Edexcel International A Level (IAL) Maths: Mechanics 1

Your notes

Working with Vectors

Contents

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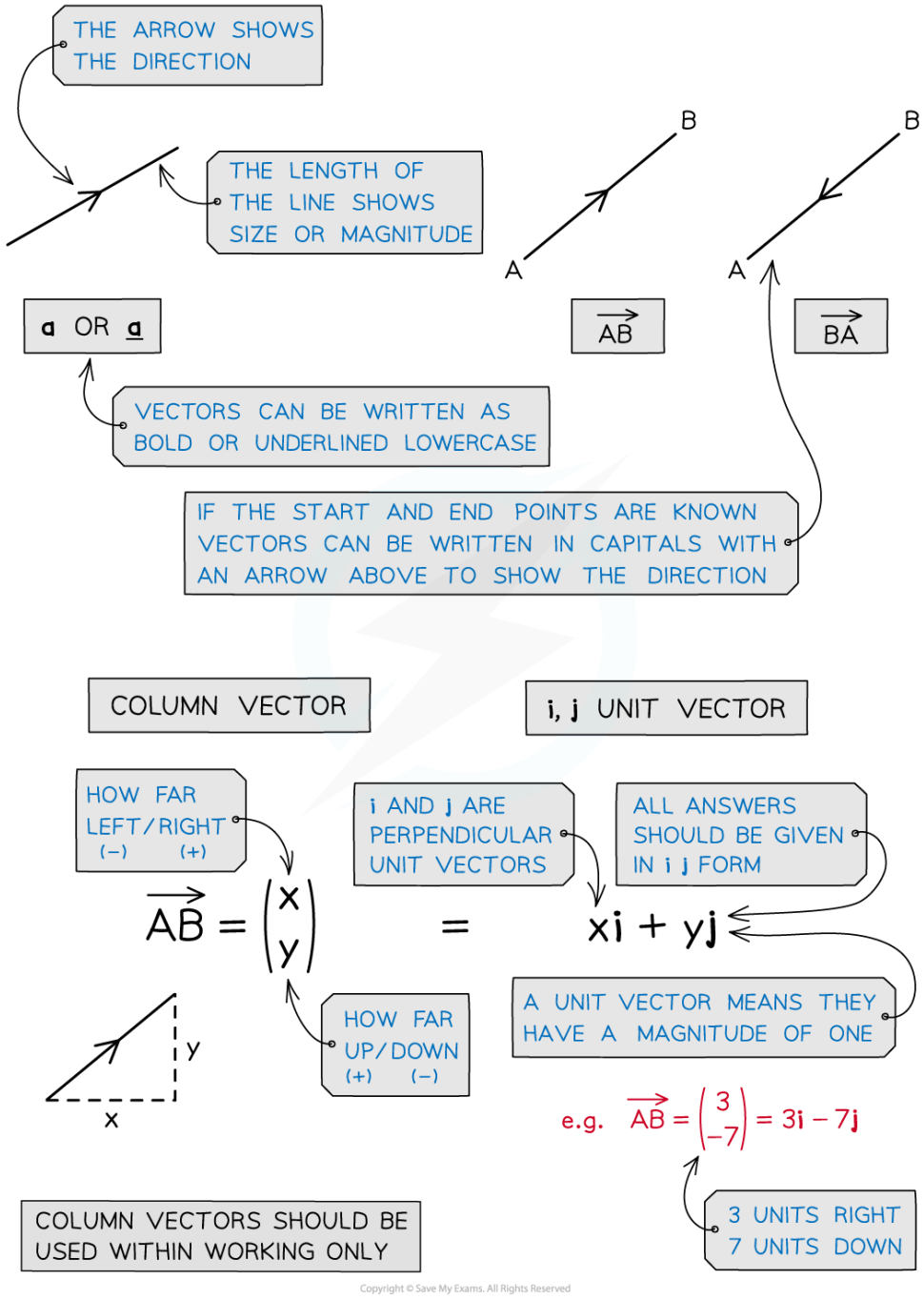
Working with Vectors

Vectors represent a movement of a certain **magnitude** (size) in a given **direction.** They are used throughout mechanics to describe forces and motion

How are Vectors used in Mechanics?

