\frac{11}{25}<e(\text{„ }1)$	A1	Or equivalent. Condone if 1 not mentioned. Allow with <1. A0 if incorrect upper limit. cso
		[7]	
7b	$x=\frac{2}{3}u$ and $y=\frac{7}{3}u$	B1	Seen or implied
	Total KE lost $=\left(\frac{1}{2}m\times 36u^2+\frac{1}{2}5m\times u^2\right)$ $-\left(\frac{1}{2}m\times x^2+\frac{1}{2}5m\times y^2\right)$	M1	Complete expression. Dimensionally correct. Correct masses connected to correct speeds. Condone subtraction in the wrong order. Allow in x and y
	$=\left(\frac{1}{2}m\times 36u^2+\frac{1}{2}5m\times u^2\right)$ $-\left(\frac{1}{2}m\times\frac{4}{9}u^2+\frac{1}{2}5m\times\frac{49}{9}u^2\right)$	A1ft	Correct unsimplified expression in m and u. Follow their x, y with e substituted
	$=\frac{20}{3}mu^2$	A1	Or single term equivalent. Accept $6.7mu^2$ or better
		[4]	
7c	velocity of Q after collision with wall $=\pm fy$ $\left(=\pm f\times\frac{7}{3}u\right)$	B1ft	Follow their y (in terms of u)
	Second collision if $fy>x$ $\frac{7}{3}fu>\frac{2}{3}u$	DM1	Correct inequality for their x, y Dependent on the B1 and P moving away from the wall
	$\frac{2}{7}<f$ „ 1	A1	Correct only Need both limits
		[3]	
		(14)	
8a	Use symmetry to find time taken: $-7=7-gt$	M1	0r equivalent complete method using *suvat* to find the time taken e.g. find the time for vertical distance = 0

	$t = \frac{14}{g} \ (= 1.428...)$	A1	Correct value seen or implied
	Horizontal distance $= 4t$	DM1	Complete method using *suvat* to find the distance. Dependent on the preceding M1
	$= 5.71(\text{m})$ or $5.7(\text{m})$	A1	3 sf or 2 sf only $\frac{40}{7}$ scores A0 $\frac{56}{g}$ scores A0 (incorrect units)