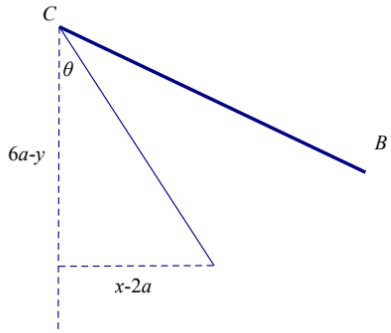
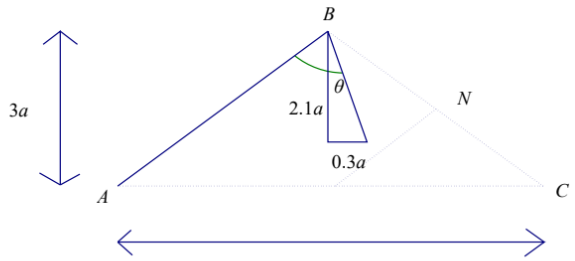
Q	Scheme	Marks	Notes
3(a)	$36a^2(6)$ | $6a^2(1)$ | $30a^2(5)$ $3a$ | $\frac{14a}{3}$ | $\bar{x}$ $3a$ | $5a$ | $\bar{y}$	B1	Mass ratios for a valid combination of shapes
		B1	Distance from *OD* or equivalent for a valid combination of shapes
		B1	Distance from *OA* or equivalent for a valid combination of shapes
	$6\times3a-1\times\frac{14a}{3}=5\bar{x}$ or $6\times3a-1\times5a=5\bar{y}$	M1	Moments equation for a horizontal or vertical axis or a vector equation combining both. For a valid dissection.
	$\Rightarrow\ \bar{x}=\frac{8a}{3}$	A1	One correct distance
	$\bar{y}=\frac{13a}{5}$	A1	Both distances correct
		(6)	
3(a) alt	$12a^2$ | $6a^2$ | $12a^2$ | $30a^2$ a | $\frac{10a}{3}$ | $4a$ | $\bar{x}$ $3a$ | $4a$ | $\frac{3a}{2}$ | $\bar{y}$		
3(b)			
	$\theta=\tan^{-1}\left(\frac{\bar{x}-2a}{6a-\bar{y}}\right)$	M1	Trig ratio of a relevant angle
	$\alpha=\tan^{-1}\left(\frac{4}{3}\right)-\tan^{-1}\left(\frac{\bar{x}-2a}{6a-\bar{y}}\right)$	DM1	expression for the required angle for their $\bar{x},\bar{y}$
	$\left(\text{or } \tan^{-1}\left(\frac{6a-\bar{y}}{\bar{x}-2a}\right)-\tan^{-1}\left(\frac{3}{4}\right)\right)$ $\left(\text{or } 90-\theta-\tan^{-1}\frac{3}{4}\right)$	A1ft	Correct unsimplified substituted expression
	$\alpha=42^\circ$	A1	The Q asks for the answer to the nearest degree.
		(4)	
			See over page

Q	Scheme	Marks	Notes
3(b) alt	Correct method for lengths of all three sides of triangle CBG	M1	$\left(\sqrt{\frac{2701}{225}}a,\ \sqrt{\frac{2536}{225}}a,\ 5a\right)$
	Correct use of cosine rule	M1	
	$\cos\alpha = \dfrac{25+\frac{2701}{225}-\frac{2536}{225}}{2\times5\times\sqrt{\frac{2701}{225}}}$	A1ft	Correct unsimplified for their values of the correct distances
	$\alpha = 42^{\circ}$	A1	The Q asks for the answer to the nearest degree.
		(4)	
		[10]	

Question Number	Scheme	Marks
4a	Area ratios: $9a^2 : \frac{9}{4}a^2 : 5\times\frac{9}{4}a^2 \quad (4:1:5)$	B1
	Distances from AC: $a, \frac{a}{2}(, \bar{y})$	B1
	Moments about AC: $4\times a + 1\times\frac{a}{2} = 5\bar{y}$	M1A1**ft**
	$\bar{y} = \frac{9}{10}a$ **(Given Answer)**	A1 **cso**
		(5)
4b	Distances from vertical through B: $0, \frac{3}{2}a(, \bar{x})$ OR:	B1
	Moments about vertical axis through B: $1\times\frac{3}{2}a = 5\bar{x}$	M1
	$\bar{x} = \frac{3}{10}a$ **N.B.** You may see $\bar{x} = 3.3a$ (if distances measured from A) or $\bar{x} = 2.7a$ (if distances measured from C)	A1
		
	Required angle $ABG: 45° + \theta$ (for their $\bar{x}$; e.g. if $\bar{x} < 0$, then $45° - \theta$)	M1
	Method for θ : $\tan\theta = \frac{\bar{x}}{3a - \frac{9a}{10}}\left(= \frac{1}{7}\right)$	M1A1**ft**
	$45° + \tan^{-1}\left(\frac{1}{7}\right) = 53°$ nearest degree	A1
		(7)
		[12]
	Notes for Qu4	
	4(a) First B1 for correct (unsimplified) area ratios seen Second B1 for correct distances from AC (or from any parallel line) First M1 for a 'moments' equation about AC (or any parallel line) First A1**ft** for a correct equation, ft on their 'distances' and 'masses'. Second A1 cso for a correct GIVEN ANSWER with no errors seen	
	N.B. **Allow equivalent vector equation for two moments equations.**	