- **Vector** questions are often embedded in a Mechanics context
- Vectors will most commonly represent **forces** , **accelerations** or **velocities**, they can also represent displacement
- Newton's Second Law ***F* = *ma*** is essential
  - Remember that ***F*** and ***a*** (force and acceleration) are **vectors**, while m (mass) is a **scalar**

When a particle is in **equilibrium**, the vector sum of the forces on it is equal to zero



What is vector notation?

- There are **two vector notations** used in A level mathematics:
  - **i** and **j** notation: **i** and **j** are **unit vectors** (they have **magnitude 1**) in the positive horizontal and positive vertical directions respectively
    e.g. The vector (-4**i** + 3**j**) would mean **4 units** in the **negative horizontal** (*x*) direction (i.e. **left**) and **3 units** in the **positive vertical** (*y*) direction (i.e. **up**)
  - **Column vectors**: This is one number written above the other enclosed in brackets,

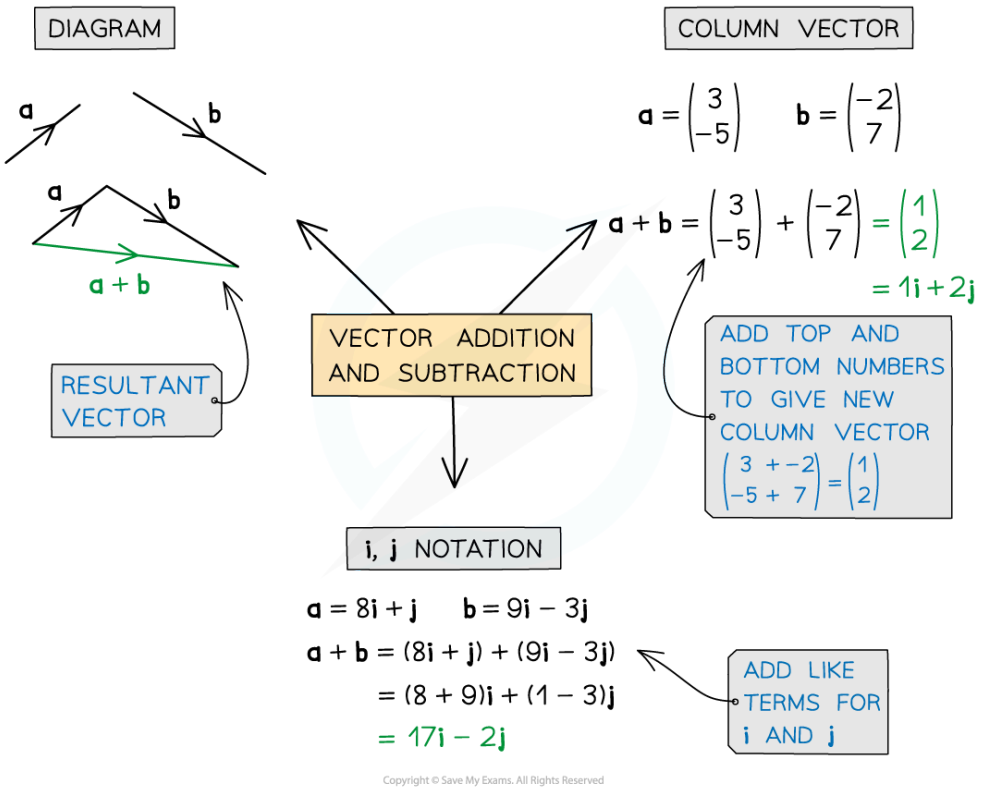
e.g. The (column) vector meaning **3 units** in the **positive horizontal** (x) direction (i.e., **right**) and **2 units** in the **negative vertical** (y) direction (i.e., **down**) can be written as:

$$\begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

- In Edexcel International A Level Mechanics you must give your **final answer** using the **i** and **j** notation form
  - You may use column vectors within your working to make calculations easier
  - Remember to convert your final answer back to the correct form
- As they are vectors, **i** and **j** are displayed **bold** in textbooks and online but in handwriting they would be underlined (i and j)

