

Chapter 3: Centre of Mass

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Figure 1

- (a) Find the distance of the centre of mass of the lamina
- (i) from OD ,
 - (ii) from OA .

(6)

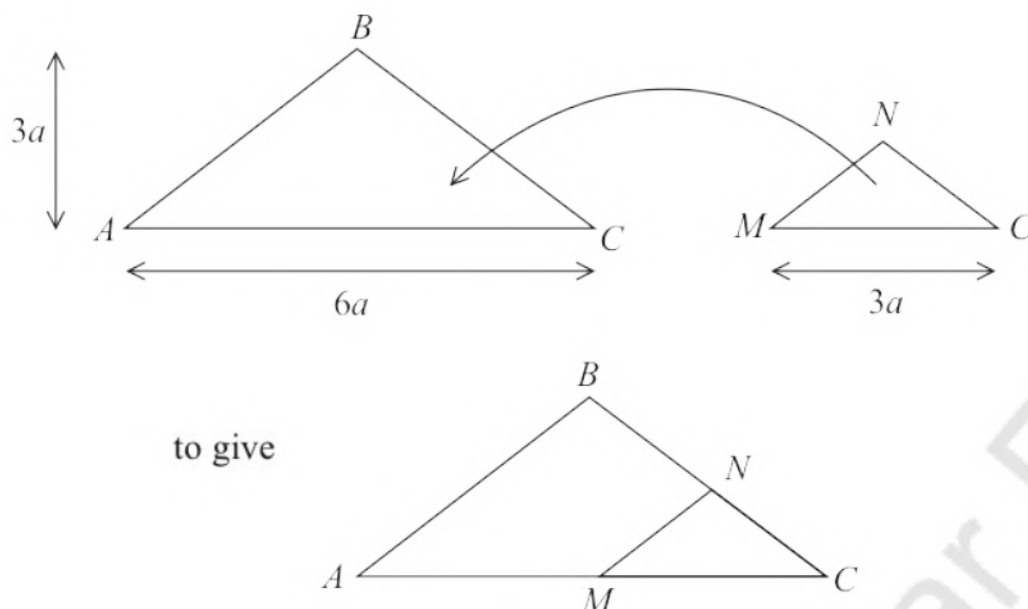
- (b) Find, to the nearest degree, the size of the angle α .

(4)

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**Figure 2**

The uniform lamina ABC is an isosceles triangle with $AB = BC$, $AC = 6a$ and the distance from B to AC is $3a$.

The uniform lamina MNC is an isosceles triangle with $MN = NC$ and $MC = 3a$. Triangles ABC and MNC are similar and are made of the same material.

The lamina L is formed by fixing triangle MNC on top of triangle ABC , as shown in Figure 2.

- (a) Show that the distance of the centre of mass of L from AC is $\frac{9}{10}a$ (5)

The lamina L is freely suspended from B and hangs in equilibrium.

- (b) Find, to the nearest degree, the size of the angle between AB and the downward vertical. (7)

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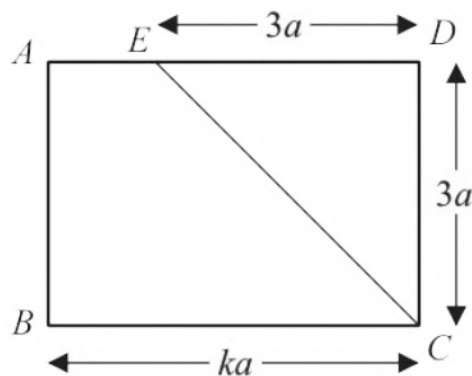


Figure 3

Figure 3 shows a uniform rectangular lamina $ABCD$ with sides of length $3a$ and ka , where $k > 3$. The point E on side AD is such that $DE = 3a$. Rectangle $ABCD$ is folded along the line CE to produce the folded lamina L shown in Figure 4.

