Edexcel International A Level (IAL) Maths: Mechanics 1

Your notes

Kinematics Graphs

Contents

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Displacement-time graphs

What is a displacement-time graph?

- **Displacement-time** graphs show the displacement of an object from a **fixed origin** as it moves in a **straight line**
- They show **displacement** (on the vertical axis) against **time** (on the horizontal axis)
- Displacement-time graphs can go **below the horizontal axis** whereas **distance-time graphs** can not
  - Distance can **not** be negative whereas displacement can be

What are the key features of a displacement-time graph?

- The **gradient** of the graph equals the velocity of the object
  - A **positive gradient** means the object is travelling **forwards**
  - A **negative gradient** means that the object is travelling **backwards**
  - The **steeper** the line, the **greater** the **speed**
- A **straight line** shows that the object is moving at a **constant velocity**
- A **curved line** shows that the object is **accelerating** or **decelerating**
- A **horizontal line** shows that the object is **stationary**
- If the graph **touches** the **x-axis,** then the object is **at the origin** at that time

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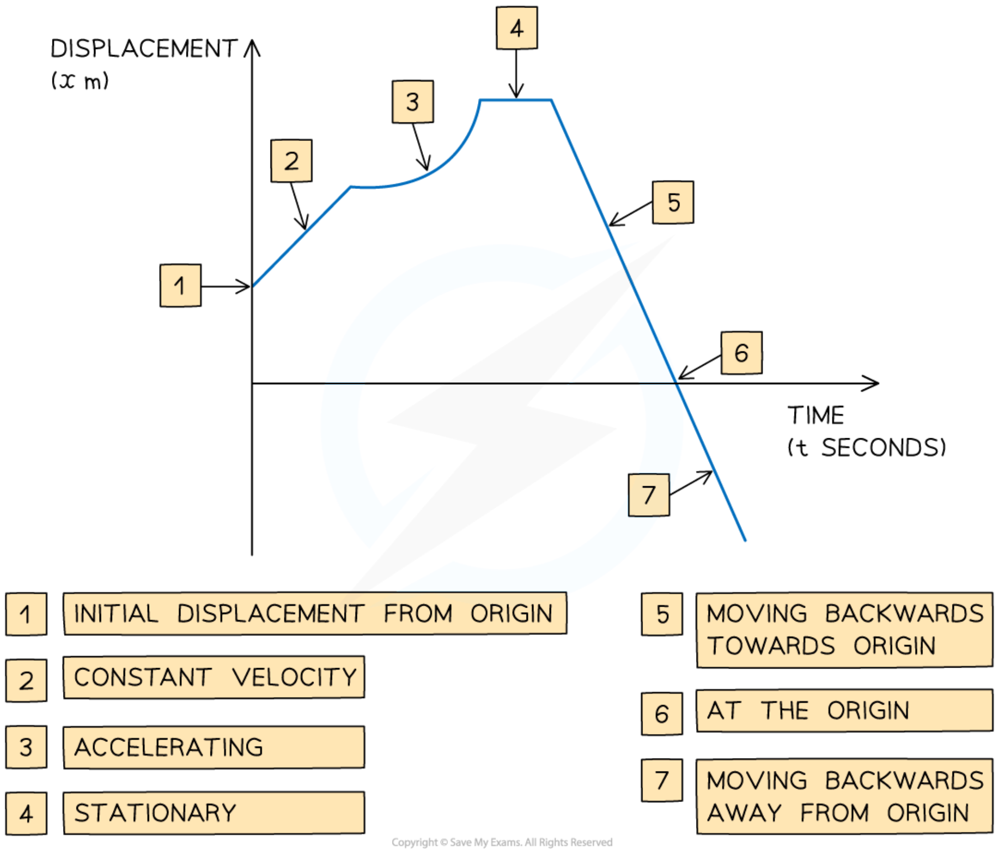


How do I find the average speed and the average velocity of a journey?

