Edexcel International A Level (IAL) Maths: Mechanics 1

Your notes

Resolving Forces, Inclined Planes & Friction

Contents

Resolving forces

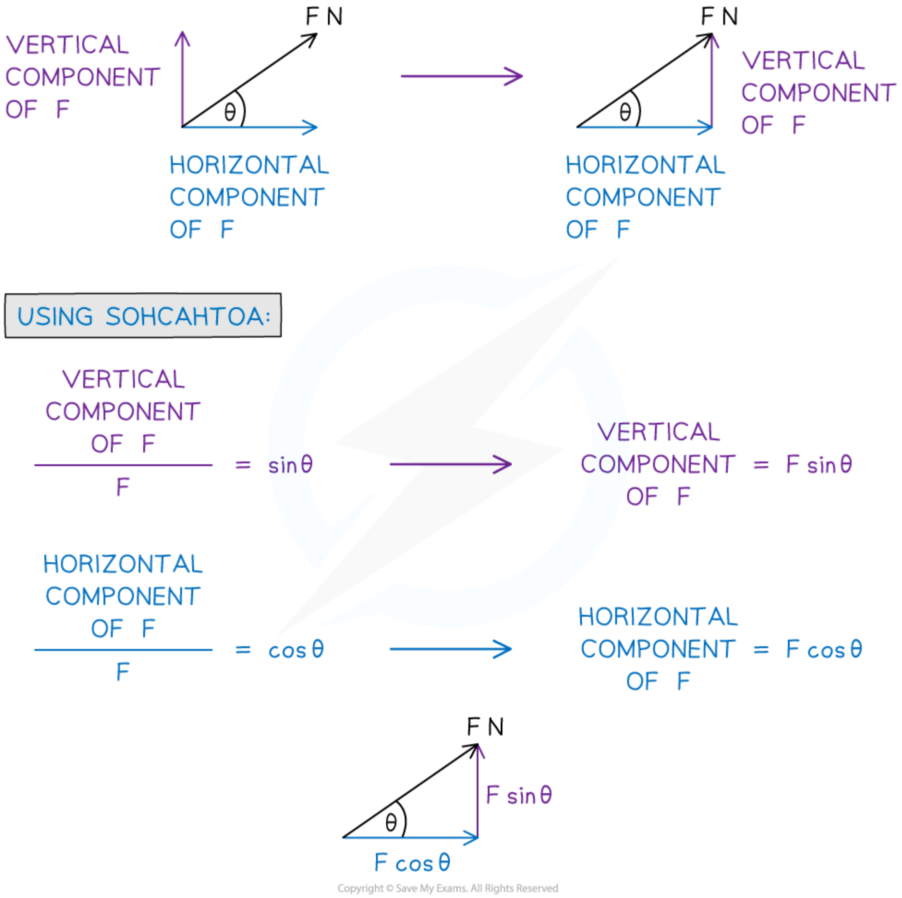
If a single force is applied at an angle to the **direction of motion** it will need to be **resolved** into **components** of the force that act **perpendicular** to each other.

Why resolve a force acting at an angle?

- Resolving a force acting at an angle often helps to simplify a problem
- The components of a force parallel and perpendicular to the line of motion allows different types of problems to be solved
  - The **parallel** component of a force acting directly on a particle will be the component that causes an effect on the particle
  - The **perpendicular** component of a force acting directly on a particle will be the component that has no effect on the particle
- Finding the horizontal and vertical components of a force can help solve problems acting in the horizontal plane
- The two components of the force will have the same combined effect as the original force

How do I resolve a force acting at an angle?