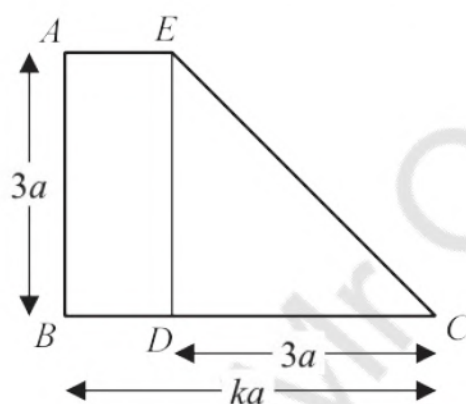


Figure 4

Find, in terms of a and k ,

(a) the distance of the centre of mass of L from AB ,

(5)

(b) the distance of the centre of mass of L from AE .

(4)

The folded lamina L is freely suspended from A and hangs in equilibrium with AB at 45° to the downward vertical.

(c) Find, to 3 significant figures, the value of k .

(4)

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- Given that the centre of mass of the three particles is at the point with coordinates $(k, 2k)$, where k is a constant, find the value of a .

(5)

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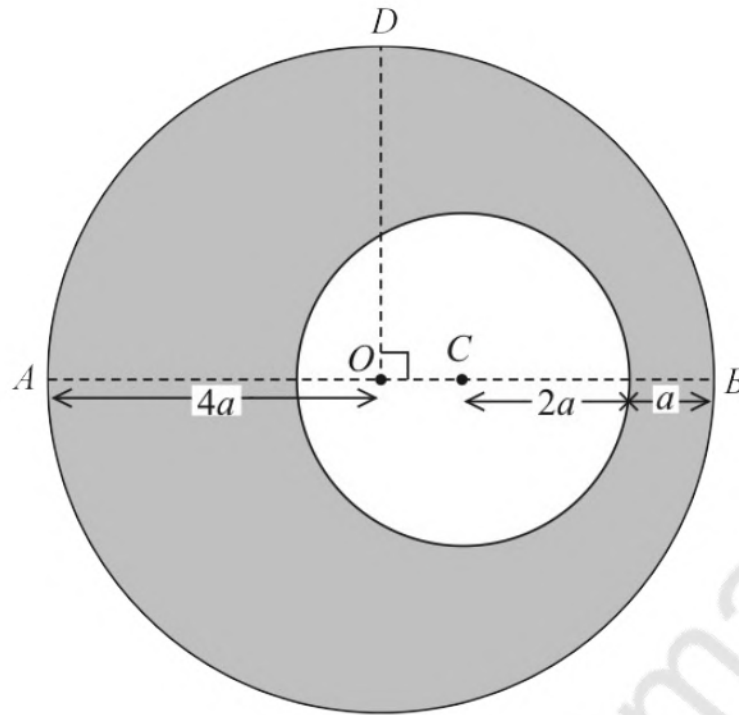


Figure 1

The uniform lamina L , shown shaded in Figure 1, is formed by removing a circular disc of radius $2a$ from a uniform circular disc of radius $4a$. The larger disc has centre O and diameter AB . The radius OD is perpendicular to AB . The smaller disc has centre C , where C is on AB and $BC = 3a$

- (a) Show that the centre of mass of L is $\frac{13}{3}a$ from B . (4)

The mass of L is M and a particle of mass kM is attached to L at B . When L , with the particle attached, is freely suspended from point D , it hangs in equilibrium with A higher than B and AB at an angle θ to the horizontal, where $\tan \theta = \frac{3}{4}$

- (b) Find the value of k . (5)

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[The centre of mass of a uniform semicircular lamina of radius r is $\frac{4r}{3\pi}$ from the centre.]

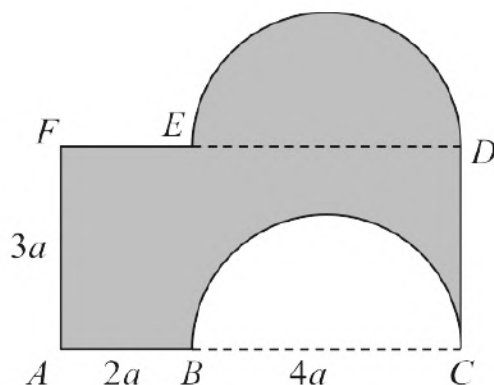


Figure 3

The uniform rectangular lamina $ABCDEF$ has sides $AC = FD = 6a$ and $AF = CD = 3a$. The point B lies on AC with $AB = 2a$ and the point E lies on FD with $FE = 2a$.

