Edexcel International A Level (IAL) Maths: Mechanics 1

Your notes

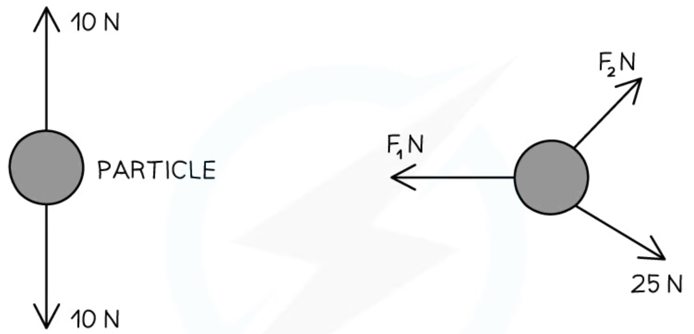
Forces

Contents

Force diagrams

Why do we need force diagrams?

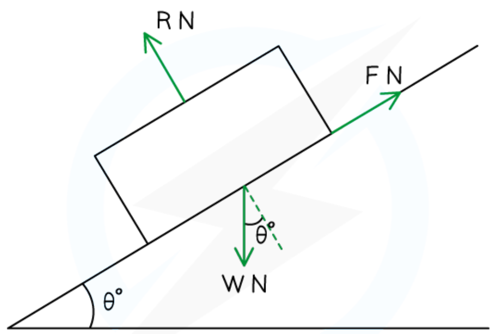
- **Force diagrams** are used to help understand a given scenario and show which **forces** are acting on which **particles** and in which **direction** they are acting
- In diagrams an **arrow** is used to represent a **force** acting on a **particle** which shows the **direction** in which the force is acting
- The **magnitude** of the **force** is normally written next to its arrow



EXAMPLES OF BASIC FORCE DIAGRAMS SHOWING DIRECTION AND MAGNITUDE. TWO OF THE FORCES IN THE SECOND DIAGRAM ARE UNKNOWN.

What types of forces could be involved?

- Specific **types of force** encountered (which may not be mentioned in the question nor labelled on a given diagram) are: **weight (*W* N)**, **tension (*T* N)**, **thrust (*T* N)**, **friction (*F* N)** and **normal reaction (*R* N)**



SPECIFIC FORCES ACTING ON A BLOCK ON A ROUGH INCLINED PLANE

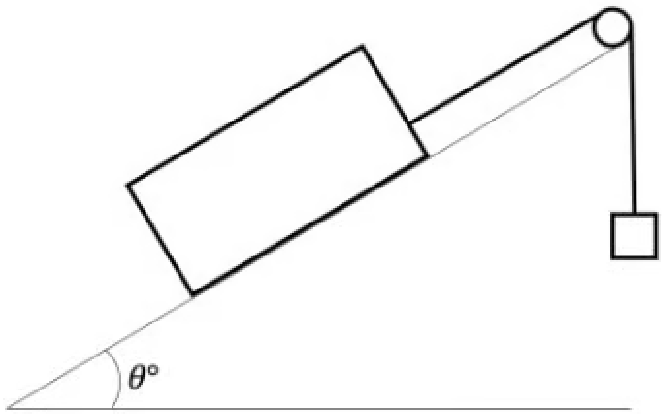
- Remember that the diagram is drawn to help understand the scenario - cars, blocks, etc are **modelled** as particles occupying a **single point** in space and so all **forces** acting on the car, block, etc act at that same **single point**
- The main forces that you will see are:
  - **tension** (a "pulling" force) acts away from a particle,

    thrust (a "pushing" force) acts towards it
  - **weight** is $W = mg$ where m kg is the **mass** of the particle and g is the **acceleration** due to **gravity** – usually g = 9.8 m s^{-2}
  - **friction** acts **parallel** to the surface in the **opposite** direction to **motion**
  - the **normal reaction** acts **perpendicular** to the surface (and **friction**)