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Time taken}}$$

- The average speed can **not** be **negative** as speed is a **scalar** quantity and can only take a positive value

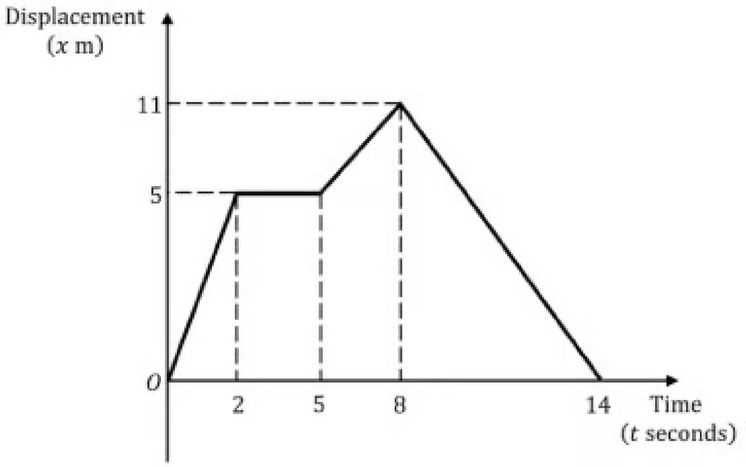
$$\text{Average velocity} = \frac{\text{Displacement from starting point}}{\text{Time taken}}$$

- The average velocity is a **vector** quantity and can be **positive**, **zero** or **negative**

Worked Example

An athlete is training by jogging in a straight line.

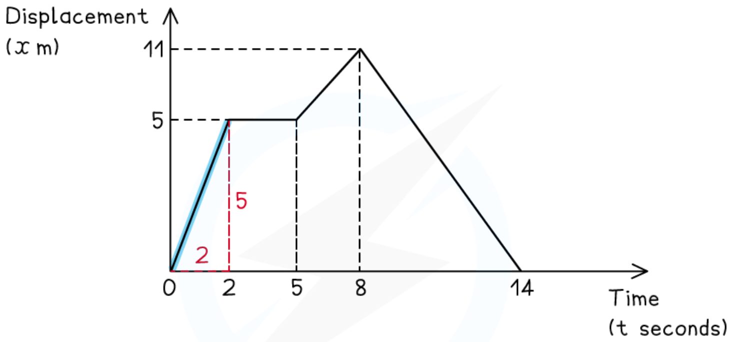
The displacement-time graph below shows the displacement of the athlete from her starting point throughout her motion.



(a) Calculate the velocity of the athlete during the first 2 seconds.

Answer:

a)



Velocity = Gradient

Velocity = $\frac{5}{2}$

Velocity = 2.5 ms^{-1}

Remember units!

(b) Describe the motion of the athlete between the times of 2 seconds and 5 seconds.

Answer:

b) Horizontal line means stationary

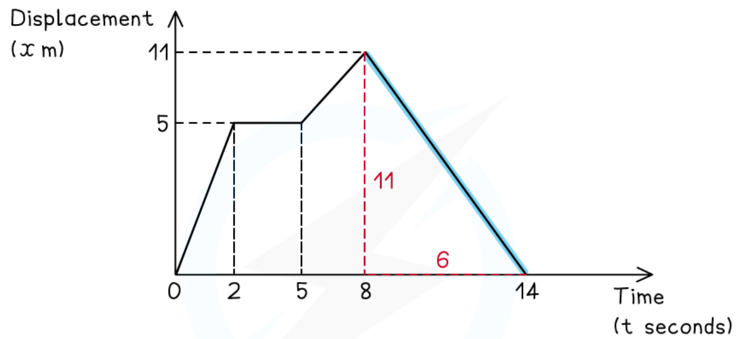
The athlete is stationary

(c) Calculate the velocity of the athlete at 10 seconds.

Answer:

Your notes

c)



Velocity = Gradient

$$\text{Velocity} = \frac{0 - 11}{14 - 8}$$

$$\text{Velocity} = -\frac{11}{6}\ \text{ms}^{-1} = -1.83\ \text{ms}^{-1}\ (3\ \text{sf})$$

Velocity is negative as the gradient is negative

(d) Find the total distance travelled by the athlete during the 14 seconds.

Answer:

d) Total distance = Distance forwards + distance backwards

Total distance = 11 + 11 = 22 m

Examiner Tips and Tricks

- Be careful to spot if you are working with a **displacement-time graph** or a **velocity-time graph**.
- Be careful to spot if you are working with a displacement-time graph or a distance-time graph
- Check where the graph starts from on the y-axis, the object does not have to start at 0. For example, if the graph starts at 100, the scenario could be a student on a walk starting 100 m from their house.
- Distance is a scalar so it can **not** be negative whereas displacement from a starting point can be.
- Think about the units when calculating, make sure they are consistent.