The template, T , shown shaded in Figure 3, is formed by removing the semicircular lamina with diameter BC from the rectangular lamina and then fixing this semicircular lamina to the opposite side, FD , of the rectangular lamina. The diameter of the semicircular lamina coincides with ED and the semicircular arc ED is outside the rectangle $ABCDEF$. All points of T lie in the same plane.

- (a) Show that the centre of mass of T is a distance $\left(\frac{9 + 2\pi}{6}\right)a$ from AC . (4)

The mass of T is M . A particle of mass kM is attached to T at C . The loaded template is freely suspended from A and hangs in equilibrium with AF at angle ϕ to the downward vertical through A .

Given that $\tan \phi = \frac{3}{2}$

- (b) find the value of k . (6)

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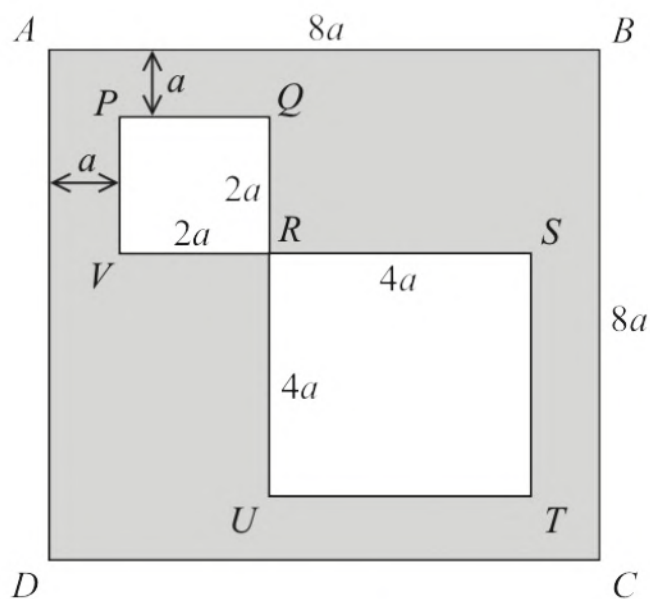


Figure 2

The uniform lamina L , shown shaded in Figure 2, is formed by removing the square $PQRV$, of side $2a$, and the square $RSTU$, of side $4a$, from a uniform square lamina $ABCD$, of side $8a$. The lines QRU and VRS are straight. The side AD is parallel to PV and the side AB is parallel to PQ . The distance between AD and PV is a and the distance between AB and PQ is a . The centre of mass of L is at the point G .

- (a) Show that the distance of G from the side AD is $\frac{42}{11}a$ (5)

The mass of L is M . A particle of mass kM is attached to L at C .

The lamina, with the attached particle, is freely suspended from B and hangs in equilibrium with BC making an angle of 45° with the horizontal.

- (b) Find the value of k . (4)

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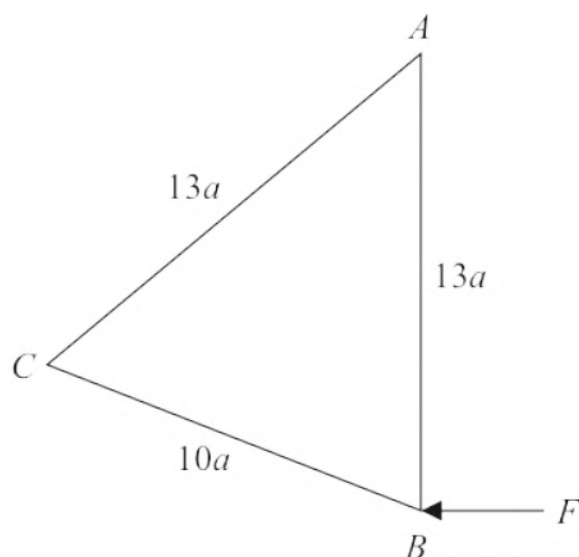


Figure 1

Find F in terms of W .

(5)

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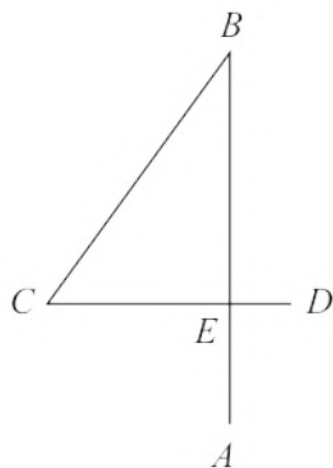


Figure 2

 $AB = 6a, BC = 5a$ and $CD = 4a$

The three rods are all made from the same material and they all lie in the same plane.