Your notes

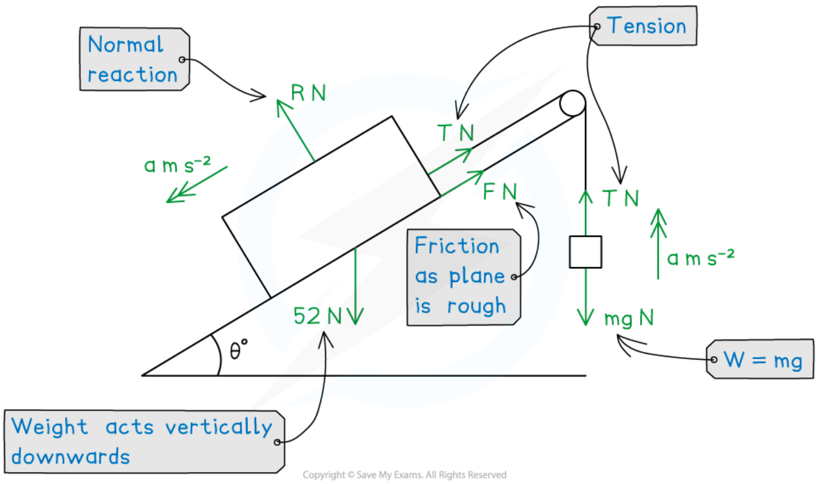
Worked Example

The diagram below shows a block, of weight 52 N, at rest on a rough plane that is inclined at $\theta°$ to the horizontal. The block is connected to a mass, m kg, via a light inextensible string that passes over a smooth pulley. The particle hangs at rest vertically below the pulley.



Add all forces (known and unknown) to the diagram that are acting on the block and the mass. Also indicate the direction of acceleration of any subsequent motion. (You may assume the block is heavier than the mass and so any subsequent motion would see the block slide down the plane.)

Answer:



Examiner Tips and Tricks

- Always draw a force diagram if appropriate.
- If a diagram is already given then add to it as you progress through the question.
- If a diagram is too small or it gets too complicated then draw a new diagram.
- You may be able to manage with just drawing the section of the diagram you are dealing with in any particular question part.

Your notes

Equilibrium in 1D

What is Newton's first law of motion?

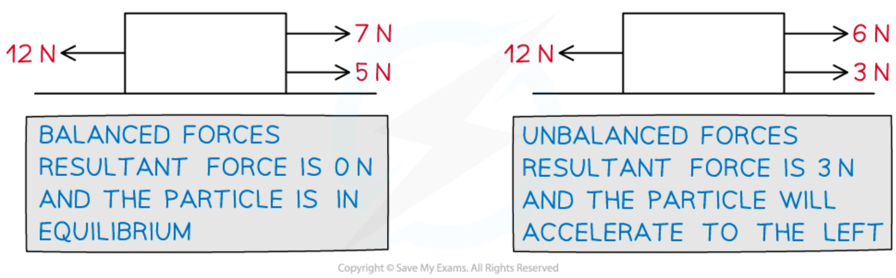
- An object at rest will stay at rest, and an object moving with constant **velocity** will continue to move with constant velocity, unless an unbalanced force acts on the object.

What is the resultant force and an unbalanced force?

- The **resultant force** is the **sum of forces** acting on a particle – consider it as a **single force** that achieves the **same result** as **all** the forces **combined**
- An **unbalanced force** is a force acting on a particle that is **not** cancelled by another force acting in the opposite direction
- So a **non-zero** resultant force will be unbalanced (hence the wording in Newton's First Law of Motion) and the particle will **accelerate**

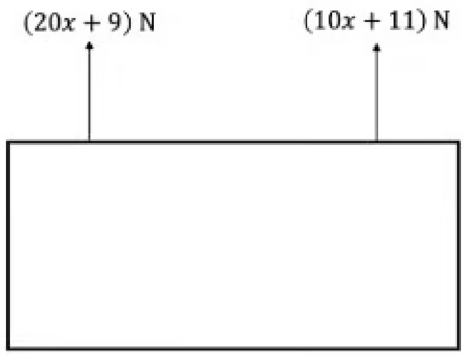
What does equilibrium mean?

- A **particle** is in **equilibrium** if the **resultant force** acting on it is **zero**
  - In other words when the **sum of the forces** acting on a particle is zero
  - For example, in the **horizontal** direction, any **forces** acting to move the particle to the **left** will be **balanced** by any **forces** acting to move the particle to the **right**.
  - There does not need to be the same number of forces in both directions – two forces could be acting to move the particle to the left but are cancelled out by only one force acting to move the particle to the right.