Question Number	Scheme	Marks
	4(b) B1 for correct distances from vertical through B (or from any parallel line) (May be seen in (a)) Allow missing a's if they recover First M1 for 'moments' equation about vertical axis through B (or from any parallel line) (May be seen in (a)) First A1 for a correct $\bar{x}$ (allow recovery of missing a's) Second M1 for use of correct angle (e.g. $45°+\theta$ or $45°-\theta$ if using tan method) oe Third M1 for appropriate equation in θ only, using their $\bar{x}$ and $\frac{9a}{10}$ Second A1**ft** for a correct equation Third A1 for 53^o (nearest degree) **ALTERNATIVE**: B1 for correct distances from vertical through B (or from any parallel line) (May be seen in (a)) Allow missing a's if they recover First M1 for a dim correct moments equation about vertical axis through B (or from any parallel line) (May be seen in (a)) First A1 for a correct $\bar{x}$ (allow recovery of missing a's) Second M1 for identifying correct angle ($ABG = \alpha$) Third M1 for use of cos rule on triangle ABG to give equation in α only, using their $\bar{x}$ and $\frac{9a}{10}$ Second A1**ft** for a correct equation Third A1 for 53^o (nearest degree) **OR**: B1 for correct distances from vertical through B (or from any parallel line) (May be seen in (a)) Allow missing a's if they recover First M1 for a dim correct moments equation about vertical axis through B (or from any parallel line) (May be seen in (a)) First A1 for a correct $\bar{x}$ (allow recovery of missing a's) Second M1 for use of angle ABG = angle ABC - angle GBC Third M1 for use of cos rule on triangle GBC to find angle GBC, using their $\bar{x}$ and $\frac{9a}{10}$ Second A1**ft** for a correct equation Third A1 for 53^o (nearest degree)	

Q	Scheme	Marks	Notes
5a	<table><tr><td></td><td>area</td><td>From AB</td><td>From AE</td></tr><tr><td>rectangle</td><td>$3ka^2$</td><td>$\frac{1}{2}ka$</td><td>$\frac{3}{2}a$</td></tr><tr><td>Triangle -</td><td>$\frac{9}{2}a^2$</td><td>$ka-3a+2a$</td><td>a</td></tr><tr><td>Triangle +</td><td>$\frac{9}{2}a^2$</td><td>$ka-3a+a$</td><td>$2a$</td></tr><tr><td>L</td><td>$3ka^2$</td><td>$\bar{x}$</td><td>$\bar{y}$</td></tr></table>		
	mass ratio: $3k:\frac{9}{2}:\frac{9}{2}:3k$	B1	
	Horizontal distances:	B1	From AB or from a parallel axis
	Moments about AB: $3k\times\frac{ka}{2}-\frac{9}{2}\times(k-1)a+\frac{9}{2}\times(k-2)a=3k\bar{x}$	M1	Need all terms and dimensionally correct Accept on vector form
		A1	Correct unsimplified equation in $\bar{x}$
	$\frac{3k^2}{2}a-\frac{9}{2}a=3k\bar{x}$, $\bar{x}=\frac{(k^2-3)}{2k}a$	A1	Or equivalent
		(5)	
5b	Vertical distances	B1	From AE or from a parallel axus
	Moments about AE: $3k\times\frac{3a}{2}-\frac{9}{2}\times a+\frac{9}{2}\times 2a=3k\bar{y}$	M1	Need all terms and dimensionally correct
		A1	Correct unsimplified equation in $\bar{y}$. Distance from $BD=\frac{3a}{2k}(k-1)$
	$\frac{9ka}{2}+\frac{9a}{2}=3k\bar{y}$, $\bar{y}=\frac{3(k+1)a}{2k}$	A1	Or equivalent
		(4)	
	If there is a triangle "missing", the total marks available for the question are B0B1M1A0A0 B1M1A0A0 M1M1M1A0 (7/13)		
			See over for alternative working

Q	Scheme	Marks	Notes
	Alternative working for parts (a) and (b)		
	<table><tr><td></td><td>mass</td><td>From AB</td><td>From AE</td></tr><tr><td>rectangle</td><td>$3(k-3)a^2$</td><td>$\frac{1}{2}(k-3)a$</td><td>$\frac{3}{2}a$</td></tr><tr><td>Triangle</td><td>$2\times\frac{9}{2}a^2$</td><td>$(k-3)a+a$</td><td>$2a$</td></tr><tr><td>L</td><td>$3ka^2$</td><td>$\bar{x}$</td><td>$\bar{y}$</td></tr></table>		
	Moments about AB: $3(k-3)\times\frac{(k-3)a}{2}+9\times(k-2)a=3k\bar{x}$		
	Moments about AE: $3(k-3)\times\frac{3a}{2}+9\times 2a=3k\bar{y}$		
5c	$\bar{x}=\bar{y}$: $(k^2-3)=3(k+1)$ for their values	M1	
	Simplify to 3 term quadratic in k: $k^2-3k-6=0$	DM1	Dependent on preceding M1
	Solve for k: $k=\frac{3\pm\sqrt{9+24}}{2}$	DM1	Dependent on preceding M1
	$k=4.37$ only	A1	The Q asks for 3 sf
		(4)	
		[13]	

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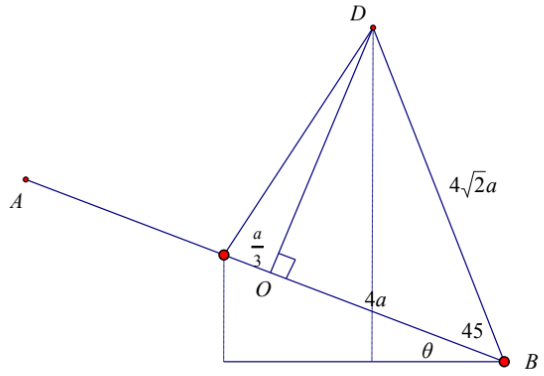
Q	Scheme	Marks	Notes
1	Moments about x-axis (or parallel axis)	M1	Require all terms. Dimensionally correct. Condone sign errors
	$3m\times 8+m\times 0+2m\times -2=6m\times 2k$	A1	Correct unsimplified equation
	Moments about y-axis (or parallel axis)	M1	Require all terms. Dimensionally correct. Condone sign errors
	$3m\times a+m\times -4+2m\times 5=6m\times k$	A1	Correct unsimplified equation
	Eliminate k and solve for a: $6a+12=20$, $a=\frac{4}{3}$	A1	Or equivalent. 1.3 or better
		(5)	
1alt	Form vector equation in k and a	M1	Require all terms. Dimensionally correct. Condone sign errors
	$3m\begin{pmatrix}a\\8\end{pmatrix}+m\begin{pmatrix}-4\\0\end{pmatrix}+2m\begin{pmatrix}5\\-2\end{pmatrix}=6m\begin{pmatrix}k\\2k\end{pmatrix}$	A1	Or equivalent. Correct unsimplified equation
	Use components from dimensionally correct equation to form two separate equations	M1	Seen or implied Condone processing errors
	$\Rightarrow 3a+6=6k$ $20=12k$	A1	Pair of correct unsimplified equations
	Eliminate k and solve for a: $6a+12=20$, $a=\frac{4}{3}$	A1	1.3 or better
		(5)	
		[5]	