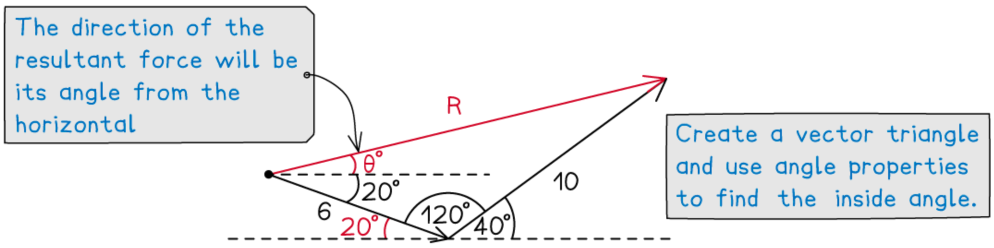
- Use **trigonometry** to resolve a force acting at an angle
- Draw a vector triangle of forces by decomposing the force into its horizontal and vertical components
- The original force will be the **hypotenuse** of the triangle and the two components will make up the **opposite** and **adjacent** sides

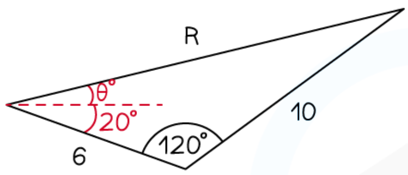


Your notes

How do I find the resultant force of two forces acting at different angles?

- Force is a **vector** quantity, so finding the **resultant** of two or more forces is the same as finding the resultant of two or more vectors
- It is possible to use geometry to find the resultant force without decomposing the forces into their horizontal and vertical components
- Use the **triangle law for vector addition** to calculate the **magnitude** and **direction** of a resultant force
- You will need to use **trigonometry** to find the missing side or angle in the triangle.





Use the cosine rule to find R:

$$R^2 = 6^2 + 10^2 - 2(6)(10)\cos 120°$$

$$= 136 - 120\cos 120°$$

$$= 196$$

$$R = 14 \text{ N}$$

Use the sin rule to find θ:

$$\frac{\sin(\theta + 20)}{10} = \frac{\sin 120°}{14}$$

$$\sin(\theta + 20) = \frac{5\sqrt{3}}{14}$$

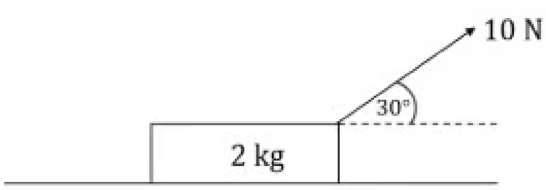
$$\theta + 20 = \sin^{-1}\left(\frac{5\sqrt{3}}{14}\right) = 38.2132...$$

$$\theta = 18.2132...$$

The resultant force is 14 N acting in a direction of 18.2° to the horizontal (3 sf)

Worked Example

A child is pulling a box of mass 2 kg along a smooth horizontal surface using a force of 10 N at an angle of 30° to the horizontal, as shown in the diagram below.



(i) Find the vertical and horizontal components of the 10 N force.

(ii) Explain why the resultant force acting to move the box is $5\sqrt{3}$ N.

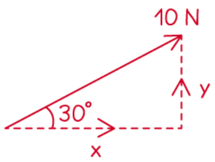
(iii) Work out the acceleration of the box.

(iv) Calculate the magnitude of the normal reaction between the box and the floor.

Your notes

Answer:

(i) Draw a diagram and add in the horizontal and vertical components:



Using trigonometry: $\sin 30° = \frac{y}{10}$ $\cos 30° = \frac{x}{10}$

$y = 10\sin 30°$ $x = 10\cos 30°$

$= 5\text{N}$ $= 5\sqrt{3}\text{ N}$

Vertical component $= 5\text{N}$
Horizontal component $= 5\sqrt{3}\text{ N}$

(ii) The horizontal component of the 10 N force is the part that acts to move the box along the floor

The resultant force is $5\sqrt{3}$ N because only the horizontal component of the force will act to move the box along the floor

(iii) The box is moving along a horizontal floor so the horizontal component of the force should be used in the equation of motion

$F = ma$

Horizontal component from part (i)

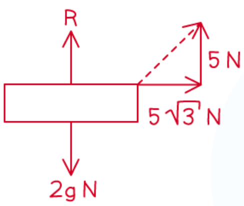
Mass of the box

$5\sqrt{3} = 2a$

$a = \frac{5\sqrt{3}}{2}\text{ ms}^{-2}$

$a = 4.33\text{ ms}^{-2}$ (3 sf)

(iv) A force diagram can help:



The box is not moving in the vertical direction so the vertical forces in the positive and negative direction will be equal

Resolving vertically: $R + 5 = 2g$

$R + 5 = 19.6$

$R = 19.6 - 5 = 14.6$

$R = 14.6\text{ N}$

Examiner Tips and Tricks

- Always draw a diagram and decompose any angled forces into their horizontal and vertical components. Make sure you are confident with basic trigonometry and angle properties.