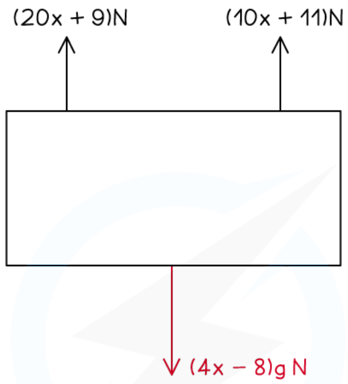
Worked Example

A rectangular shop sign, of mass $(4x - 8)$ kg is held in equilibrium by two light inextensible strings as shown in the diagram below. The tensions in the two strings are $(20x + 9)$ N and $(10x + 11)$ N.



The sign is modelled as a particle.
Taking the acceleration due to gravity (g) as 10 m s^{-2}, find the value of x.

Answer:



Sign (particle) is in equilibrium

$\therefore (20x + 9) + (10x + 11) = (4x - 8)g$

$g = 10$

$\therefore 30x + 20 = 40x - 80$

$10x = 100$

$x = 10$

Examiner Tips and Tricks

- It is unlikely you will get an exam question that only deals with one dimension at AS and A level. However two-dimensional problems can often be broken down into two one-dimensional problems so the principles in this note are important to understand.

Your notes

Equilibrium in 2D

How do I model particles in two dimensions?

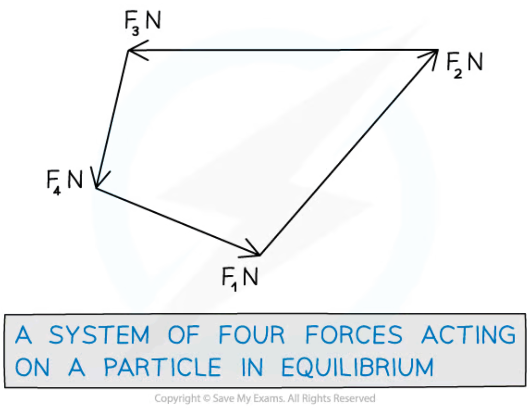
- In **two dimensions**, a particle may be considered as existing and being able to move around two-dimensional space rather than in a single straight line (1D)
  - This 2D space is called a **plane** – so you may see the $\boldsymbol{Oxy}$ **plane** mentioned
- The directions of the two dimensions are chosen so that they are perpendicular to each other
- The two directions are usually
  - **horizontal** and **vertical**, or
  - **parallel** and **perpendicular** to an inclined plane

What is the resultant force and an unbalanced force in 2D?

- The **resultant force** in 2D is the **sum of forces** acting on a particle but this will now need to be considered in two parts – one for each direction
- An **unbalanced force** is a force acting on a particle where one or both parts are **not** cancelled by another force acting in the opposite direction
- So a **non-zero** resultant force will be non-zero in at **least one** of the two dimensions, is therefore **unbalanced** and the particle will **accelerate**

What does equilibrium in 2D mean?

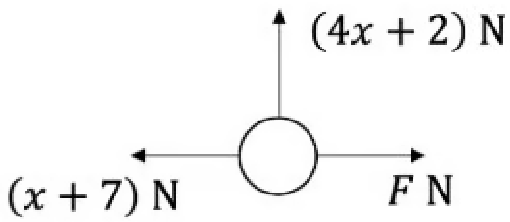
- In two dimensions a **particle** is in **equilibrium** if the **resultant force** acting on it is **zero**
  - For example, if the two dimensions involved are the **horizontal** and **vertical** directions, a particle will be in equilibrium if any forces acting **left** are **balanced** by any forces acting **right** <u>and</u> any forces acting **up** are **balanced** by any forces acting **down**
- When a particle has **several** forces (at least 3 – have a think as to why!) acting on it – and it is in **equilibrium** – the forces can be drawn "nose-to-tail" such that they form a polygon



Your notes

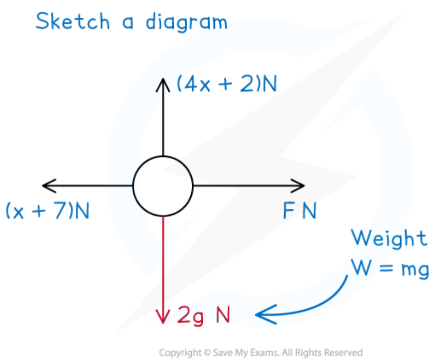
Worked Example

A particle of mass 2 kg is suspended in the air by three strings, one vertical and two horizontal. The particle is in equilibrium and the magnitudes of the tensions in the strings are illustrated in the diagram below.



