Edexcel International A Level (IAL) Maths: Mechanics 1

Your notes

Moments

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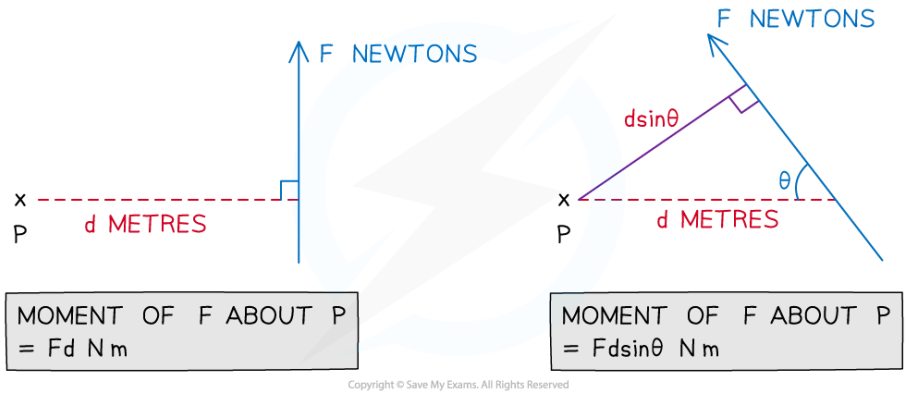
Moments diagrams

When an object is modelled as a **particle** we do **not** need to consider the **rotation** of the object due to forces. When we model an object as a **rigid body** then we need to consider the rotation of the object.

What is a moment?

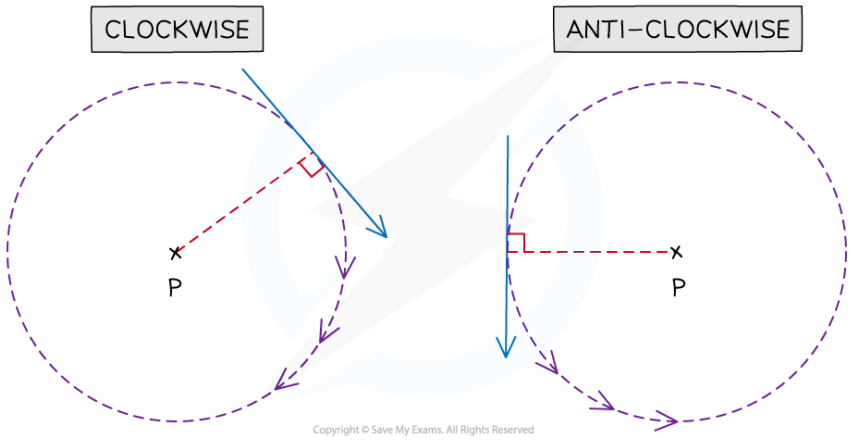
- The **moment** of a force is the measure of its ability to cause a body to **rotate** about a specific point (usually called the **pivot**)
- The **units** for moments are **newton metres** (N m)
- A force can cause a body to rotate **clockwise** or **anti-clockwise**
- The moment of a force about a point is calculated by **multiplying** the magnitude of the **force** by the **perpendicular distance** from the line of action of the force to the point
  - We can choose the **positive direction** to be **clockwise** or **anti-clockwise** and the **negative direction** is the **opposite**
  - A **zero moment** will not cause a body to rotate
- The **moment** of a force about a point is **zero** if the line of action of the force goes through the point
- If the force is not acting perpendicular to the distance given you will need to use right-angled **trigonometry** to find the perpendicular distance



How do I know if a moment is clockwise or anti-clockwise?

- A clockwise moment will cause a body to rotate in the clockwise direction
- To visualise this:

- Imagine that the perpendicular line from the point to the force is the radius of a circle
- The direction of the force tells you which way the circle goes round



What is the resultant moment?

- The **resultant** moment is the **sum** of **all** the moments acting on a body (both positive and negative)
- To find the resultant moment:
  - Define the positive direction (clockwise or anti-clockwise)
  - **Add together** all the moments in **that direction**
  - **Subtract all** the moments in the **opposite direction**
  - The overall value is the resultant moment
- The **sign** of the **resultant** moment indicates which direction the body will rotate
  - Positive will be the direction chosen as positive
  - Negative will be the opposite direction

Worked Example

Calculate the resultant moment about P of the forces indicated in the following diagram.