The framework is suspended from B and hangs in equilibrium with BA at an angle θ to the downward vertical.

Find θ to the nearest degree.

(9)

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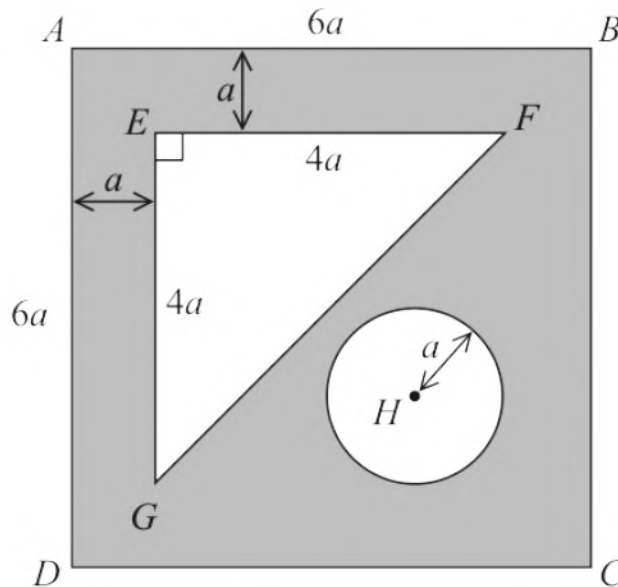


Figure 1

The uniform lamina $ABCD$ is a square of side $6a$. The template T , shown shaded in Figure 1, is formed by removing the right-angled triangle EFG and the circle, centre H and radius a , from the square lamina.

Triangle EFG has $EF = EG = 4a$, with EF parallel to AB and EG parallel to AD . The distance between AB and EF is a and the distance between AD and EG is a .

The point H lies on AC and the distance of H from BC is $2a$.

- (a) Show that the centre of mass of T is a distance $\frac{4(67 - 3\pi)}{3(28 - \pi)}a$ from AD . (5)

The template T is suspended from the ceiling by two light inextensible vertical strings. One string is attached to T at A and the other string is attached to T at B so that T hangs in equilibrium with AB horizontal.

The weight of T is W . The tension in the string attached to T at B is kW , where k is a constant.

- (b) Find the value of k , giving your answer to 2 decimal places. (3)

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7. In this question you may use, without proof, the formula for the centre of mass of a uniform sector of a circle, as given in the formulae book.

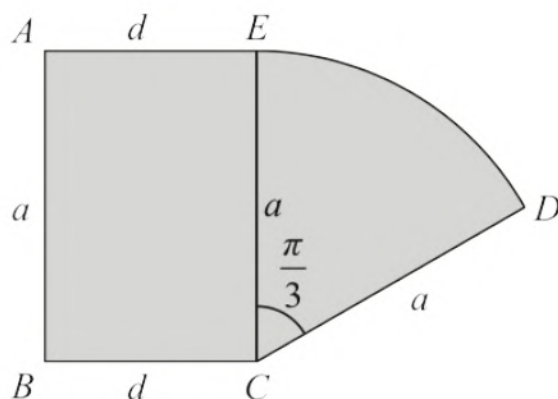


Figure 3

The uniform lamina $ABCDE$, shown shaded in Figure 3, is formed by joining a rectangle to a sector of a circle.

- The rectangle $ABCE$ has $AB = EC = a$ and $AE = BC = d$
- The sector CDE has centre C and radius a
- Angle $ECD = \frac{\pi}{3}$ radians

The centre of mass of the lamina lies on EC .

- (a) Show that $a = \sqrt{3}d$ (4)

The lamina is freely suspended from B and hangs in equilibrium with BC at an angle β radians to the downward vertical.

- (b) Find the value of β (7)

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6. [The centre of mass of a semicircular arc of radius r is $\frac{2r}{\pi}$ from the centre.]