Your notes

Inclined planes

How do I use F = ma on an inclined plane?

- Problems involving inclined planes will always be acting along the **line of greatest slope** of the plane
  - The line of greatest slope is the most direct route with which it is possible to move from the bottom to the top of the plane
  - A force said to be acting in the same **vertical plane** as the line of greatest slope will be acting in the same plane as this most direct route
- It is usually easier to resolve **parallel** and **perpendicular** to the plane
- The **normal reaction force** always acts perpendicular to the plane and will not need to be resolved
- **Weight (mg)** always acts vertically downwards and so it is easier to use when resolved into components perpendicular and parallel to the plane
- When an object is moving up or down the inclined plane, it has **acceleration** only in the parallel direction
  - The perpendicular forces will be equal in the positive and negative directions
  - Newton's Second Law (**F = ma**) can be used in the parallel direction to determine acceleration or missing forces
- On an inclined plane **friction** will always act in the opposite direction to the direction the object is moving in or on the point of moving in

Worked Example

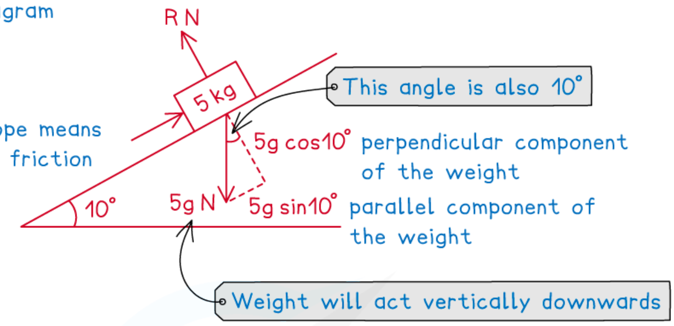
A box of mass 5 kg is being pushed up a smooth ramp inclined at $10°$ to the horizontal by a force of 12 N.

(i) Find the magnitude of the reaction force acting on the box.

(ii) Find the acceleration of the box up the slope.

Answer:

Draw a diagram



(i) The box is moving parallel to the slope so the perpendicular forces will have no effect and will be equal

Perpendicular to the plane (↖): $R = 5g\cos 10°$

$$= 5 \times 9.8 \times \cos 10° = 48.255\ldots \text{ N}$$

$$R = 48.3 \text{ N} \quad (3 \text{ sf})$$

(ii) Apply Newton's Second Law parallel to the plane:

Parallel to the plane (↗): $F_{(total)} = ma$

$$12 - 5g\sin 10° = 5a$$

$$12 - 5 \times 9.8 \times \sin 10° = 5a$$

$$3.49123\ldots = 5a$$

$$a = \frac{3.49123\ldots}{5}$$

$$a = 0.698 \text{ ms}^{-2} \quad (3 \text{ sf})$$

Examiner Tips and Tricks

- Use a triangle of components on your force diagrams to help you work out the components of the force but be careful not to count the components as two separate forces. Make sure you are confident with angle properties and always check you are using the correct angle between the force and the direction you are resolving in.

Your notes

Coefficient of friction

What is friction?