Calculating Resultant Vectors:

- **Adding** vectors together gives the **resultant vector**
- The vectors can be placed nose to tail for a diagram of the resultant vector
- Both **column vectors** and **i, j notation** can be used for calculating **resultant vectors**
  - Remember to give final answer using **i, j** notation
- This is the same when adding force vectors; the **resultant force** is simply the force vectors added together
- Forces in **equilibrium** have a resultant force equal to **zero**



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Calculating Magnitude and Direction:

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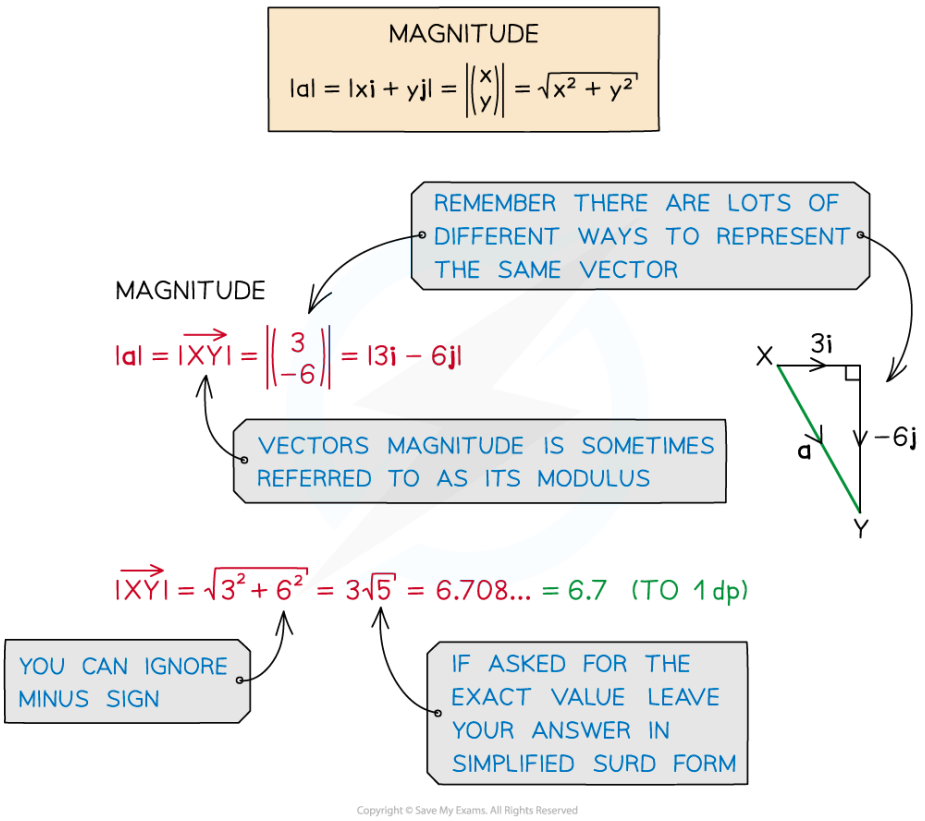
- Pythagoras is used to find the **magnitude** of a vector.
  - The magnitude of a displacement vector is the distance
  - The magnitude of a velocity vector is the speed
- Trigonometry is used to find the **direction** of a vector
  - Always draw a diagram, the **i** and **j** components will be the opposite and adjacent sides of the right-angled triangle
  - The direction is given as the angle the vector makes with the horizontal
- You may be asked for the direction as a **bearing.** The **unit vector j** can be used to represent north and the unit vector **i** can be used to represent east.
  - A unit vector has a magnitude of one
  - In the case where $k > 0$
- If a particle is moving north then its velocity will have a vector of $k\mathbf{j}$
- if a particle A is due south of another particle B then the displacement vector from B to A will have the form $-k\mathbf{j}$
- If a particle is moving east then its velocity will have a vector of $k\mathbf{i}$
- if a particle A is due west of another particle B then the displacement vector from B to A will have the form $-k\mathbf{i}$
- If the position vectors of two particles have the same **j** component, then the particle with the greater **i** component will be positioned due east of the other
- If the position vector of a particle has equal **i** and **j** components then it is positioned due north - east of the origin
  - if a particle A is north – east of another particle B then the displacement vector from B to A will have equal **i** and **j** components