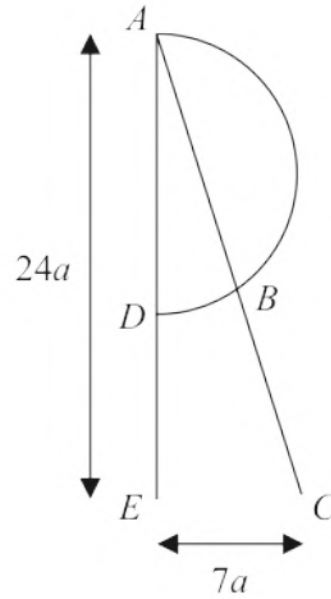


Figure 2

Uniform wire is used to form the framework shown in Figure 2.

In the framework,

- ABC is straight and has length $25a$
- ADE is straight and has length $24a$
- ABD is a semicircular arc of radius $7a$
- $EC = 7a$
- angle $AEC = 90^\circ$
- the points A, B, C, D and E all lie in the same plane

The distance of the centre of mass of the framework from AE is d .

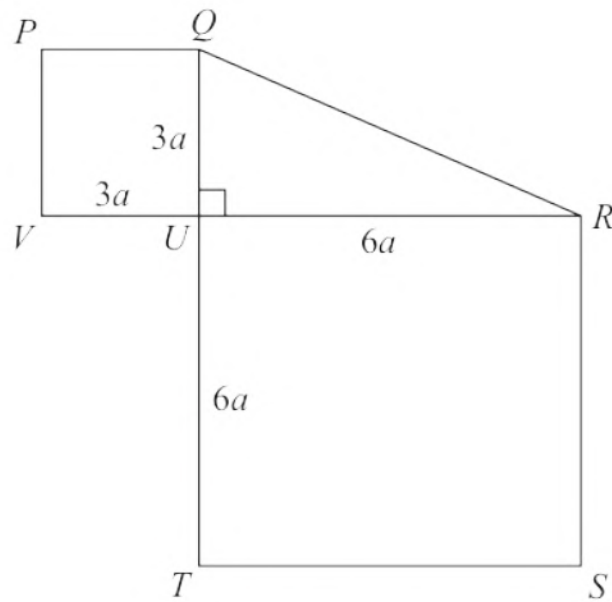
- (a) Show that $d = \frac{53}{2(7 + \pi)}a$ (4)

The framework is freely suspended from A and hangs in equilibrium with AC at angle α° to the downward vertical.

- (b) Find the value of α . (7)

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**Figure 3**

The template shown in Figure 3 is formed by joining together three separate laminae. All three laminae lie in the same plane.

- $PQUV$ is a uniform square lamina with sides of length $3a$
- $URST$ is a uniform square lamina with sides of length $6a$
- QRU is a uniform triangular lamina with $UQ = 3a$, $UR = 6a$ and angle $QUR = 90^\circ$

The mass per unit area of $PQUV$ is k , where k is a constant.

The mass per unit area of $URST$ is k .

The mass per unit area of QRU is $2k$.

The distance of the centre of mass of the template from QT is d .

- (a) Show that $d = \frac{29}{14}a$ (5)

The template is freely suspended from the point Q and hangs in equilibrium with QR at θ° to the downward vertical.

- (b) Find the value of θ (7)

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- (a) Show that $\bar{x} = \frac{2 + 12k}{9}$

(2)

The centre of mass of the three particles lies at a point on the straight line with equation $x + 2y = 3$

- (b) Find the value of k .

(4)

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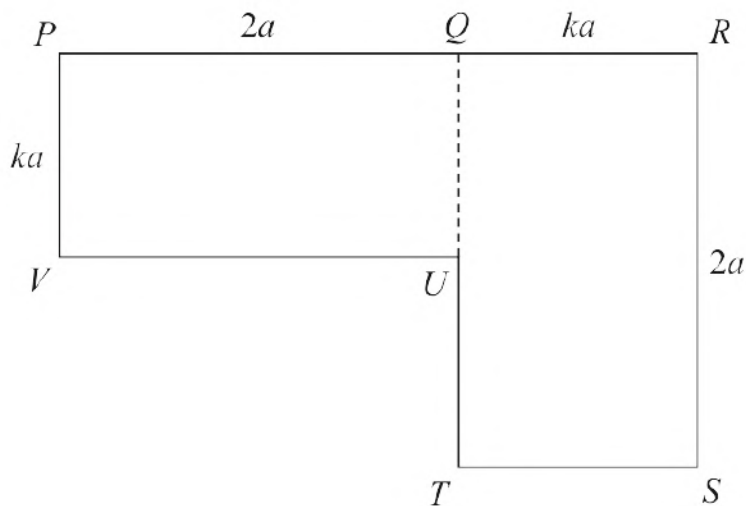


Figure 2

The rectangles have sides $PQ = RS = 2a$ and $PV = QR = ka$.