Answer:

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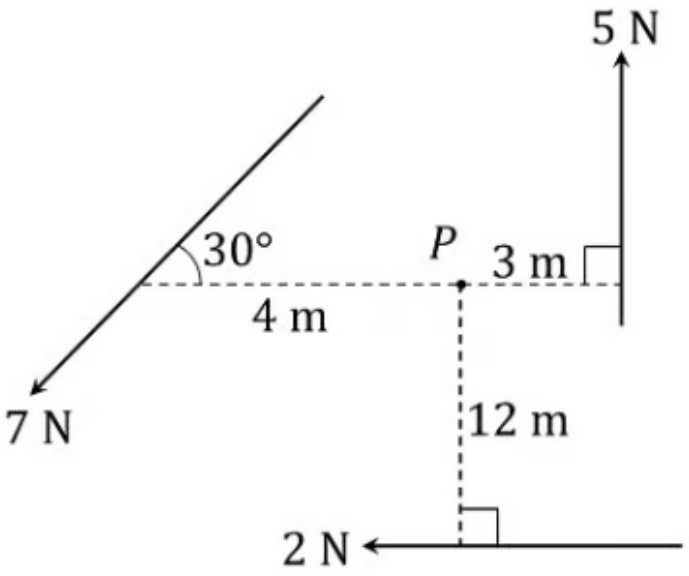
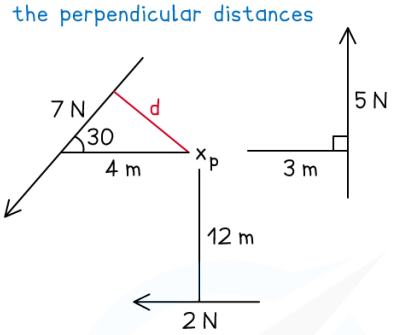
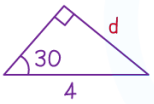


Find the perpendicular distances



Find d by using SOHCAHTOA



$\sin 30 = \frac{d}{4}$

$d = 4\sin 30$

Let clockwise be the positive direction

$2 \times 12 = 24$ Nm Clockwise ↻

$5 \times 3 = 15$ Nm Anti-clockwise ↺

$7 \times 4\sin 30 = 14$ Nm Anti-clockwise ↺

To find the resultant moment

$24 - 15 - 14 = -5$ Nm

Negative means anti-clockwise

Resultant moment 5 Nm anti-clockwise

Examiner Tips and Tricks

- Always define a positive direction as either clockwise or anti-clockwise.
- It is best to state the magnitude of a moment and write its direction when giving a final answer.

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Using moments with equilibrium

What is the resultant moment for a body in equilibrium?

- If a rigid body is in **equilibrium** then:
  - The **resultant force** is 0 N
  - The **resultant moment** about **any point** is 0 N m
- The sum of the magnitudes of the **clockwise moments equals** the sum of the magnitudes of the **anti-clockwise moments**

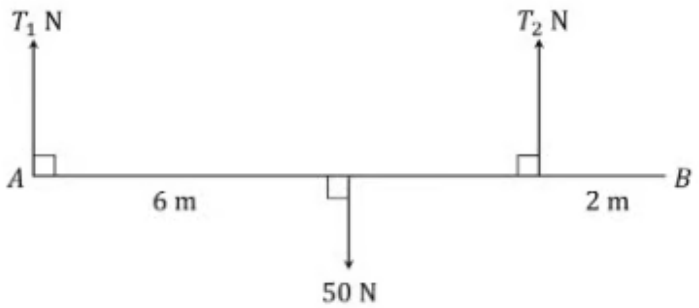
How do I use F = ma and moments to solve equilibrium problems?

- Step 1: Identify **all forces** and **perpendicular distances** from the pivot
- Step 2: Use the fact that the **resultant force is 0 N** to form an equation
- Step 3: Use the fact that the **resultant moment** about any point is **0 N m** to form equations
  - If there is an unknown force you can choose a pivot that is on the line of action of that force – this will mean that the moment of that force is 0 N m
- Step 4: **Solve** the equations

Worked Example

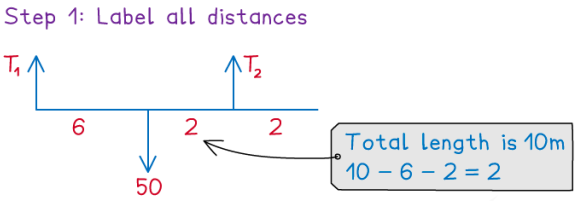
A light rod AB of length 10 m is held in equilibrium by three forces: 50 N, T_1 N and T_2 N.

T_1 N acts vertically upwards at A, 50 N acts vertically downwards 6 m away from A and T_2 N acts vertically upwards 2 m away from B as shown in the diagram.



Find the values of T_1 and T_2.