Q	Scheme	Marks	Notes
4a	<table><tr><td></td><td>large</td><td>small</td><td>L</td></tr><tr><td>Mass ratio</td><td>16π</td><td>4π</td><td>12π</td></tr><tr><td>c of m from B</td><td>$4a$</td><td>$3a$</td><td>x</td></tr></table>	B1	Correct ratios and distances
	Moments about B: $16\pi\times 4a-4\pi\times 3a=12\pi\times x$	M1	Or equivalent from another point. Require all terms. Condone sign errors. Dimensionally correct.
		A1	Correct unsimplified
	$x=\dfrac{64a-12a}{12}=\dfrac{52a}{12}=\dfrac{13}{3}a$ ***Given Answer***	A1	Obtain **given answer** from correct working
		(4)	
4b			For all alternative moments equations the distances must relate to the stated point
	Moments about B: $\dfrac{13}{3}a\cos\theta\times M=4\sqrt{2}a\cos(45+\theta)\times M(1+k)$	M1	Require all terms. Dimensionally correct. Condone sign errors
		A1	Unsimplified equation with at most one error
		A1	Correct unsimplified
	Substitute for trig and solve: $\dfrac{13}{3}\times\dfrac{4}{5}=(1+k)\times 4\sqrt{2}\left(\dfrac{1}{\sqrt{2}}\times\dfrac{4}{5}-\dfrac{1}{\sqrt{2}}\times\dfrac{3}{5}\right)$	DM1	
	$\dfrac{13}{3}=1+k$, $k=\dfrac{10}{3}$	A1	(3.3 or better)
		[5]	
4b alt	Moments about B:	M1	Require all terms. Dimensionally correct. Condone sign errors
	$\dfrac{13}{3}a\cos\theta\times M=a\cos\theta\times M(1+k)$	A1	Unsimplified equation with at most one error
		A1	Correct unsimplified
	(Substitute for trig and) solve for k: $\dfrac{13}{3}=1+k$,	DM1	
	$k=\dfrac{10}{3}$	A1	(3.3 or better)

Q	Scheme	Marks	Notes
4b alt	Moments about O:	M1	Require all terms. Dimensionally correct. Condone sign errors
	$-\frac{1}{3}a\times M+kM\times 4a=(kM+M)\bar{x}$	A1	Unsimplified equation, at most one error
	$(k+1)\bar{x}=\left(4k-\frac{1}{3}\right)a$	A1	Correct unsimplified
	$\frac{\bar{x}}{OD}=\frac{3}{4}$	DM1	
	$k=\frac{10}{3}$ (3.3 or better)	A1	
4b alt	New c of m at G where $\frac{OG}{OD}=\tan\theta$, $OG=3a$	M1	
	Moments about G: $Mg\left(3a+\frac{a}{3}\right)=kMg(4a-3a)$	DM1	Require all terms. Dimensionally correct. Condone sign errors
		A1	Unsimplified equation, at most one error
		A1	Correct unsimplified
	$k=\frac{10}{3}$ (3.3 or better)	A1	
4b alt	New c of m at G where $\frac{OG}{OD}=\tan\theta$, $OG=3a$	M1	
	Moments about D: $Mg\left(3a+\frac{a}{3}\right)\cos\theta=kMg(4a-3a)\cos\theta$	DM1	Require all terms. Dimensionally correct. Condone sign errors
		A1	Unsimplified equation, at most one error
		A1	Correct unsimplified
	$k=\frac{10}{3}$ (3.3 or better)	A1	
4b alt	Moments about G: $M\left(\frac{a}{3}+OG\right)=kM(4a-OG)$	M1	Require all terms. Dimensionally correct. Condone sign errors
		A1	Unsimplified equation, at most one error
	$OG(1+k)=a\left(4k-\frac{1}{3}\right)$, $\left(OG=\frac{a(12k-1)}{3(k+1)}\right)$	A1	Correct unsimplified
	$\frac{OG}{OD}=\frac{3}{4}$,	DM1	
	$k=\frac{10}{3}$	A1	
	Other alternatives:		Moments equation: M1A1A1
			Use angle and solve for k: M1A1
		[9]	

Question Number	Scheme	Marks
4(a)	Moments about AC	M1
	$18\times\frac{3a}{2}-2\pi\times\frac{8a}{3\pi}+2\pi\left(3a+\frac{8a}{3\pi}\right)=18\bar{y}$	A1
	NB: valid to use $18\times\frac{3a}{2}-2\pi\times d+2\pi\left(3a+d\right)=18\bar{y}$ for $d\neq 0$ without stating value for d Use of $d=0\Rightarrow$ M0	
	$\left(27a+6\pi a=18\bar{y}\right)$	A1
	The same incorrect distance used twice in place of $\frac{8a}{3\pi}$ is one error The same incorrect area for the semicircle used twice is one error.	A1 (4)
	$27a+6\pi a=18\bar{y}\ \Rightarrow\bar{y}=\frac{9+2\pi}{6}a$ *	
4b	$M\bar{x}+kM\times 6a=\left(1+k\right)M\bar{x}_T$	M1
	$3a+6ak=\left(1+k\right)\bar{x}_T$ o.e.	A1
	$M\bar{y}=\left(1+k\right)M\bar{y}_T$	M1
	$\left(1+k\right)\bar{y}_T=\frac{9+2\pi}{6}a$	A1
NB	For their second equation they could use $\tan\phi$ and their $\bar{x}_T$ or $\bar{y}_T$ to form an expression for $\bar{y}_T$ or $\bar{x}_T$	
	$\tan\phi=\frac{3}{2}=\frac{\bar{x}_T}{\bar{y}_T}\ \Rightarrow\ \frac{3}{2}=\frac{6\left(3a+6ak\right)}{\left(9+2\pi\right)a}$	DM1
	$\Rightarrow k=\frac{\pi}{12}-\frac{1}{8}$ or equivalent	A1 (6)