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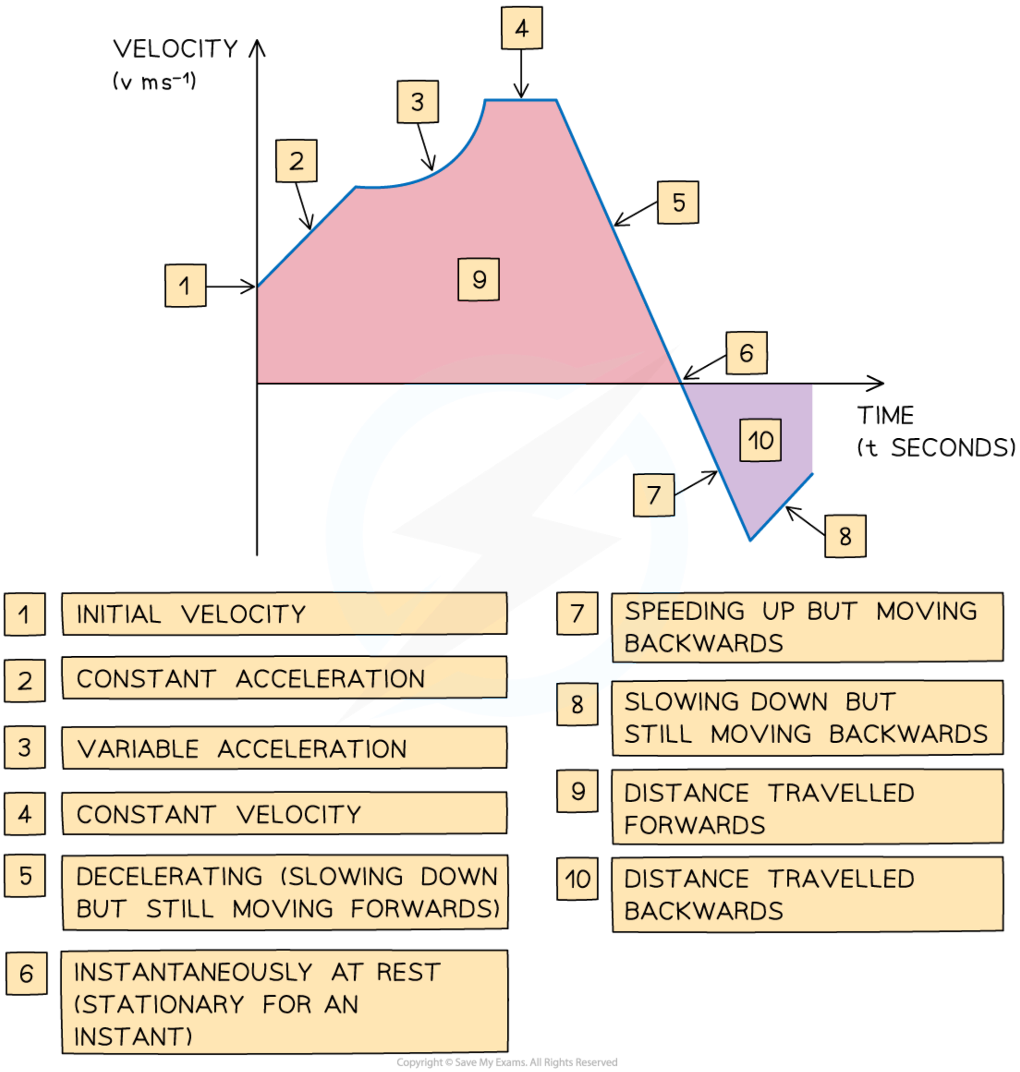
Velocity-time graphs

What is a velocity-time graph?

- **Velocity- time** graphs show the **velocity** of an object as it moves in a **straight line**
- They show **velocity** (on the vertical axis) against **time** (on the horizontal axis)
- Velocity-time graphs can go **below the horizontal axis** whereas **speed-time graphs** can not

What are the key features of a velocity-time graph?