Answer:



Step 2: Form a force equation

R (↕) $T_1 + T_2 = 50$

Step 3: Form a moment equation

M(A) $50 \times 6 = 300$ Clockwise ↻

$T_2 \times 8 = 8T_2$ Anti-clockwise ↺

$8T_2 = 300$

Taking moments about A allows us to eliminate T_1

Step 4: Solve

$8T_2 = 300$

$T_2 = 37.5$

$T_1 + T_2 = 50$

$T_1 + 37.5 = 50$

$T_1 = 12.5$

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Examiner Tips and Tricks

- If you see a force diagram that involves distances then you can be fairly sure that moments will be involved! A clear diagram showing all the distances is key.
- If you pick a pivot and form a moment equation which involves more than one unknown, try choosing another pivot to form simultaneous equations.
- Pick your pivot wisely as unknown and unwanted forces can be eliminated from the equation by choosing the point to be on the line of action of a force.

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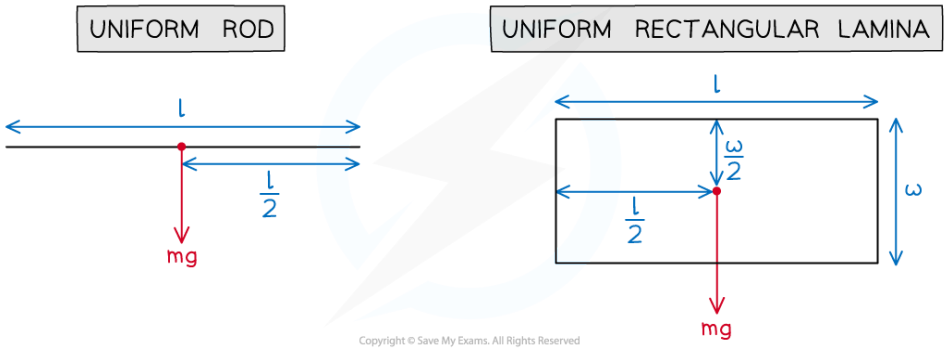
Uniform rods & laminae

What is the centre of mass of an object?

- The centre of mass of an object is **the point at which the weight of the object may be considered to act**
- For a **uniform object** the centre of mass is at the centre of the object where the lines of symmetry intersect
  - For a **uniform rod** this will be at its midpoint
  - For a **uniform rectangular lamina** this will be where the diagonals intersect
- For a **non-uniform** object the centre of mass is not necessarily at the centre of the object

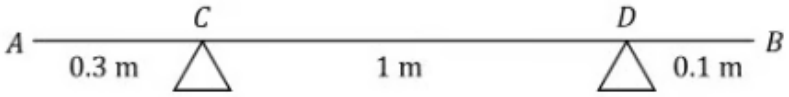


Where is the centre of mass of a uniform rod?

- If you are told that a **rod is uniform** then you can draw the weight at the **midpoint of the rod**
- If a rod lies on a **support or peg** then there will be a **normal reaction force** which acts **perpendicular to the rod** at that point
- If the rod is **suspended** by **strings** or **cables** then there will be **tensions** in the strings which keep the **rod in place**

Worked Example

A uniform rod AB of mass 20 kg and length 1.4 m rests in equilibrium on two supports at points C and D. AC = 0.3 m and BD = 0.1 m.

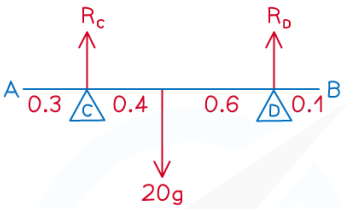


Find the magnitudes of the normal reaction forces at C and D.

Answer:

Let R_C and R_D be the normal reaction forces.
Uniform rod $\Rightarrow$ centre of mass is 0.7m from A

Step 1: Draw a diagram with all forces and distances



Step 2: Form a force equation

R (↕) $R_C + R_D = 20g$

Step 3: Form a moment equation

M(C) $20g \times 0.4 = R_D \times 1$

$R_D = 8g$

Step 3: Form a moment equation

$R_C + R_D = 20g$

$R_C + 8g = 20g$

$R_C = 12g$

Reaction force at C = 12g N = 120 N (2 sf)
Reaction force at D = 8g N = 78 N (2 sf)

Examiner Tips and Tricks

- If there are two supports with unknown reaction forces then choosing the pivot to be at one of the supports will help to find the force at the other support. The same method works with strings too.

Non-uniform rods

How do I find the centre of mass of a non-uniform rod in equilibrium?