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Resolving Vectors:

- A single **vector** can be broken down into its parts, or components, that will be perpendicular to each other
- **Resolving** a vector means writing it in **component** form (as **i** and **j** components)
- Given the **magnitude** and **direction** of a vector you can work out its components and vice versa

Worked Example

Two forces $\mathbf{F_1}$, $\mathbf{F_2}$ act on a particle P. $\mathbf{F_1} = -7\mathbf{i} + 3\mathbf{j}$ and $\mathbf{F_2} = 4\mathbf{i} + 6\mathbf{j}$.

a) Find the magnitude of the resultant force **R** acting on the particle.

Leaving your answer as a simplified surd.

Answer:

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a) Find the resultant force $R = F_1 + F_2$

$R = (-7\underline{i} + 3\underline{j}) + (4\underline{i} + 6\underline{j}) = \begin{pmatrix} -7 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ 9 \end{pmatrix} = (-3\underline{i} + 9\underline{j})N$

Using column vector notation makes addition and subtraction easier

Magnitude of $R = |R| = \sqrt{(-3)^2 + 9^2} = \sqrt{90} = 3\sqrt{10}$ N

Find magnitude using Pythagoras

Leave answer in simplified surd form

Magnitude $= 3\sqrt{10}$

b) Find the bearing of the resultant force **R**.

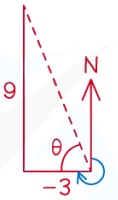
Give your answer to the nearest degree.

Answer:

b) Draw a sketch of the resultant vector to help calculate the correct angle:

$R = (-3\underline{i} + 9\underline{j})N$

3 units to the left
9 units up



The bearing is measured from north

The bearing is measured in the clockwise direction

Ignore any negatives when calculating the angle

$\tan\theta = \frac{9}{3}$

$\theta = \tan^{-1}(3) = 71.565...$

Bearing $= 270 + 71.565... = 341.565...$

Bearing $= 342°$

A third force $\mathbf{F_3} = s\mathbf{i} - t\mathbf{j}$ brings the particle into equilibrium.

c) Find s and t and state the force for $\mathbf{F_3}$ in terms of **i** and **j**.

Answer:

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c) Forces in equilibrium have a resultant force equal to zero:

$F_1 + F_2 + F_3 = 0 \Rightarrow R + F_3 = 0$

$(-3\underline{i} + 9\underline{j}) + (s\underline{i} + t\underline{j}) = 0\underline{i} + 0\underline{j}$

Find the values of s and t such that the i and j components add up to zero

Using R from part (a)

$\begin{pmatrix} -3 \\ 9 \end{pmatrix} + \begin{pmatrix} s \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-3 + s = 0 \quad 9 + t = 0$

$s = 3 \quad t = -9$

$s = 3$

$t = -9$

$F_3 = 3\underline{i} - 9\underline{j}$

Examiner Tips and Tricks

- When working with vectors pay attention to accuracy, leaving magnitude in surd form or correct to 3.s.f.
- In your exam you can't write in **bold** so should underline your vector notation