- The **gradient** of the graph equals the **acceleration** of an object
- A **straight line** shows that the object is **accelerating** at a **constant rate**
- A **horizontal** line shows that the object is moving at a **constant velocity**
- The **area** between the graph and the x-axis tells us the **change in displacement** of the object
  - Graph **above** the x-axis means the object is moving **forwards**
  - Graph **below** the x-axis means the object is moving **backwards**
- The **total displacement** of the object from its starting point is the sum of the **areas above** the x-axis **minus** the sum of the **areas below** the x-axis
- The **total distance travelled** by the object is the sum of **all** the **areas**
- If the graph **touches** the **x-axis** then the object is **stationary** at that time
- If the graph is **above** the **x-axis** then the object has positive velocity and is **travelling forwards**
- If the graph is **below** the **x-axis** then the object has negative velocity and is **travelling backwards**



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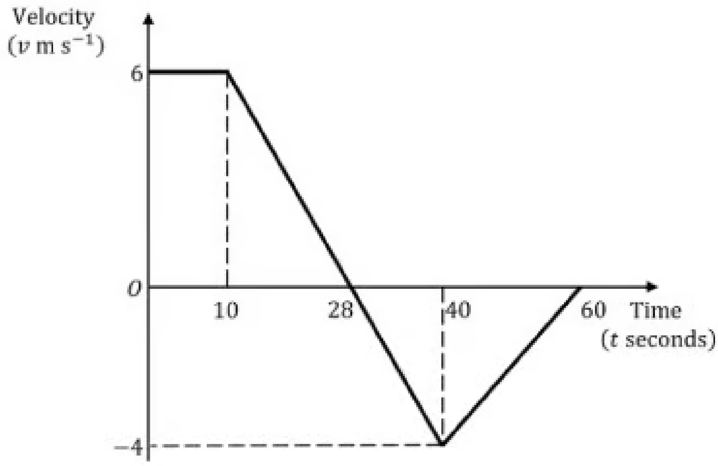
Worked Example

The motion of a bird travelling in a straight line is tracked for 60 seconds.

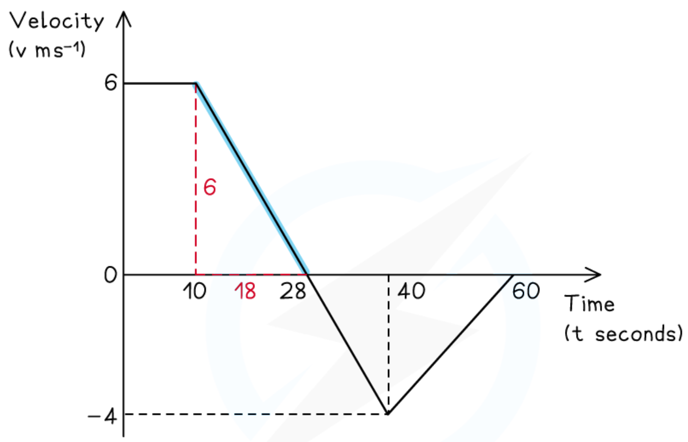
This information is displayed in the velocity-time graph below.



(a) Calculate the acceleration of the bird at 20 seconds.

Answer:

a)



Acceleration = gradient

Acceleration = $-\frac{6}{18}$ negative as gradient is negative

Acceleration = $-\frac{1}{3}$ ms^{-2} = -0.333 ms^{-2}

Remember units!

(b) Calculate the distance travelled in the first 28 seconds.

Answer:

b) Displacement = area

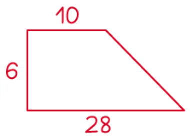
Step 1: Identify the shape

Step 2: Identify the lengths

Step 3: Find the area above x−Axis

So distance = displacement

Trapezium



$\frac{1}{2}(10 + 28)(6) = 114$

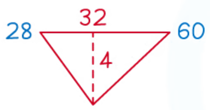
Distance = 114m

(c) Calculate the displacement of the bird from its starting point after 60 seconds.

Answer:

c) Distance forwards = 114 m

Step 1: Find the distance backwards by finding the area beneath the x−axis



Area $= \frac{1}{2}(32)(4) = 64$ m

THE BIRD HAS TRAVELLED BACKWARDS 64 m

Step 2: Calculate displacement by distance forwards − distance backwards

114 − 64

Displacement = 50 m from starting point

Examiner Tips and Tricks

- Be careful to spot if you are working with a distance-time graph or a velocity-time graph.
- Be careful to spot if you are working with a speed-time graph or a velocity-time graph.
- Check where the graph starts from on the y-axis, the velocity does not have to start at 0. For example, the scenario could be a car driving at a constant speed and the driver sees a hazard.
- Be extra vigilant when working with negative gradients or with graphs under the x-axis, it is easy to make mistakes with these.
- Speed is a scalar so it can **not** be negative whereas velocity can be.

Make sure your units are consistent.