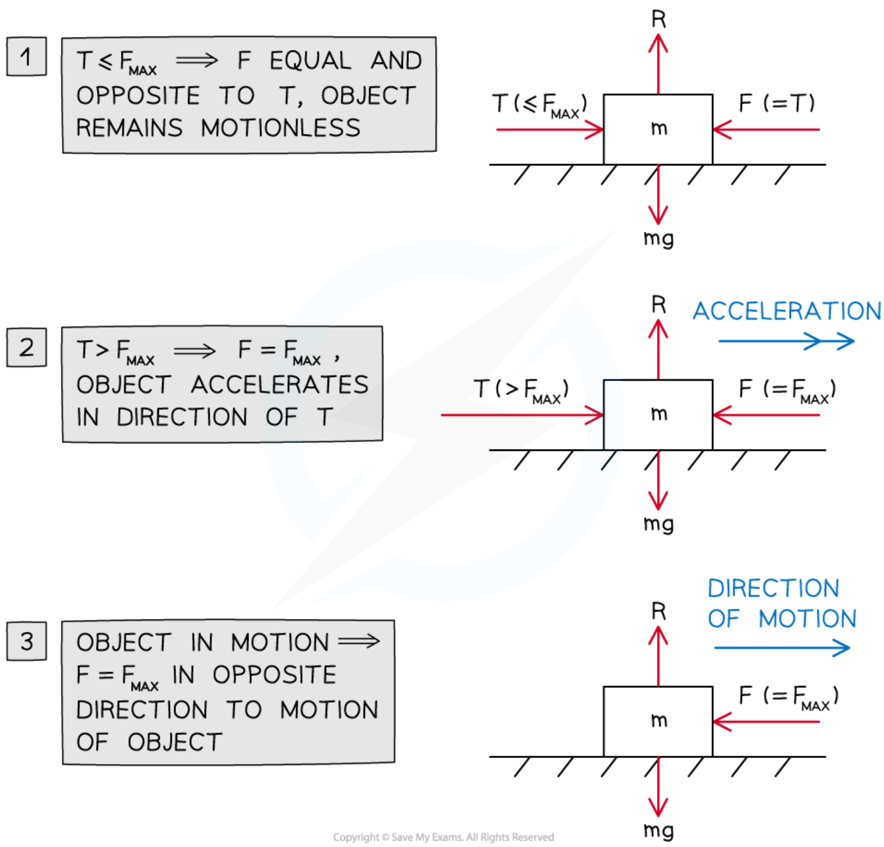
- Friction is a **force** between a surface and an object that acts to resist motion of the object across the surface
- On a horizontal surface with no forces acting on a stationary object other than **gravity** and the surface's **normal reaction force**, there is no force of friction
- If an additional force acts on the object **parallel** to the surface then friction manifests as a force in the **opposite direction**
    - Friction always acts in a direction **parallel** to the **surface**
    - Friction acts in the **opposite direction** to the **resultant force** that would cause the object to move **parallel** to the surface
    - Up to a certain **limiting value** (see next section) the force of friction **prevents motion** entirely
    - Beyond that **limiting value** friction is a force of **constant magnitude** in the **opposite** direction to the motion of the object

What is the coefficient of friction?

- The magnitude of the frictional force satisfies the inequality:
    - $F \leq \mu R$
    - where F is the force of friction
    - R is the normal reaction force between the object and the surface
    - and μ is the **coefficient of friction** between the object and the surface
        - If there is no friction then $\mu = 0$ and the **surface** is **smooth**
        - Otherwise $\mu > 0$
- The maximum possible frictional force is given by the equation
    - $F_{MAX} = \mu R$
    - where F_{MAX} is the maximum possible force of friction, and R and μ are as above
- F_{MAX} serves as a **limiting value** for mechanics problems involving friction
    - **If the object is stationary** and the resultant of other parallel forces to the surface is **less than or equal to F_{MAX}** then
        - the object **remains stationary**

- the frictional force is always **equal** and **opposite** to the resultant of the other horizontal forces
- if the resultant of other horizontal forces is **equal** to F_{MAX} then the object is said to be in **limiting equilibrium**

- **If the object is stationary** and the resultant of other horizontal forces becomes **greater than** F_{MAX} then
  - the object begins to **accelerate** in the direction of the resultant force
  - the frictional force is equal to F_{MAX} with a direction always opposite to that of the object's motion
- **If the object is in motion parallel to the surface** then (regardless of any other forces) the frictional force will always be **equal** to F_{MAX} with a direction opposite to that of the object's motion

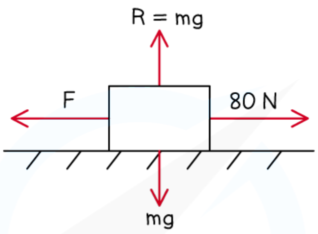


Worked Example

A box with a mass of 12 kg is at rest on a horizontal floor. The coefficient of friction between the box and the floor is 0.7. A horizontal force of 80 N is applied to the box. Describe the force of friction in this situation and determine whether or not the box will begin to move.