- (a) Show that the centre of mass of the lamina is $\left(\frac{6+k}{4}\right)a$ from PV (3)

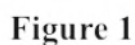
The lamina is freely suspended from P and hangs in equilibrium with PR at an angle of α to the downward vertical.

Given that $\tan \alpha = \frac{7}{15}$

- (b) find the value of k . (6)

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(a) Show that the centre of mass of the template is $\frac{14}{5}a$ from AE .

(5)

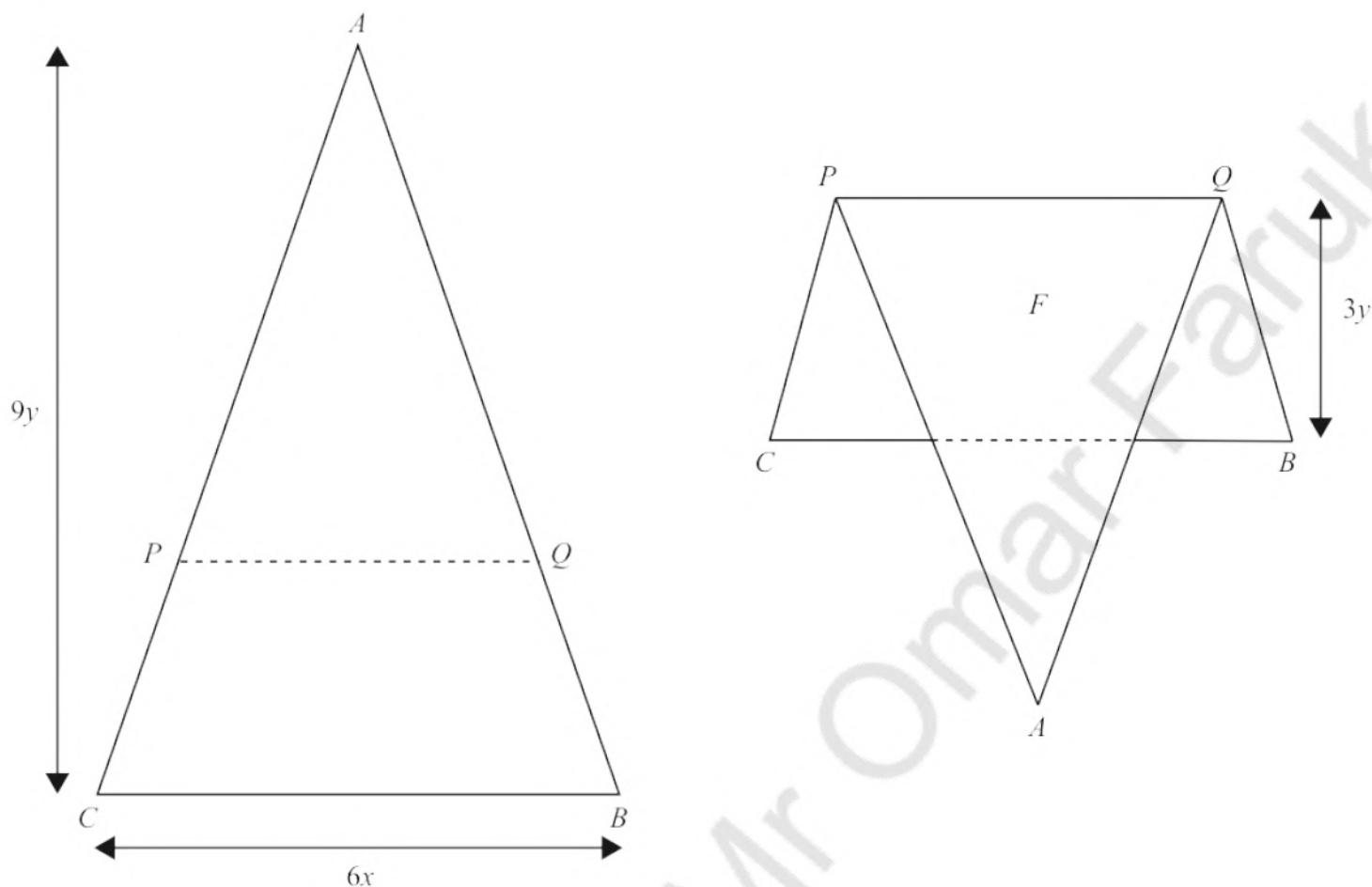
(b) Find the value of θ , giving your answer to the nearest whole number.