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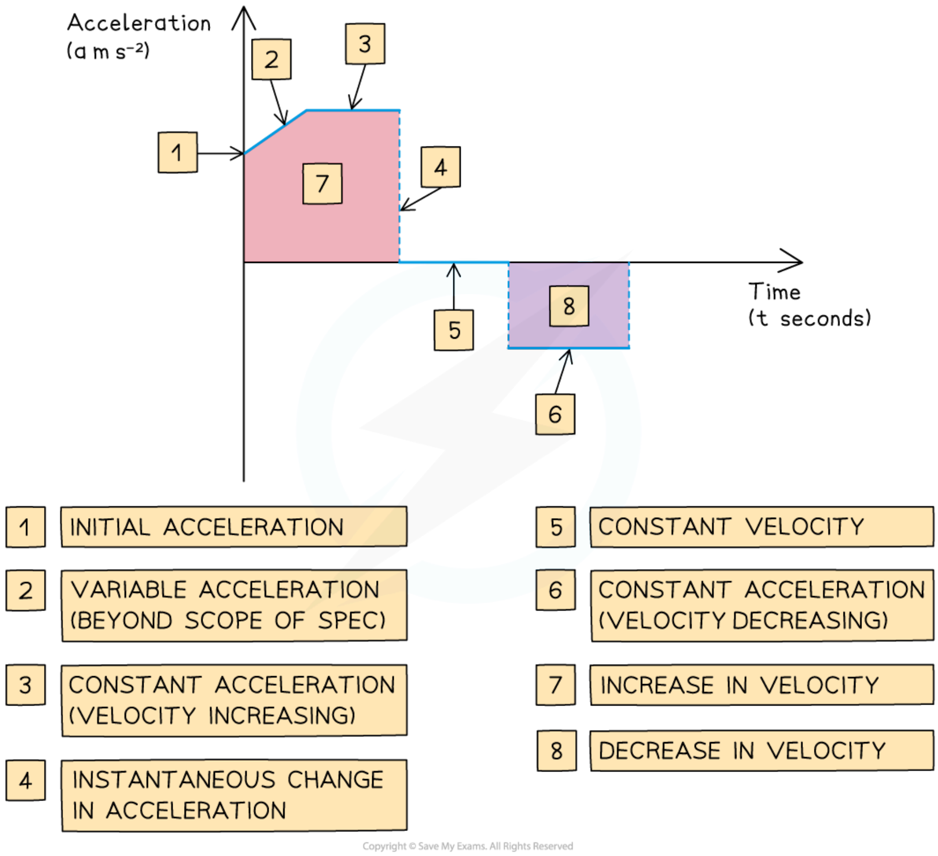
Acceleration-Time Graphs

What is an acceleration-time graph?

- **Acceleration-time** graphs show the **acceleration** of an object as it moves in a **straight line**
- They show **acceleration** (on the vertical axis) against **time** (on the horizontal axis)
- Acceleration-time graphs can go **below the horizontal axis**

What are the key features of an acceleration-time graph?

- A **horizontal** line shows that the object is accelerating at a constant rate
- If the graph **touches** the **x-axis** then the object is **moving with constant velocity** at that time
- If the graph is **above** the **x-axis** then the object has positive acceleration and **its velocity is increasing** which means
  - It is speeding up if it is moving forward
  - It is slowing down if it is moving backwards
- If the graph is **below** the **x-axis** then the object has negative acceleration and **its velocity is decreasing** which means
  - It is slowing down if it is moving forward
  - It is speeding up if it is moving backwards
- The **area** between graph and the x-axis tells us the **change in velocity** of the object
  - Graph **above** the x-axis means the **velocity** of the object is **increasing**
  - Graph **below** the x-axis means the **velocity** of the object is **decreasing**
- The **total change in velocity** of the object from its starting point is the sum of the **areas above** the x-axis **minus** the sum of the **areas below** the x-axis