- **Step 1**: Label the weight of the rod at a random point on the rod
- **Step 2**: **Call** the perpendicular **distance** between the pivot and the weight *x*

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- Or any letter of your choice

- **Step 3**: **Form** a **moment equation** using the fact that the rod is in **equilibrium**
- **Step 4**: **Solve** the equation to find *x*

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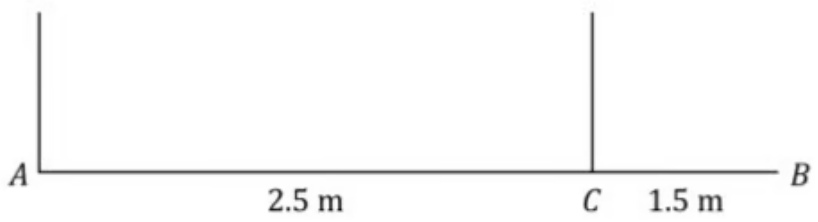
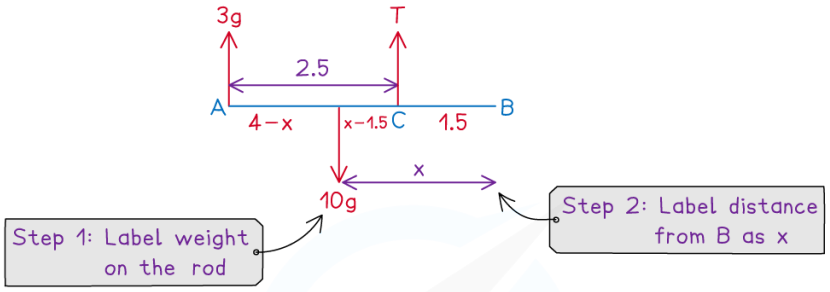
Worked Example

A non-uniform rod AB of mass 10 kg and length 4 m is suspended by two strings at points A and C with AC = 2.5 m. When the tension in the string at A is 3g N the rod hangs horizontally in equilibrium as shown in the diagram.



Find the distance of the centre of mass of the rod from B.

Answer:



Step 3: Form a moment equation

M(C) $3g \times 2.5 = 10g \times (x - 1.5)$

$7.5g = 10g(x - 1.5)$

Step 4: Solve

$10g(x - 1.5) = 7.5g$ Cancel the g's

$x - 1.5 = \frac{7.5}{10}$

$x - 1.5 = 0.75$

$x = 2.25$

Distance from centre of mass to B is 2.25 m

Examiner Tips and Tricks

- Make sure you read carefully which distance you are asked to find.

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Tilting

What does it mean if a rod is on the point of tilting?

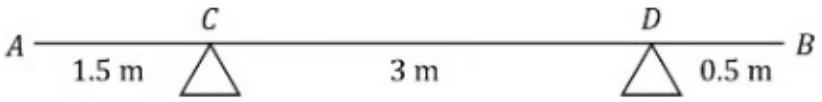
- A rod will **tilt** (rotate) about a **pivot** if the **resultant moment** about that pivot is **non-zero**
- Examples of tilting:
  - A person walking a plank on a ship will get to a certain distance before the plank tilts
  - Children playing on a seesaw

How do I solve problems involving a rod on the point of tilting?

- If a rod is on the **point of tilting** about a pivot (at a support) then the **reaction forces** at the **other** supports are **zero**
- The rod is still in **equilibrium** at the point of tilting
- **Taking moments about the pivot** is the key step as it allows you to ignore the reaction force
- The question might not use the phrase "tilting"
  - It could ask you how far along a plank that it is safe to walk (it is safe as long as the plank does not tilt)
  - It could ask you to find the maximum mass that could be hung from a rod that would keep it horizontal (as soon as it begins to tilt it is no longer horizontal)

Worked Example

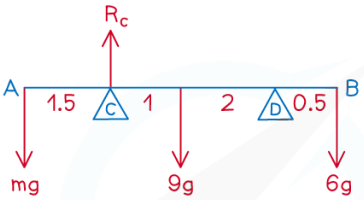
A uniform rod AB of mass 9 kg and length 5 m rests on two supports at two points C and D as shown in the diagram below.



A mass of 6 kg is placed at B and a mass of *m* kg is placed at A. Given that the rod is on the point of tilting about C, find the value of m.

Answer:

Point of tilting about C so reaction force at D is 0 N



Take moments about C so we can ignore R_C

Anti-clockwise Clockwise

M(C) $mg \times 1.5 = 9g \times 1 + 6g \times 3.5$

$1.5mg = 30g$ cancel the g's

$m = \frac{30}{1.5}$

$m = 20$

Examiner Tips and Tricks

- Remember to use common sense! If a rod was supported by two supports and a mass was placed on the rod in between these supports then the rod would never tilt (in real-life it might break though).

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