(3)

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**Figure 1**

The uniform triangular lamina ABC , shown in Figure 1, has height $9y$, base $BC = 6x$, and $AB = AC$

The points P and Q are such that $AP : PC = AQ : QB = 2 : 1$

The lamina is folded along PQ to form the folded lamina F

The distance of the centre of mass of F from PQ is d

(a) Show that $d = \frac{16}{9}y$ (5)

The folded lamina is suspended from P and hangs freely in equilibrium with PQ at an angle α to the downward vertical.

Given that $\tan \alpha = \frac{64}{81}$

(b) find x in terms of y (3)

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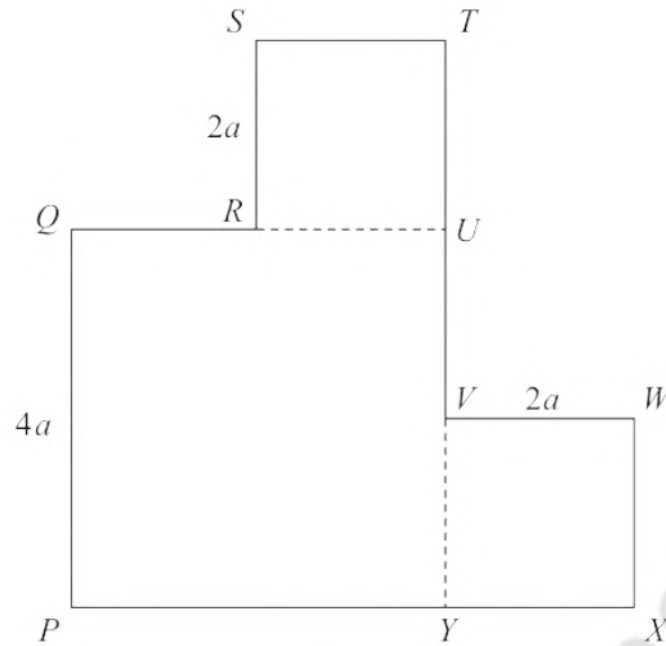


Figure 1

Figure 1 shows a template where

- $PQUY$ is a uniform square lamina with sides of length $4a$
- $RSTU$ is a uniform square lamina with sides of length $2a$
- $VWXY$ is a uniform square lamina with sides of length $2a$
- the three squares all lie in the same plane
- the mass per unit area of $VWXY$ is **double** the mass per unit area of $PQUY$
- the mass per unit area of $RSTU$ is **double** the mass per unit area of $PQUY$
- the distance of the centre of mass of the template from PX is d

(a) Show that $d = \frac{5}{2}a$ (5)

The template is freely pivoted about Q and hangs in equilibrium with PQ at an angle of θ to the downward vertical.

(b) Find the value of $\tan \theta$ (6)

The mass of the template is M

The template is still freely pivoted about Q , but it is now held in equilibrium, with PQ vertical, by a horizontal force of magnitude F which acts on the template at X . The line of action of the force lies in the same plane as the template.