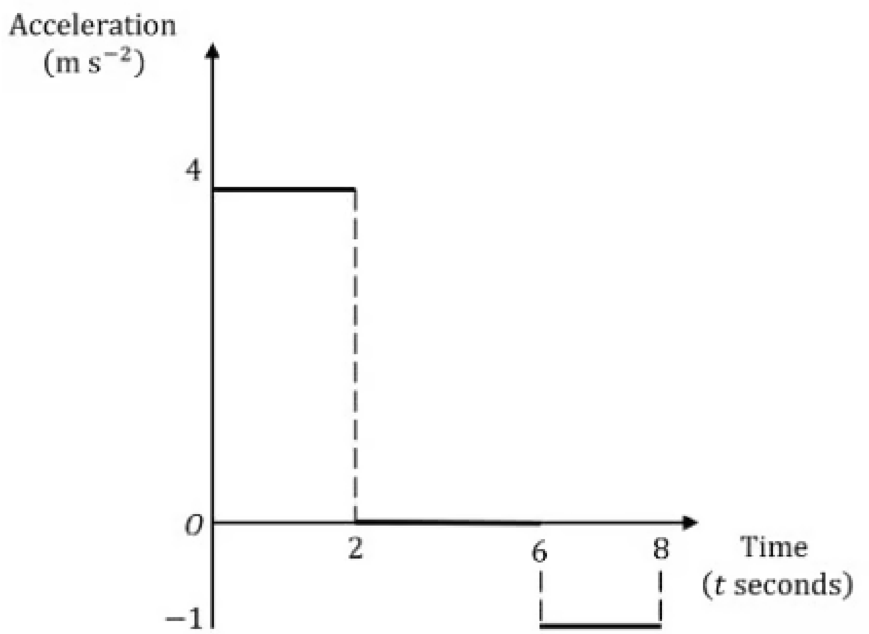


What does an acceleration-time graph NOT show?

- You can **not tell the velocity** of the object at a **specific time** just by looking at an acceleration-time graph
  - You can **calculate** this using the **areas** between the graph and the x-axis if you know the **initial velocity** of the object
  - If the acceleration-time graph is **positive** this does **not guarantee** that the velocity will be positive

Worked Example

A traffic tracks a car on the road for an 8-second period. The acceleration-time graph shows the journey of the car.

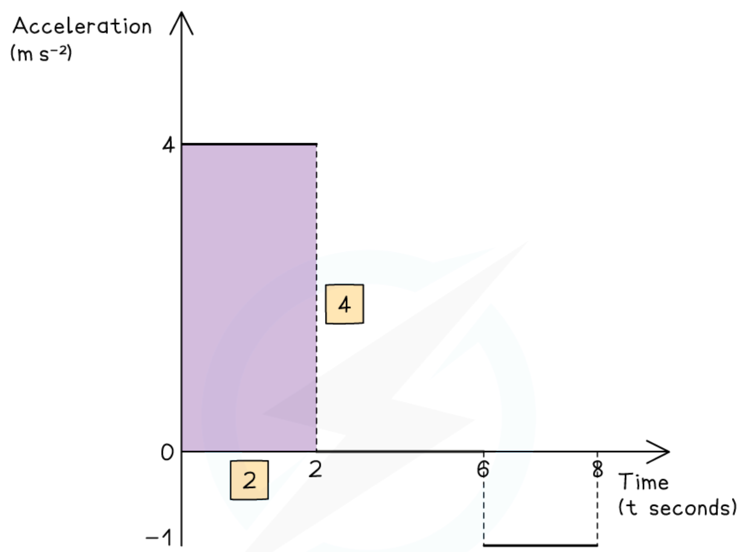


Given that the car is travelling with speed 25 m s^{-1} when the traffic officer starts tracking the car, find:

(i) the speed of the car after 2 seconds,

Answer:

a)



Area = change in velocity

Area = $4 \times 2 = 8$

Velocity increases by 8 ms^{-1}

$25 + 8 = 33$

Initial velocity

Speed = 33 ms^{-1}

(ii) the distance travelled between the times $t = 2$ and $t = 6$,

Answer:

b) Acceleration is zero $\Longrightarrow$ constant velocity

Constant speed $\Longrightarrow$ distance = speed × time

Distance = 33 × 4

Speed when t = 2

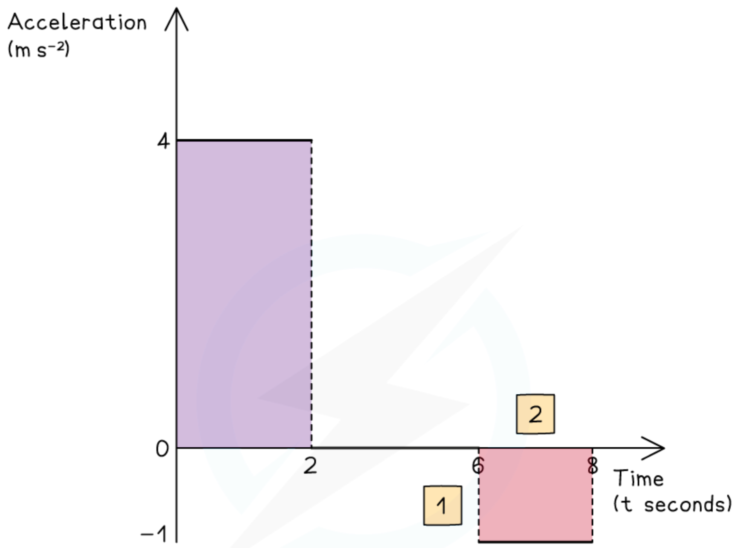
Time between t = 2 and t = 6

Distance = 132 m

(iii) the speed of the car after 8 seconds.

Answer:

c)



Area = change in velocity

Area = 2 × 1 = 2 Between t = 6 and t = 8

Velocity decreases by 2 ms^{-1}

33 − 2 = 31

Speed = 31 ms^{-1}

Examiner Tips and Tricks

- Most of the time the acceleration-time graphs you will encounter will comprise of three disjointed horizontal lines, acceleration-time graphs will not have curved or diagonal lines for this course. Though in real life acceleration is likely to vary.

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Drawing Travel Graphs

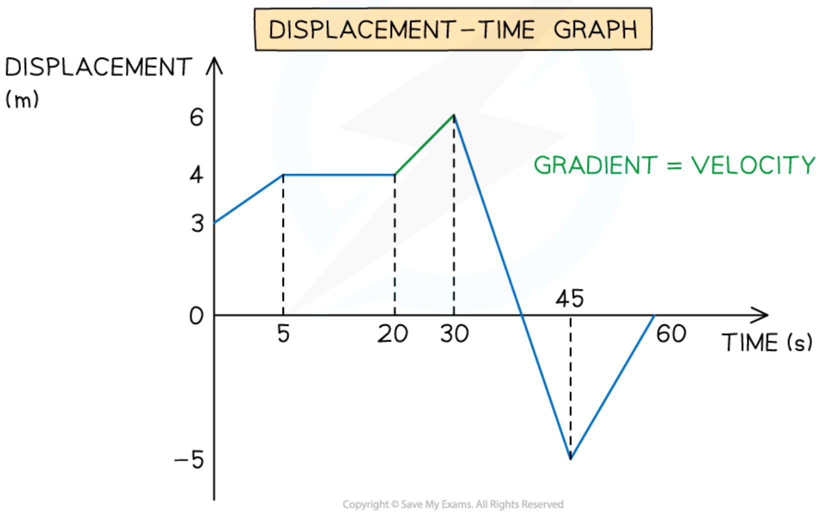
How do I draw a displacement-time graph or a velocity-time graph?

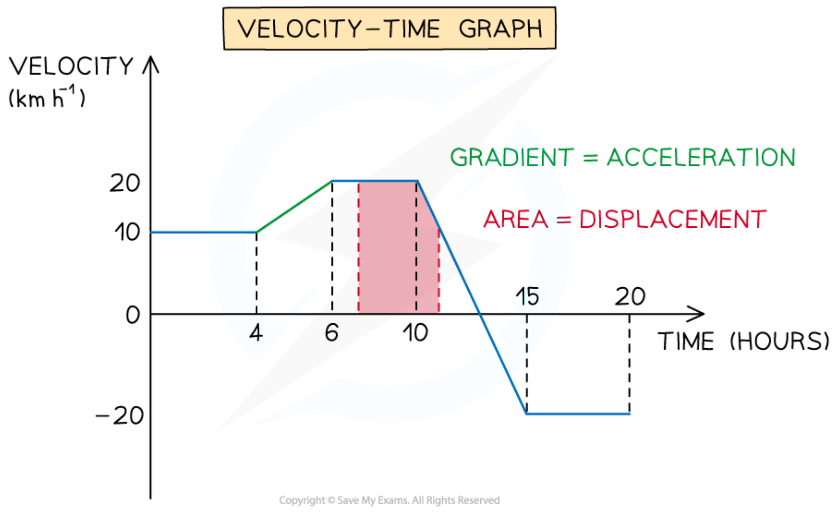
- You may be asked to draw a **displacement-time** graph or a **velocity-time** graph
- You will be given information about different sections of a journey and must draw each section and **label** the **points** on the axes
- Remember that time will always be on the x-axis
- Always remember to include **units** when you label the axis
- Use appropriate facts about the **gradient** and **area** under the graph to work backwards and find relevant or missing information
  - The **gradient** of a **displacement-time** graph is the **velocity** and the **gradient** of a **velocity-time** graph is the **acceleration**
  - The **area** between a **velocity-time** graph and the x-axis is the **displacement**

STEP 1: DRAW APPROPRIATE AXES
(LABEL DISPLACEMENT / TIME OR VELOCITY / TIME)

STEP 2: MARK ON KEY POINTS GIVEN IN THE QUESTION
(LABEL THESE ON THE AXES)

STEP 3: DRAW IN THE LINES TO COMPLETE THE GRAPH





How do I draw an acceleration-time graph?