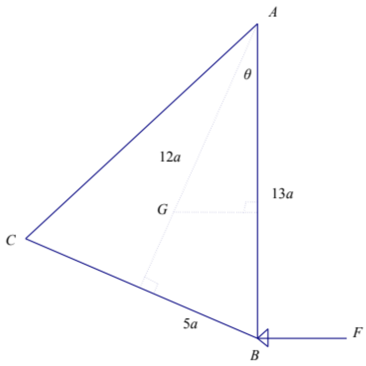
Notes

M1	All terms. Dimensionally correct. Condone sign errors
A1	Unsimplified equation with at most one error.
A1	Correct unsimplified equation
A1	Obtain **given answer** from sufficient correct exact working. Must see a separate conclusion for $\bar{y}$.
NB	e.g. $\bar{y}_T=\frac{2\left(3+6k\right)a}{3\left(1+k\right)}$
DM1	Form equation in k and solve for k.Dependent on the previous 2 M marks.
A1	$k=0.137\ \ (0.14)$ or better See over for alternative solution to 4(b)

Question Number	Scheme	Marks
4(b) alt	Distance of original c of m from vertical through A	M1
	$\left(\frac{9+2\pi}{6}a-2a\right)\times\sin\phi\left(=\frac{\sqrt{13}(2\pi-3)a}{26}\right)$	A1
	Distance of additional particle from vertical through A	M1
	$6a\times\cos\phi\left(=\frac{12a}{\sqrt{13}}\right)$	A1
	$mg\times\frac{\sqrt{13}(2\pi-3)a}{26}=kmg\times\frac{12a}{\sqrt{13}}$	DM1
	$k=0.137\quad(0.14)$	A1 **(6)**
	Notes	**[10]**
M1		
A1	Or equivalent	
M1	Distance of additional particle from vertical through A	
M1 A1	Or equivalent	
DM1	Moments about A Dependent on the 2 previous M marks	
A1	(0.14 or better)	

Question Number	Solution	Marks	Notes
4a	<table><tr><td></td><td>*ABCD*</td><td>*PQRV*</td><td>*RSTU*</td><td>*L*</td></tr><tr><td>Mass ratio</td><td>64</td><td>4</td><td>16</td><td>44</td></tr><tr><td>c of m from *AD*</td><td>4*a*</td><td>2*a*</td><td>5*a*</td><td>(*d*)</td></tr></table>	B1 B1	Correct mass ratios for their split Correct distances from vertical axis for their split Must be multiples of *a*
	M(*AD*)	M1	Moments about *AD* or a parallel axis. Need all terms and dimensionally consistent.
	$64\times 4a-4\times 2a-16\times 5a=44d$	A1	Correct unsimplified equation Accept as part of a vector equation
	$\Rightarrow d=\frac{168}{44}a=\frac{42}{11}a$ *	A1*	Obtain **given answer** from correct working
		(5)	
4b	C of M of *L* lies at midpt of *AC*	B1	Seen or implied
	M(Mid pt *AB*)	M1	Use of moments to form equation in *k*.
	$\left(4-\frac{42}{11}\right)aM=4akM$	A1	Correct unsimplified equation. Allow with *a* not seen
	$k=\frac{1}{22}$	A1	0.05 or better (0.0454545…) Allow with *a* not seen
		(4)	
4b alt	C of M of *L* lies at midpt of *AC*	B1	Seen or implied by use of $\bar{x}=\bar{y}$ or $\tan 45°=1$
	Find $\bar{x}$ and $\bar{y}$ for system	M1	
	From *AB*: $\frac{42}{11}Ma+8akM=(1+k)M\bar{y}$ From *BC*: $\frac{46}{11}aM=(1+k)M\bar{x}$	A1	Correct unsimplified equations in $\bar{x}$ and $\bar{y}$ Allow with *a* not seen
	$\bar{x}=\bar{y}\Rightarrow\frac{42}{11}+8k=\frac{46}{11}\Rightarrow k=\frac{1}{22}$	A1	Allow with *a* not seen
4b alt	C of M of *L* lies at midpt of *AC*	B1	Seen or implied in moments equation
	If *G* is c of m of *L* then $\tan ABG=\frac{42}{46}$ and take moments about *B*	M1	Complete method for moments about *B*
	$8a\sin 45°\times kM$ $=\frac{Ma\sqrt{46^2+42^2}}{11}\sin(45°-ABG)$	A1	Correct unsimplified equation in *k* Allow with *a* not seen
	$\Rightarrow k=\frac{1}{22}$	A1	Allow with *a* not seen
4b alt	C of M of *L* lies at midpt of *AC*	B1	Seen or implied in moments equation

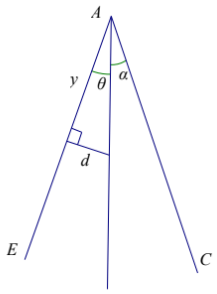
	Take moments about the centre of *ABCD*	M1	
	$M \times \frac{2\sqrt{2}}{11}a = kM \times 4\sqrt{2}a$	A1	Correct unsimplified equation in *k* Allow with *a* not seen
	$\Rightarrow k = \frac{1}{22}$	A1	Allow with *a* not seen
		[9]	