(a) Write down the resultant forces acting on the particle in both the horizontal and vertical directions.

Answer:



Your notes

a) Particle is in equilibrium, so the resultant force is zero in both directions

$\therefore$ Resultant force in horizontal direction is 0 N.
Resultant force in vertical direction is 0 N.

(b) Find the value of *F*.

Answer:

b) Horizontally: $(\rightarrow)$ $F - (x + 7) = 0$

Vertically: $(\uparrow)$ $(4x + 2) - 2g = 0$

Solving the second equation

$4x = 2g - 2$

$x = \dfrac{g - 1}{2}$

Using $g = 9.8$ $x = \dfrac{8.8}{2} = 4.4$

Substitue $x = 4.4$ into the first equation

$F - (4.4 + 7) = 0$

$F = 11.4$

$\therefore F = 11.4$ N

Examiner Tips and Tricks

- F_{net} is often used for **resultant force** in equations.
- Sketching diagrams, or adding to any given in a question, can help you to understand a problem and pick up some marks.

Your notes

Forces in 2D – Vector Notation

How are forces and vectors linked?

- **Forces** are **vectors** – they have **magnitude** and **direction**.
- The **magnitude** of a force is measured in Newtons and the **direction** of a force is an **angle**, usually measured in **degrees**, **anti-clockwise** from the **horizontal**
- There are other ways to talk about direction – in particular **Cartesian coordinates**(x, y) are used to describe the position of a point in two-dimensional space (plane), relative to a fixed **origin O.**
- You may see questions start with a sentence along the lines of "Relative to the frame of reference ***Oxy*** ..." – this means the two-dimensional space (plane) and distances within it will be based on **horizontal** (x) and **vertical** (y) components relative to the origin *O*.

What is vector notation?

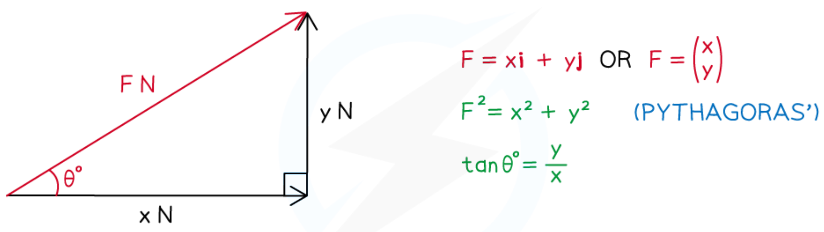
- Forces in **2D** should be given in the form p**i** +q**j** where **i** and **j** are unit vectors
- **Both column vectors** and **i, j notation** can be used for calculating **resultant vectors**
  - Column vectors can be useful when calculating with vectors as mistakes are less common, but you must remember to change the answer back to **i, j notation** form
- A force written as a vector is written with its two components separated so the **magnitude** and **direction** of the vector are **not** directly known

What are the notations for magnitude and direction?

- The **magnitude** of the force **F N** would be denoted by |**F**| N or *F N* – notice the use of bold and italics in particular
- **Direction** is an angle, usually measured in degrees anti-clockwise from the horizontal, $\theta°$ is usually used

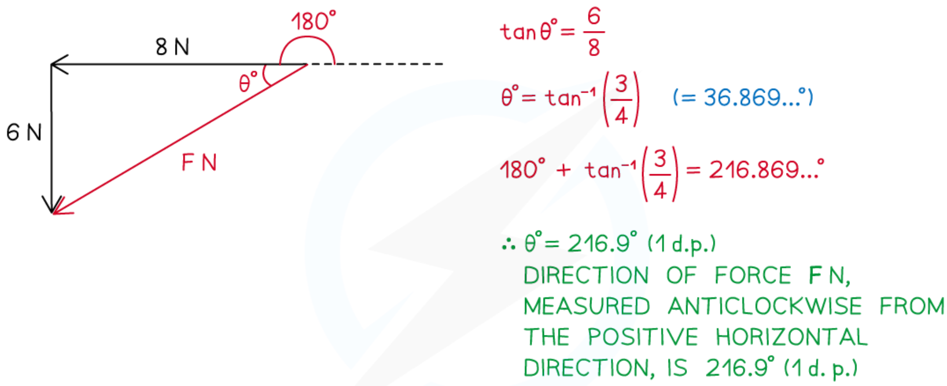
How do I find the magnitude and direction of a force from its components?