- Most of the time the acceleration will be **constant** so you have to draw **horizontal** lines
- There will be **discontinuities** in the graph if the object instantaneously changes from **acceleration at a constant rate** to moving with **constant velocity**
- The **area** between an acceleration-time graph and the x-axis is the **change in velocity**
- You might not be given the acceleration but instead expected to calculate it
  - Remember acceleration is the **rate of change of velocity**
  - If **acceleration is constant** then:
    - $$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}}$$

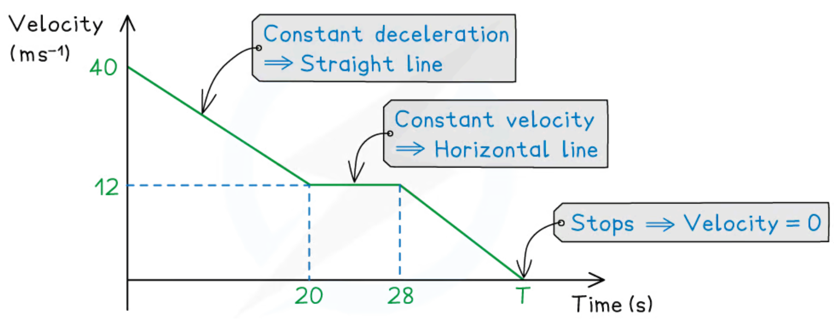
Worked Example

The driver of a train, travelling at 40 m s^{-1} on a straight horizontal track, is informed of debris on the track 0.74 km ahead and immediately applies the brakes.
The train decelerates uniformly for 20 seconds to a speed of 12 m s^{-1}.
The train then maintains this speed for another 8 seconds.
The driver reapplies the brakes to slow down the train at a constant rate and bring it to a complete stop 10 m before the debris.

(a) Sketch a velocity-time graph to show the motion of the train.

Answer:

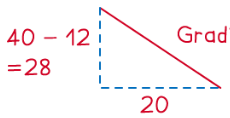
a)



(b) Find the deceleration of the train from the moment the brakes were first applied to the moment its speed first reached 12 m s^{-1}.

Answer:

b) Gradient of velocity − time graph is the acceleration



$\text{Gradient} = -\dfrac{28}{20} = -1.4$

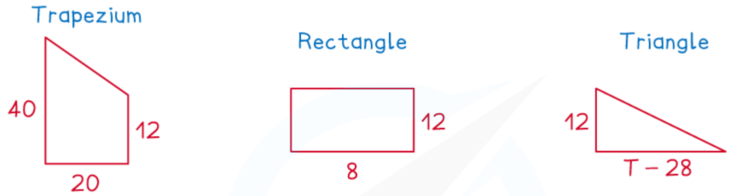
Question says deceleration already so we don't need the negative sign

Deceleration = 1.4 ms^{-2}

(c) Calculate the total time from the moment the brakes were first applied to the moment the train came to rest.

Answer:

c) Area under velocity − time graph is the displacement



$\frac{1}{2}(12 + 40)(20) = 520$ $\quad (8)(12) = 96$ $\quad \frac{1}{2}(12(T - 28)) = 6(T - 28)$

Stops 10 m before debris

0.74 km − 10 m = 730 m

Areas should add to 730

$520 + 96 + 6(T - 28) = 730$

$6(T - 28) = 114$

$T - 28 = 19$

$T = 47$

Time = 47 seconds

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Examiner Tips and Tricks

- As in the worked example, examiners can use other words, such as uniformly, to mean constant.
- Remember that displacement and velocity can be negative whereas distance and speed can **not** be negative.
- Take care when a velocity-time graph is below the x-axis, if it has a negative gradient then it is speeding up and moving backwards. If it has a positive gradient below the x-axis then it is still moving backwards but it is slowing down.