(c) Find F in terms of M and g (3)

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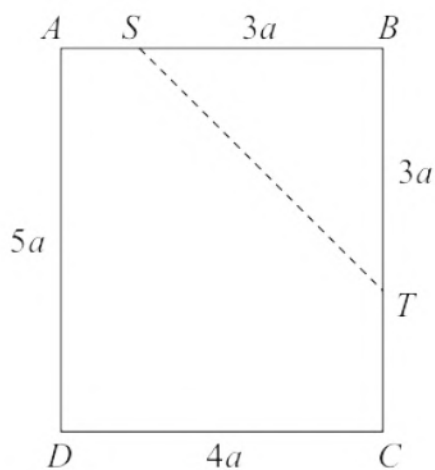


Figure 2

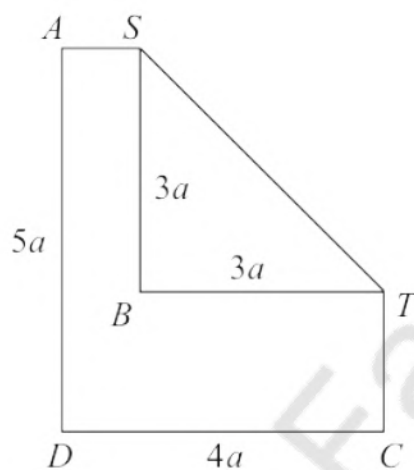


Figure 3

The uniform rectangular lamina $ABCD$, shown in Figure 2, has $DC = 4a$ and $AD = 5a$

The points S on AB and T on BC are such that $SB = BT = 3a$

The lamina is folded along ST to form the folded lamina L , shown in Figure 3.

The distance of the centre of mass of L from AD is d .

(a) Show that $d = \frac{71}{40}a$

(5)

The weight of L is $4W$. A particle of weight W is attached to L at C .

The folded lamina L is freely suspended from S .

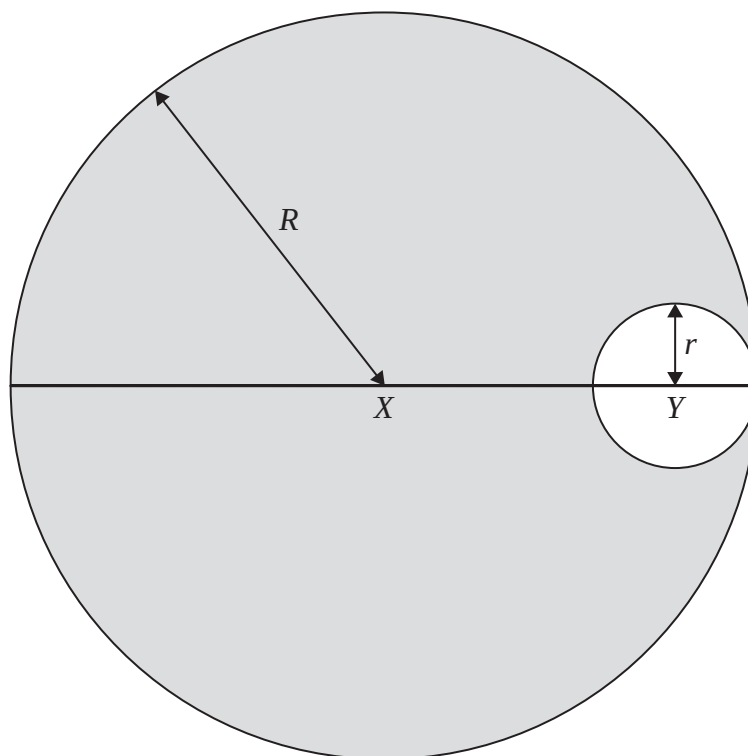
A force of magnitude F , acting parallel to DC , is applied to L at D so that AD is vertical.

(b) Find F in terms of W

(4)

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**Figure 1**

A uniform circular disc C has centre X and radius R .

A disc with centre Y and radius r , where $0 < r < R$ and $XY = R - r$, is removed from C to form the template shown shaded in Figure 1.

The centre of mass of the template is a distance kr from X .

(a) Show that $r = \frac{k}{1-k}R$ (4)

(b) Hence find the range of possible values of k . (2)

The point P is on the outer edge of the template and PX is perpendicular to XY .

The template is freely suspended from P and hangs in equilibrium.

Given that $k = \frac{4}{9}$

(c) find the angle that XY makes with the vertical. (3)

The mass of the template is M .

(d) Find, in terms of M , the mass of the lightest particle that could be attached to the template so that it would hang in equilibrium from P with XY horizontal. (3)

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