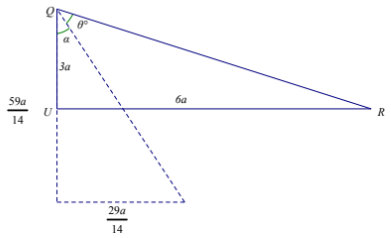
Question Number	Scheme	Marks	Notes
2.			
	Centre of mass of triangle is at *G*, where $AG = 8a$	**B1**	Or equivalent. Seen or implied e.g. $\frac{2}{3}\times 5a\cos\theta\left(=\frac{40a}{13}\right)$ from *AB*
	$\sin\theta = \frac{5}{13}$	**B1**	Or equivalent. Any correct trig ratio for an angle in the triangle. Seen or implied
	M(*A*): $13aF = W\times 8a\times\frac{5}{13}$	**M1**	Dimensionally correct with resolved component of their $8a$ Condone sin/cos confusion. If *g* appears, mark as an accuracy error
		A1	Correct substituted equation (any form)
	$F = \frac{40W}{169}\ (\text{N})$	**A1**	$0.24W$ or better
		[5]	

Question Number	Scheme	Marks	Notes
4	$6m\times 3a+4m\times 4a+5m\times 2a=15m\times y$	M1	Moments about a horizontal axis. Terms dimensionally consistent. Condone slip with a Needs all terms. Condone sign errors
	$\left(44ma=15my\right)$	A1	Correct unsimplified
	$y=\frac{44a}{15}$ from B	A1	Or equivalent. Correct for their axis $\frac{46a}{15}$ from A $\frac{16a}{15}$ from E (CD)
	$5m\times\frac{3a}{2}+4m\times a=15mx$	M1	Moments about a vertical axis Terms dimensionally consistent. Condone slip with a Needs all terms. Condone sign errors
	$\left(\frac{23ma}{2}=15mx\right)$	A1	Correct unsimplified
	$x=\frac{23a}{30}$ from E (AB)	A1	Or equivalent. Correct for their axis $\frac{67a}{30}$ from C
	$\tan\theta=\frac{\frac{23}{30}}{\frac{44}{15}}$	M1	Find a relevant angle using distances measured from B (Allow for $\tan\theta=\frac{88}{23}$)
	$=\frac{23}{88}\left(=0.261...\right)$	A1ft	Correct for their distances from B. $\left(\frac{\text{horizontal}}{\text{vertical}}\right)$
	$\theta=14.64....\simeq 15°$	A1	From correct working. The question asks for the answer to the nearest degree.
		[9]	
	SR1: If a candidate has not used a in their working at all, marks as a misread. Maximum marks available: M1A0A0 M1A1A1 M1A1A1		
	SR2: If a candidate has a in their working, but not as part of their values for $\bar{x}$ and $\bar{y}$ the maximum marks available are M1A1A0 M1A1A0 M1A1A1		

3a	<table><tr><td></td><td>square</td><td>triangle</td><td>circle</td><td>T</td></tr><tr><td>mass</td><td>36</td><td>8</td><td>π</td><td>$28-\pi$</td></tr><tr><td>c from AD</td><td>$3a$</td><td>$\frac{7}{3}a$</td><td>$4a$</td><td>d</td></tr></table>		
	Mass ratio	B1	
	Distances from AD or a parallel axis	B1	
	M(AD or parallel axis):	M1	Moments equation. Need all terms and dimensionally correct. Condone sign errors.
	$36\times 3a-8\times\frac{7}{3}a-\pi\times 4a=(28-\pi)d$	A1	Correct unsimplified equation for their parallel axis
	$\left(108a-\frac{56}{3}a-4\pi a=(28-\pi)d\right)$		
	$d=\frac{324-56-12\pi}{3(28-\pi)}a=\frac{4(67-3\pi)}{3(28-\pi)}a$ *	A1*	Obtain **given answer** from correct working
			Distance from BC is $\frac{(236-6\pi)a}{3(28-\pi)}$ Allow 4/5 if seen.
		(5)	
3b	M(A):	M1	Complete method to form an equation in k and W only Dimensionally correct.
	$W\times\frac{4(67-3\pi)}{3(28-\pi)}a=kW\times 6a$	A1	Correct unsimplified equation
	$k=0.51$	A1	Q asks for 2dp
		(3)	
		[8]	

7a	Use of $\dfrac{2a\times\frac{1}{2}}{3\times\frac{\pi}{6}}\left(=\dfrac{2a}{\pi}\right)$	B1	Seen or implied
	Moments about *EC*:	M1	Dimensionally correct Condone use of a parallel axis
	$ad\times\dfrac{d}{2}=\dfrac{1}{6}\pi a^2\times\dfrac{2a}{\pi}\times\sin\dfrac{\pi}{6}$	A1	Correct unsimplified equation
	$\Rightarrow\left(d^2=\dfrac{a^2}{3}\right)\quad a=\sqrt{3}d$ *	A1*	Obtain **given answer** from correct working
		(4)	
7b	Mass ratios $\dfrac{a^2}{\sqrt{3}}:\dfrac{\pi a^2}{6}:\dfrac{a^2}{\sqrt{3}}+\dfrac{\pi a^2}{6}$	B1	Or equivalent. Seen or implied
	Moments about *BC*	M1	Dimensionally correct Condone use of a parallel axis
	$\dfrac{1}{\sqrt{3}}\times\dfrac{a}{2}+\dfrac{\pi}{6}\times\dfrac{2a}{\pi}\times\dfrac{\sqrt{3}}{2}=\left(\dfrac{1}{\sqrt{3}}+\dfrac{\pi}{6}\right)y$	A1ft	Correct unsimplified. Follow their $\dfrac{2a}{\pi}$
	Distance from *BC* = $y=\dfrac{6a}{6+\sqrt{3}\pi}$	A1	Or equivalent $\left(y=\dfrac{6d}{2\sqrt{3}+\pi}\right)$
	Use of trig to find a relevant angle	M1	
	$\tan\beta^{\mathrm{C}}=\dfrac{6}{6+\sqrt{3}\pi}\times\sqrt{3}\qquad\left(\dfrac{\bar{y}}{d}\right)$	A1ft	Or equivalent correct unsimplified equation for the required angle
	$\beta=0.737\quad(0.74)$	A1	0.74 or better $42.2°$ implies correct method
		(7)	
		[11]	