Answer:

START BY SKETCHING A FORCE DIAGRAM!



$R = mg = 12 \times 9.8 = 117.6$ N

SO $F_{MAX} = \mu R = 0.7 \times 117.6 = 82.32$ N

$80 \text{ N} < F_{MAX}$, SO THE BOX WILL REMAIN STATIONARY.

THE FORCE OF FRICTION $F = 80$ N, EQUAL IN MAGNITUDE AND OPPOSITE IN DIRECTION TO THE HORIZONTAL FORCE OF THE BOX.

Examiner Tips and Tricks

- Always draw a force diagram and label it clearly.
- Look out for the words **smooth** and **rough** in mechanics problems involving an object moving (or potentially moving) along a surface:
  - If the surface is described as **smooth** then you can ignore friction in the problem (ie $\mu = 0$)
  - If the surface is described as **rough** then you need to include the force of friction in solving the problem.
- If a friction question states that an object is **on the point of moving** that means that the object is in **limiting equilibrium.**

Your notes

Your notes

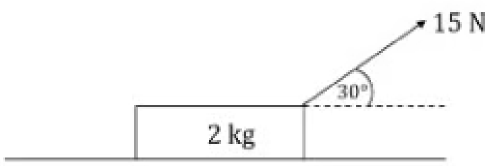
Coefficient of friction & F = ma

How do I use F = ma in problems involving friction?

- The **coefficient of friction** combined with ***F = ma*** allows you to determine an object's motion where friction is involved in a problem
- For problems where the surface is horizontal:
- Step 1. If necessary, **resolve** any angled forces into vertical and horizontal components
- Step 2. Calculate the normal reaction force
  - Be careful – if there are vertical forces other than gravity these will affect the value of ***R***
  - with a horizontal surface ***R*** will always be directed vertically upwards
  - the magnitude of ***R*** will be such as to make the **total** vertical force on the object zero
- Step 3. Calculate $F_{MAX} = \mu R$ and find the **resultant** (total force) of all the horizontal forces on the object
  - Remember – if the resultant of the other horizontal forces is less than or equal to F_{MAX} then friction will exactly balance those forces out and the object will remain stationary
- Step 4. Use ***F = ma*** to determine the acceleration of the object
- For non-horizontal surfaces see the notes on **inclined planes**

Worked Example

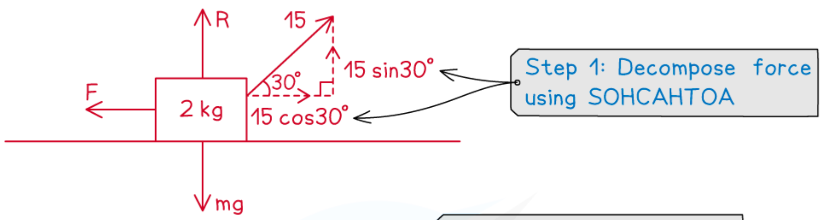
A wooden block of mass 2 kg is at rest on a rough horizontal floor, where the coefficient of friction between the block and the floor is 0.6. A constant force of 15 N is applied to the block at an angle of 30° to the horizontal, as shown in the diagram below.



Find the acceleration of the block.