- For a force **F** = x**i** + y**j** N to find
  - its **magnitude**, |**F**| N or F N, use Pythagoras' theorem
  - its **direction** (as an angle), use a diagram and trigonometry



FINDING THE MAGNITUDE AND DIRECTION OF A FORCE

- If either (or both) components are **negative** then still use a diagram and $\tan\ \theta° \ = \ \frac{y}{x}$ but
  - treat *x* and *y* as **positive** (so |x| and |y| strictly speaking)
  - the angle found from $\tan\ \theta° \ = \ \frac{y}{x}$ may need **adjusting** depending on where, and which way, the **direction** is being **measured** from
    e.g. Find the direction at which the force **F** = (-8**i** - 6**j**) N, giving your answer as an angle measured in degrees anti-clockwise from the positive horizontal direction



FINDING THE MAGNITUDE AND DIRECTION OF A FORCE WHEN COMPONENTS ARE NEGATIVE

What does equilibrium with vectors mean?

- In two dimensions a **particle** is in **equilibrium** if the **resultant force** acting on it in both directions is **zero**
  - For vectors in **i-j notation**, a **resultant force** of zero would look like (0**i** + 0**j**) N
  - Both forms may be written as **0** N (called 'the **zero vector**')

Worked Example

Your notes

Two forces, (3**i** + 8**j**) N and (—5**i** — ll**j**) N act on a particle.

(i) Find the resultant force.

(ii) Find the magnitude of the resultant force and its direction as an angle measured anti-clockwise from the **i**-direction.

(iii) A third force is applied to the particle such that it is brought into equilibrium. Find the third force, giving your answer in the form $(x\mathbf{i} + y\mathbf{j})$ N

Answer:

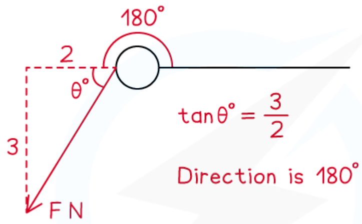
(i) Let the resultant force be $\underline{F}$ N.

$\underline{F} = (3\underline{i} + 8\underline{j}) + (-5\underline{i} - 11\underline{j})$

$\therefore\ \underline{F}\text{ N} = (-2\underline{i} - 3\underline{j})\text{ N}$

(ii) $F^2 = (-2)^2 + (-3)^2 = 13$

Draw a diagram to help find the required direction



$\tan\theta° = \dfrac{3}{2}$

Direction is $180° + \tan^{-1}\left(\dfrac{3}{2}\right)$

$\therefore$ Magnitude of $\underline{F}$ N is $F\text{ N} = \sqrt{13}\text{ N}$
The direction of $\underline{F}$ N is 236.3° (1 d.p.)
anti-clockwise from the $\underline{i}$-direction

(iii) From part (a) we know the first two forces give a resultant force $\underline{F} = (-2\underline{i} - 3\underline{j})$N

$\therefore\ \underline{F} + (x\underline{i} + y\underline{j}) = 0 \qquad (\underline{0} = 0\underline{i} + 0\underline{j}$, the 'zero vector')

$(-2 + x) = 0, \quad x = 2$
$(-3 + y) = 0, \quad y = 3$

$\therefore$ The third force is $(2\underline{i} + 3\underline{j})$N

Your notes

Examiner Tips and Tricks

- Ideally you should stick to the **i-**,**j-** vector notation used in a question but if you prefer to use column vectors within calculations you can use a mixture within the same question. Just be careful your final answer is in the correct format.
- Draw diagrams – including 'mini'-diagrams of individual forces/vectors – this can help in understanding a problem and being accurate.