6a	Mass ratio $24a : 25a : 7\pi a : 7a(7+\pi)$	B1	Correct ratio seen or implied
	Moments about AE	M1	Need all terms, with their masses and horizontal distances Allow use of a parallel axis.
	$25a \times \frac{7}{2}a + 7\pi a \times \frac{14a}{\pi}$ $= 7a(7+\pi)d$	A1	Correct unsimplified equation
	$\frac{371}{2}a^2 = 7a(7+\pi)d$ $\Rightarrow d = \frac{53}{2(7+\pi)}a$ *	A1*	Obtain **given answer** from correct working Condone if they call it $\overline{x}$
		[4]	
6b	Centre of mass of semicircle lies $7a$ "vertically below" A	B1	Seen or implied e.g. $17a$ above E
	Moments about "horizontal" axis through A:	M1	Or a parallel axis. Need all terms, with their masses and distances.
	$24a \times 12a + 25a \times 12a + 7\pi a \times 7a$ $= 7a(7+\pi)y$	A1	Correct unsimplified equation
	$y = \frac{49a(12+\pi)}{7(7+\pi)}\left(= \frac{7a(12+\pi)}{7+\pi}\right)$	A1	Any equivalent form. Accept $\frac{84+17\pi}{7+\pi}a$ from E
	NB: A candidate might have a vector equation in (a) which provides evidence for some or all or the first 4 marks in (b).		
			
	$\theta° = \tan^{-1}\frac{d}{y} = \tan^{-1}\frac{53}{14(12+\pi)}(= 14.037...°)$	DM1	Use trig to find relevant angle $(\theta$ or $90-\theta)$ in a triangle with d and A (must now be working with vertical distance of C of M from A) Dependent on first M
	$\alpha° = \tan^{-1}\frac{7}{24} - \theta°$	DM1	Dependent on the previous M1. Complete method for the required angle
	$\alpha = 2.2$	A1	2.2 or better (2.22…)
		[7]	
		(11)	

7a	<table><tr><td></td><td>*PQUV*</td><td>*URST*</td><td>*QRU*</td><td>total</td></tr><tr><td>Mass ratio</td><td>$9a^2$</td><td>$36a^2$</td><td>$18a^2$</td><td>$63a^2$</td></tr><tr><td>Displacement From *QT*</td><td>$-\frac{3a}{2}$</td><td>$3a$</td><td>$2a$</td><td>d</td></tr></table>	B1 B1	Correct mass ratios (1:4:2:7) Correct displacements from *QT* or a parallel axis seen or implied. Signs consistent
	Equation for moments about *QT* $18\times 2a+36\times 3a-9\times\frac{3a}{2}=63d$ $\left(4a+12a-\frac{3a}{2}=7d\right)$	M1 A1	(or a parallel axis) Dimensionally correct. Condone sign errors Or equivalent Correct unsimplified equation Check consistent in *a*.
	$d=\frac{29a}{2}\Big/7\left(=\frac{261a}{2}\Big/63\right)=\frac{29a}{14}$ *	A1*	Obtain **given answer** from correct working. Need to see at least one interim step with all the *a* terms collected. Check *a* is in final answer.
		[5]	
7b	Condone if "*a*" is missing throughout the working in part (b) because they have not been asked for the distance here.		
	Vertical distances from *Q*: $\frac{3a}{2}, 6a\left(=3a+3a\right), 2a, \left(v\right)$ From *T*: $7.5a, 3a, 7a$	B1	Seen or implied
	Equation for moments about *PQ* $9\times\frac{3a}{2}+18\times 2a+36\times 6a=63v$ $\left(\frac{3a}{2}+2\times 2a+4\times 6a=7v\right)$	M1 A1	(Or a parallel axis) Dimensionally correct. Condone sign errors Correct unsimplified equation
	$v=\frac{59a}{14}$ $\left(\frac{67}{14}a \text{ above } T, \frac{17}{14}a \text{ below } U\right)$	A1	$4.2a$ or better (4.214…)
	The working for (a) and (b) might be combined in a vector equation. The marks for (b) are scored if the work is used in (b).		
			
	$\tan\alpha=\frac{29}{59}$ $\left(=26.175...°\right)$	M1	Use trig and their *v* to find a relevant angle Allow for $90°-26.17...°$

	$\theta° = \tan^{-1} 2 - \tan^{-1}\left(\frac{29}{59}\right)$	M1	Use their v to find the required angle $(63.43..° - 26.175..°)$
	$\theta = 37.3$	A1	37 or better
		[7]	
		(12)	
	$\sqrt{\quad}$		
	–		–
			$\sqrt{\quad}$
	– –		
	– $\sqrt{\quad}$		
	$\sqrt{\quad}$		
	Final A1 as above		$\frac{}{35\times25} - \frac{}{\sqrt{37}} = 0$
		(14)	

Q	Mark Scheme	Marks	Marking guidance
1 (a)	M(x axis)	M1	Need all terms. Dimensionally consistent. Condone if m missing throughout. Accept as part of a vector equation
	$2m \times - + \times + \times = \times(2)\ 3m\ 24m\ 3\bar{k} \quad 9m \quad x$ $\bar{x} = \frac{2+12k}{9} *$	A1*	Obtain given result
		2	
(b)	M(y axis)	M1	Need all terms. Dimensionally consistent. Might be seen as part of a vector equation in (a). It does not score any marks until referred to in part (b). Condone if m missing throughout.
	$2m \times + \times - + \times = \times 5\ 3m\ (3)\ 4m\ \bar{k} \quad 9m \quad y$ $\bar{y} = \frac{14+k}{9}$	A1	Correct unsimplified equation. Allow if m missing throughout.
	Form and solve equation in k $(2+12k + + = 2 \quad 8k \quad 27)$	DM1	Use their $\bar{y}$ and $\bar{x} + = 2\bar{y} \quad 3$ Dependent on the two preceding M marks
	$k = \frac{23}{20}\ (1.15)$	A1	Correct answer only
		4	
		(6)	