Answer:

Start by drawing a diagram



Vertical forces up equal to vertical forces down

Step 2: $R + 15\sin 30° = mg$

$R = mg - 15\sin 30° = 2 \times 9.8 - 15 \times \frac{1}{2}$

$R = 12.1\text{ N}$

Step 3: $F_{max} = \mu R = 0.6 \times 12.1 = 7.26\text{ N}$

$15\cos 30 = \frac{15\sqrt{3}}{2} = 12.99...\text{ N} > F_{max}$

so $F = F_{max}$ and total horizontal force is

$12.99... - 7.26 = 5.73...\text{ N}$ (to the right)

Step 4: $F_{total} = ma$

$5.73... = 2a$

$a = 2.87\text{ ms}^{-2}$ (3 sf)

Examiner Tips and Tricks

- Always draw a force diagram and label it clearly. Look out for the words **smooth** and **rough** in mechanics problems involving an object moving (or potentially moving) along a surface:
  - If the surface is described as **smooth** then you can ignore friction in the problem (ie $\mu = 0$)
  - If the surface is described as **rough** than you need to include the force of friction in solving the problem
- Be aware of whether the question is on a horizontal surface or an **inclined plane.**

Your notes

Your notes

Coefficient of friction & inclined planes

How do I solve friction problems on inclined planes?

- On an **inclined plane** the basic principles are the same as for **coefficient of friction** on horizontal surfaces
- The important directions are **parallel** to the plane and **perpendicular** to the plane (instead of horizontal and vertical)
- Step 1. Resolve weight (and any other forces if necessary) into components parallel and perpendicular to the plane
- Step 2. Calculate the normal reaction force ***R***
  - Be careful – ***R*** will never simply be equal to ***mg*** in an inclined plane problem!
  - ***R*** will always be directed perpendicular to and away from the plane
  - the magnitude of ***R*** will always be such as to make the **total** perpendicular force zero
- Step 3. Calculate $F_{MAX} = \mu R$ and find the **resultant** (total force) of all the forces on the object that are parallel to the plane
  - The force of friction ***F*** will always act parallel to the plane in the direction opposite to any movement (or potential movement) of the object
  - Remember – if the resultant of the other parallel forces is less than or equal to F_{MAX} then friction will exactly balance those forces out and the object will remain stationary
- Step 4. Use ***F = ma*** to determine the acceleration of the object

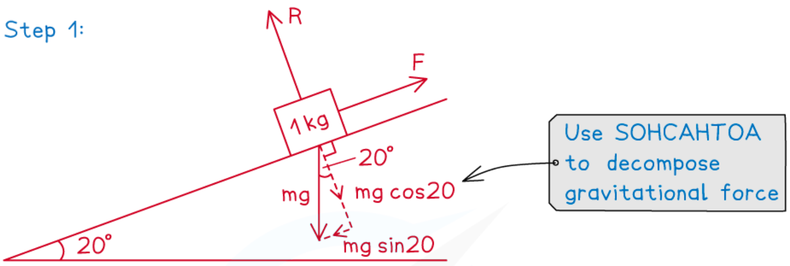
Worked Example

A wooden block of mass 1 kg is released from rest on a rough plane that is inclined at 20° above the horizontal. The coefficient of friction between the block and the plane is 0.2. Find the acceleration of the block.

Answer:

Start by drawing a diagram

Step 1:



Step 2: $R = mg\cos 20 = 1 \times 9.8 \times \cos 20$

$R = 9.208...$ N

Step 3: $F_{max} = \mu R = 0.2 \times 9.208... = 1.841...$

The component of gravity down the plane is $mg\sin 20 = 1 \times 9.8 \times \sin 20 = 3.351...$

$3.351... > F_{max}$, so $F = F_{max}$ and the total force parallel to the plane is

$3.351... - 1.841... = 1.509...$ N (down the plane)

Step 4: $F_{total} = ma$

$1.509... = 1 \times a$

$a = 1.51$ ms^{-2} (2 d.p.)

Your notes

Examiner Tips and Tricks

- Always draw a force diagram and label it clearly.
- Look out for the words **smooth** and **rough** in mechanics problems involving an object moving (or potentially moving) along a surface:
  - If the surface is described as **smooth** then you can ignore friction in the problem (ie $\mu = 0$)
  - If the surface is described as **rough** than you need to include the force of friction in solving the problem
- If a friction question states that an object is **on the point of moving** that means that the object is in **limiting equilibrium**.

Your notes

Coefficient of friction (equilibrium)

How do I model friction when a body is in equilibrium?

- When a body is in static equilibrium under the action of a number of forces, including friction, the value of friction will be less than or equal to the value of μR
- When the value of friction is equal to μR, the body is in **limiting equilibrium** and is on the point of moving
- When a body is stationary on an inclined plane then it is either in equilibrium or in limiting equilibrium, we cannot assume that $F = \mu R$

How do I model friction when a body is moving?

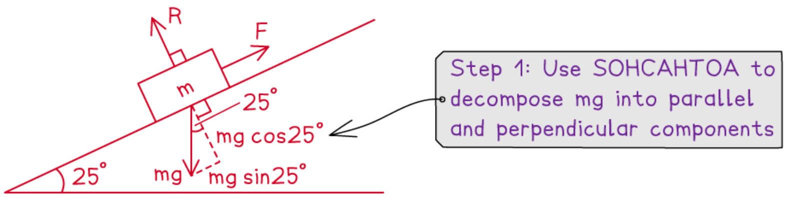
- When a body is moving under the action of a number of forces, including friction, the value of friction will be equal to the value of μR

Worked Example

A wooden block of mass m is stationary on a plane inclined at 25° above the horizontal.
The coefficient of friction between the block and the plane is μ. Find the least possible value of μ.

Answer:

Start by drawing a diagram:



Step 2: Find the normal reaction force:

$R = mg\cos25°$ We don't have a vale for m, so it is easiest to keep R in this exact form

The block is in equilibrium, so the resultant of the forces acting parallel to the plane is zero and $F \leqslant F_{max}$

Step 3: Find F_{max}

$F_{max} = \mu R = \mu(mg\cos25)$

Step 4: The block is stationary so we do not need to use $F = ma$. The forces acting parallel to the plane will be equal.

$R\ (\nearrow)$ $F = mg\sin25°$

$F_{max} \geqslant F$: $\mu(mg\cos25) \geqslant mg\sin25$ (the block will slide down the plane at the point when $\mu R < mg\sin25$)

$$\mu \geqslant \frac{mg\sin25}{mg\cos25}$$

$$\mu \geqslant \tan25$$

The least possible value of μ is 0.466 (3 sf)

Your notes

Examiner Tips and Tricks

- Always draw a force diagram and label it clearly.
- Look out for key vocabulary such as rough, smooth, equilibrium, on the point of moving and limiting equilibrium and know how to use each.

Coefficient of friction (connected particles)

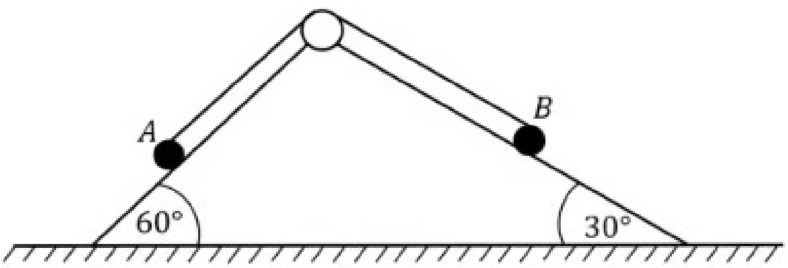
How do I model friction for connected particles?

- When two particles are connected by means of a light, inextensible string, friction will act only on any particle that is resting on a rough surface
- On an inclined slope, friction will act in the direction opposite to the direction the particle is moving in or on the point of moving in
- The direction they will move in depends on both the weight of the particles and the angles of the slopes they are resting on

- Compare the components of the weight in the direction of the motion the particles will move in
- The particle with the heavier component will move down the slope

Worked Example

Two particles A and B, of identical mass, are connected by means of a light inextensible string. Particle A is held motionless on a rough fixed plane inclined at 60° to the horizontal. Particle B is resting on a rough fixed plane inclined at 30° to the horizontal. The string passes over a smooth light pulley fixed at the top of the two planes as shown in the diagram below.