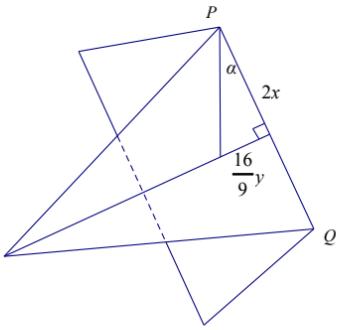
6 (a)	M(PV)	M1	Allow use of a parallel axis. Terms dimensionally consistent. Could be seen as part of a vector equation. Condone error(s) in distance(s).
	$a\times 2ka^2+\left(2a+\frac{1}{2}ka\right)2ka^2=\bar{x}\times\left(2ka^2+2ka^2\right)$	A1	Correct unsimplified equation
	$2\bar{x}=a+2a+\frac{1}{2}ka\Rightarrow\bar{x}=\frac{6+k}{4}a$ *	A1*	Obtain **given answer** from correct working
		3	
6 (b)	M(PR)	M1	Allow use of a parallel axis. Terms dimensionally consistent. Could be seen as part of a vector equation in (a) but needs to be used here to score mark(s) in (b). Condone error(s) in distance(s). If working from VU they might assume that c of m of $QRST$ lies on their axis. So long as they say that this is what they have done (e.g. in a table of values) this can score M1A0A0M1A1ftA0.
	$\frac{1}{2}ka\times 2ka^2+a\times 2ka^2=\bar{y}\times 4ka^2$	A1	Correct unsimplified equation
	$\bar{y}=\frac{k+2}{4}a$	A1	Correct answer $(\ )\pm$ seen or implied Accept distance from $VU=\pm\frac{3k-2}{4}a$ $6-k$

			Or distance from TS = ± $\frac{a}{4}$
	Use given ratio to form equation in k	M1	Correct use of given ratio. Allow reciprocal
	$\frac{7y}{15x} = \frac{(k+2)a}{4} \times \frac{4}{(6+ka)}$	A1ft	Correct unsimplified equation using given x and their y $\frac{ka}{2a}$ or $\frac{y}{x}$ e.g. $\frac{VU}{TS}$ or $\frac{y}{x}$
	$\Rightarrow \frac{3}{2} = k(1.5)$	A1	Correct only
		6	
		(9)	

3a	<table><tr><td></td><td>area</td><td>distance from AE</td></tr><tr><td>rectangle</td><td>$48a^2$</td><td>$4a$</td></tr><tr><td>triangle</td><td>$18a^2$</td><td>$8a-2a\left(=6a\right)$</td></tr><tr><td>lamina</td><td>$30a^2$</td><td></td></tr></table>	B1 B1	Mass ratio correct Distances from AE (or parallel axis) correct
	M(AE)	M1	Allow use of a parallel axis. The moments equation should include a but condone if the mass ratio does not include a factor of a^2. Dimensionally correct.
	$48a^2\times4a-18a^2\times6a=30a^2\bar{x}$	A1	Correct unsimplified equation for their axis. Accept as part of a vector equation.
	$\bar{x}=\frac{84}{30}a=\frac{14}{5}a$ *	A1*	Obtain **given answer** from correct working (including correct use of a)
			If they take moments about BD they get $d=5.2a$ Allow B1B1M1A1A0 if they get this far.
		5	
3b	Find trig ratio of a relevant angle	M1	Correct use of trig.
	$\tan\theta°=\frac{3a}{2.8a}$	A1	Correct equation for the required angle. (DO NOT ISW: If they obtain 47 and then use 90 – 47 = 43 they score M1A0A0)
	$\theta=47$	A1	The Q asks for a whole number of degrees. 0.82 radians scores M1A1A0
		3	
		(8)	

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>Notes</th></tr>
<tr><td colspan="4">They must have a dissection for which they should know or find the position of the centre of mass (e.g. triangles and rectangles). A false assumption about the position of the centre of mass of a trapezium results in 0/5.</td></tr>
<tr><td rowspan="4">3a</td><td><table>
<tr><td></td><td>Large tri</td><td>Small tri</td><td>Small tri</td><td>Whole</td></tr>
<tr><td>Dist PQ</td><td>0</td><td>-2y</td><td>2y</td><td>d</td></tr>
<tr><td>Mass ratio</td><td>27xy</td><td>12xy</td><td>12xy</td><td>27xy</td></tr>
</table></td><td>B1

B1</td><td>Correct distances from PQ or a parallel axis for their complete dissection
Correct mass ratios for a complete dissection</td></tr>
<tr><td>Moments about PQ:</td><td>M1</td><td>Or a parallel axis. Dimensionally correct. Need all non-zero terms and no extras. Condone sign error(s). Allow for $\pm d$
Check the logic carefully.</td></tr>
<tr><td>$(27xy\times 0)-12xy\times(-2y)+12xy\times 2y=27xyd$</td><td>A1</td><td>Correct unsimplified equation. Allow for $\pm d$
Allow for correct distance from a parallel axis</td></tr>
<tr><td>$d=\frac{48}{27}y=\frac{16}{9}y$ *</td><td>A1*</td><td>Obtain given result from fully correct working.</td></tr>
<tr><td colspan="4">There are many different approaches to this.
NB If they are using a trapezium they must show method for the distance. For PQBC the correct value for distance centre of mass from PQ is $\frac{8y}{5}$
Possible alternative moments equations include:
$15xy\times\frac{8y}{5}+9xy\times\frac{4y}{3}+3xy\times 4y=27xyd$ using PQBC, PQDE and DEA
$12xy\times 2y+15xy\times\frac{8y}{5}=27xyd$ using PQA and PQBC
$2\times 3xy\times y-3xy\times y+2\times 6xy\times 1.5y+2\times 3xy\times 2y=27xyd$ working from BC for the folded figure.
$2\times 3xy\times 2y+4\times\frac{1}{2}3xy\times y+2\times 6xy\times 1.5y+3xy\times 4y=27xyd$ working down from PQ</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
</table>

Question	Scheme	Marks	Notes
3b			
	Use of trigonometry	M1	Trig ratio for a relevant angle In their working they need a valid attempt to find α or $90°-\alpha$.
	$\tan\alpha=\frac{\frac{16}{9}y}{2x}=\frac{64}{81}$	A1	Correct unsimplified equation in x and y
	$\Rightarrow x=\frac{9}{8}y$	A1	Correct only. $\left(x=1.125y\right)$ (Accept $x=1.1y$ or better)
		(3)	