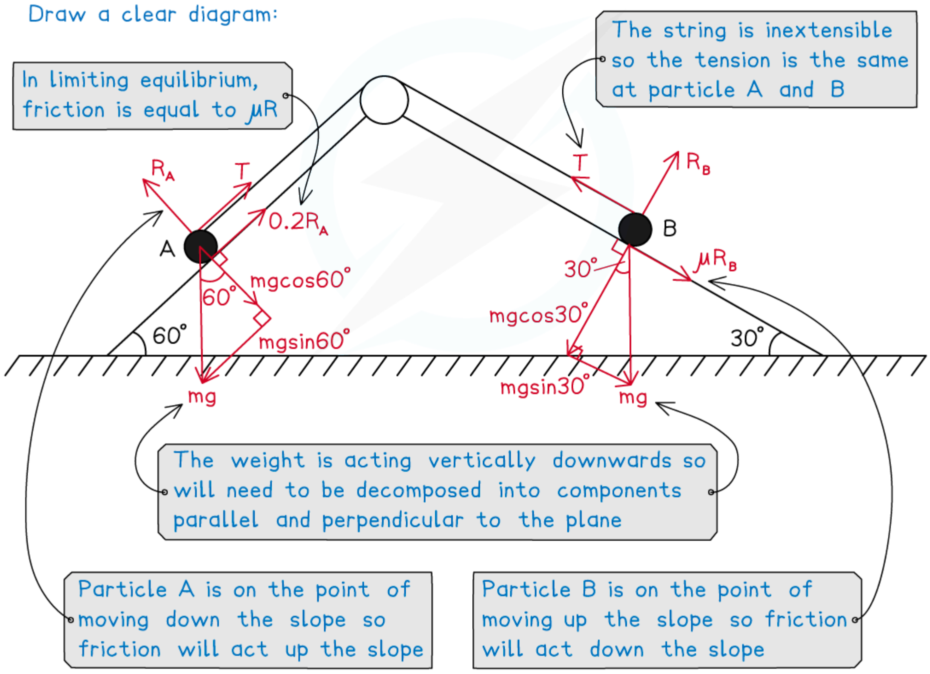
Given that the coefficient of friction between particle A and the plane is 0.2 and that the particles are in limiting equilibrium, show that particle A is on the point of moving down the slope and find the value of the coefficient of friction between particle B and the plane.

Answer:

For each particle the component of weight acting down the slope will be the force acting to pull the particle downwards

mgsin60 < mgsin30 so particle A is about to move down the slope and B is about to move up the slope

Draw a clear diagram:



Your notes

Considering particle A:

$(\nwarrow)\quad R_A = mg\cos 60° = \frac{1}{2}mg$

$(\nearrow)\quad 0.2R_A + T = mg\sin 60°$

$0.2\left(\frac{1}{2}mg\right) + T = \frac{\sqrt{3}}{2}mg$

$T = \left(\frac{\sqrt{3}}{2} - 0.1\right)mg$

Considering particle B:

$(\nearrow)\quad R_B = mg\cos 30° = \frac{\sqrt{3}}{2}mg$

$(\searrow)\quad T = \mu R_B + mg\sin 30°$

$= \frac{\sqrt{3}}{2}mg\mu + \frac{1}{2}mg$

$T = \left(\frac{\sqrt{3}}{2}\mu + 0.5\right)mg$

Equating tensions: $\left(\frac{\sqrt{3}}{2} - 0.1\right)\cancel{mg} = \left(\frac{\sqrt{3}}{2}\mu + 0.5\right)\cancel{mg}$

$\frac{\sqrt{3}}{2} - 0.1 = \frac{\sqrt{3}}{2}\mu + 0.5$

$\frac{\sqrt{3}}{2}(1 - \mu) = 0.6$

$1 - \mu = \frac{0.6}{\frac{\sqrt{3}}{2}}$

$\mu = 1 - \frac{2\sqrt{3}}{5} = 0.30717...$

$\mu = 0.307 \quad (3\text{ sf})$

Examiner Tips and Tricks

- Be careful not to assume that the system will move in the direction of the heavier particle, remember to consider the angle too. Make sure your diagram is clear enough to work with, sometimes there can be a lot of information on the diagram.

Your notes