Question	Scheme	Mark	Notes
2a	<table><tr><td></td><td>$PQUY$</td><td>$RSTU$</td><td>$VWXY$</td></tr><tr><td>Mass</td><td>$16a^2$</td><td>$2\times4a^2$</td><td>$2\times4a^2$</td></tr><tr><td>From PX</td><td>$2a$</td><td>$5a$</td><td>a</td></tr></table>	B1 B1	Correct mass ratios (accept 2:1:1) Correct vertical distances
	Moments about PX or a parallel axis	M1	Dimensionally correct equation. All terms required Allow for an equation within a vector equation.
	$16a^2\times2a+8a^2\times5a+8a^2\times a=32a^2d$ $\left(=(16+8+8)a^2\times d\right)$ or equivalent for a parallel axis	A1	Correct unsimplified equation Allow for an equation within a vector equation. Could have y for d here o.e.
	$80a=32d\Rightarrow d=\frac{5}{2}a$ *	A1*	Obtain given answer from correct working. At least one stage of simplifying the moments equation is required. e.g. $32a^3+40a^3+8a^3$ seen, or they might have simplified the mass ratios at the start. Must get to $d=\ldots$ in the final line
		[5]	
2b	Moments about PQ or a parallel axis	M1	Dimensionally correct equation. All terms required
	$16a^2\times2a+8a^2\times3a+8a^2\times5a$ $=(16+8+8)a^2\times h$ or equivalent for a parallel axis	A1ft A1	Unsimplified equation with at most one error. Follow their mass ratios. Correct unsimplified equation
	$\Rightarrow h=3a$ from PQ	A1	a from YT, $3a$ from XW
	The working for the first 4 marks must be seen or used in part (b)		
	Correct use of trig. to find the tangent of a relevant angle.	M1	With *their* $3a$ e.g. $\tan\theta=\frac{3}{4-\frac{5}{2}}$
	$\tan\theta=2$	A1	Correct only
		[6]	
2c	Complete method to obtain an equation in M and F	M1	e.g. *M*oments about Q Dimensionally correct equation.
	$3a\times Mg=4a\times F$	A1	Correct unsimplified equation Condone if a missing throughout
	$F=\frac{3}{4}Mg$	A1	Correct only
		[3]	
		(14)	

<table>
<tr><td>4a</td><td colspan="3">If the division of the shape involves non-standard shapes (e.g. a trapezium) the centres of mass must be quoted correctly or a correct method used to find the position to score any marks.</td></tr>
<tr><td></td><td>
<table>
<tr><td>rectangle</td><td>-triangle</td><td>+triangle</td></tr>
<tr><td>$20a^2$</td><td>$-\frac{9}{2}a^2$</td><td>$\frac{9}{2}a^2$</td></tr>
<tr><td>$2a$</td><td>$3a$</td><td>$2a$</td></tr>
</table>
or
<table>
<tr><td>rectangle</td><td>rectangle</td><td>Double triangle</td></tr>
<tr><td>$3a^2$</td><td>$8a^2$</td><td>$9a^2$</td></tr>
<tr><td>$\frac{1}{2}a$</td><td>$2a$</td><td>$2a$</td></tr>
</table>
or
<table>
<tr><td>rectangle</td><td>trapezium</td><td>triangle</td></tr>
<tr><td>$5a^2$</td><td>$\frac{21}{2}a^2$</td><td>$\frac{9}{2}a^2$</td></tr>
<tr><td>$\frac{1}{2}a$</td><td>$\frac{48}{21}a$</td><td>$2a$</td></tr>
</table>
</td><td>B1

B1</td><td>Correct mass ratios for a correct division of the folded template and correct total of $20a^2$
Correct distances from AD seen or implied.
B0B1 is possible if they have incorrect masses but a full set of correct distances. e.g. if they use the second alternative but have not doubled the triangle. Or they might have a correct split with an error in one of the areas or an incorrect (or missing) total

or equivalent</td></tr>
<tr><td></td><td>Moments about AD or a parallel axis.</td><td>M1</td><td>Dimensionally consistent.
All terms for a correct division of L and no extras.
Accept as part of a vector equation</td></tr>
<tr><td></td><td>$40a^3-\frac{27}{2}a^3+9a^3=20a^2d$
or $\frac{3}{2}a^3+16a^3+18a^3=20a^2d$
or $\frac{2}{2}a^3+\frac{48}{2}a^3+9a^3=20a^2d$</td><td>A1</td><td>Correct unsimplified equation for their axis. Allow for correct component in a vector equation.</td></tr>
<tr><td></td><td>$d=\frac{71}{40}a$ *</td><td>A1*</td><td>Obtain given answer from correct working. Need at least one line of working to collect like terms e.g. $20d=\frac{71}{2}a$ Final answer must be as printed i.e. $d=\ldots$.</td></tr>
<tr><td></td><td></td><td>[5]</td><td></td></tr>
<tr><td>4b</td><td>Moments about S</td><td>M1</td><td>A complete method to get an equation in W and F only.
Need all terms and no extras.
Dimensionally consistent.</td></tr>
<tr><td></td><td colspan="3">NB If they start by finding the centre of mass for the system they do not score marks until they form the moments equation.
If they are clearly using moments about A (e.g. d and $4a$ used as distances in their equation) this is M0 unless they include the reaction at S and resolve to form the required equation.
If they say they are using moments about S and have just one incorrect distance allow M1A1A0A0</td></tr>
<tr><td></td><td>$4W\times\frac{31}{40}a+W\times3a=F\times5a$
or $(4W+W)(2.22a-a)=5aF$</td><td>A1

A1</td><td>Unsimplified equation with at most one error
Correct unsimplified equation</td></tr>
<tr><td></td><td>$F=\frac{61}{50}W$</td><td>A1</td><td>Accept $1.22W$ or $1.2W$</td></tr>
<tr><td></td><td></td><td>[4]</td><td></td></tr>
<tr><td></td><td></td><td>(9)</td><td></td></tr>
</table>