Questions

Q1.

The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 3 & 0 & 1 \\ 1 & 2 & 2 \\ 4 & 0 & 3 \end{pmatrix}$$

(a) (i) Show that 5 is an eigenvalue of **M**.

(ii) Find the other two eigenvalues of **M**.

(5)

(b) Find an eigenvector corresponding to the eigenvalue 5

(3)

The transformation represented by the matrix **M** maps the straight line l_1 onto the straight line l_2

The equation of l_1 is $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$ where $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$

(c) Find an equation for the line l_2, giving your answer in the form $(\mathbf{r} - \mathbf{c}) \times \mathbf{d} = \mathbf{0}$ where **c** and **d** are constant vectors.

(5)

(Total for question = 13 marks)

Q2.

The plane Π_1 has equation $x + y + z = 3$ and the plane Π_2 has equation $2x + 3y - z = 4$

The planes Π_1 and Π_2 intersect in the line L.

(a) Find a cartesian equation for the line L.

(6)

The plane Π_3 has equation

$$\mathbf{r} . \begin{pmatrix} 5 \\ -4 \\ 4 \end{pmatrix} = 12$$

The line L meets the plane Π_3 at the point A.

(b) Find the coordinates of A.

(3)

(c) Find the acute angle between $\overrightarrow{OA}$ and the line L, where O is the origin.

Give your answer in degrees to one decimal place.

(3)

(Total for question = 12 marks)

Q3.

The plane Π_1 has equation

$$x - 5y + 3z = 11$$

The plane Π_2 has equation

$$3x - 2y + 2z = 7$$

The planes Π_1 and Π_2 intersect in the line l.

(a) Find a vector equation for l, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a scalar parameter.

(5)

The point $P(2, 0, 3)$ lies on Π_1

The line m, which passes through P, is parallel to l.

The point $Q(3, 2, 1)$ lies on Π_2

The line n, which passes through Q, is also parallel to l.

(b) Find, in exact simplified form, the shortest distance between m and n.

(5)

(Total for question = 10 marks)

Q4.

The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Find a vector perpendicular to Π

(2)

The line l passes through the point A with coordinates $(2, -4, 0)$ and meets Π at the point with coordinates $(3, 2, -1)$.

The acute angle between the plane Π and the line l is α.

(b) Find α, giving your answer to the nearest degree.

(4)

(c) Find the perpendicular distance from A to Π

(4)

(Total for question = 10 marks)

Q5.

The plane Π_1 contains the point (3, 3, −2) and the line $\dfrac{x-1}{2}=\dfrac{y-2}{-1}=\dfrac{z+1}{4}$

(a) Show that a cartesian equation of the plane Π_1 is

$$3x - 10y - 4z = -13$$

(5)

The plane Π_2 is parallel to the plane Π_1

The point (α, 1, 1), where α is a constant, lies in Π_2

Given that the shortest distance between the planes Π_1 and Π_2 is $\dfrac{1}{\sqrt{5}}$

(b) find the possible values of α.

(6)

(Total for question = 11 marks)

Q6.

The coordinates of the points A, B and C relative to a fixed origin O are (1, 2, 3), (–1, 3, 4) and (2, 1, 6) respectively. The plane Π contains the points A, B and C.

(a) Find a cartesian equation of the plane Π.

(5)

The point D has coordinates (k, 4, 14) where k is a positive constant.

Given that the volume of the tetrahedron $ABCD$ is 6 cubic units,

(b) find the value of k.

(4)

(Total for question = 9 marks)

Q7.

With respect to a fixed origin O, the points $A(-1, 5, 1)$, $B(1, 0, 3)$, $C(2, -1, 2)$ and $D(3, 6, -1)$ are the vertices of a tetrahedron.

(a) Find the volume of the tetrahedron $ABCD$.

(4)

The plane Π contains the points A, B and C.

(b) Find a cartesian equation of Π.

(4)

The point T lies on the plane Π.

The line DT is perpendicular to Π.

(c) Find the exact coordinates of the point T.

(4)

(Total for question = 12 marks)

<u>Examiner's Report</u>

Q1.

This matrix question was a good source of marks, particularly in parts (a) and (b). The methods for finding eigenvalues and an eigenvector were well known and few slips were seen. The most common source of errors was in the handling of the equations in part (b).
There were many correct solutions to part (c) but again despite this transformation being a task that has been seen many times in exams for this module, some were unsure about the method. A common pitfall was to calculate the cross product of **a** and **b** and to apply **M** to the resulting vector. Some others succumbed to confusion over the points and directions of the lines.

Q2.

Many students scored all 6 marks in part (a). Those who used Way 2 were almost always correct. It was noticeable that many students obtained a correct equation in vector form but could not convert this to Cartesian form.

Part (b) was very well understood and answered. Almost every student scored at least the method marks here.

Part (c) was also generally very well understood and answered. A small minority of students appeared to know that the scalar product was required but used sine in place of cosine and some students used the normal to the plane rather than the direction of their line and so were looking for the wrong angle.

Q3.

Most students made some progress in part (a) and the two methods, finding the direction of the line and a point on the line or solving the equations of the planes simultaneously were equally popular.
Evaluation of vector product $(1\ -5\ 3)^T \times (3\ -2\ 2)^T$ was generally accurate. Calculation of a point on the line starting with either x or y or $z = 0$ was also generally accurate though starting with different values saw some arithmetical slips. The final accuracy mark was occasionally lost due to the omission of "**r** =" from the equation.
The method where students aimed to find a formula for z in terms of y and z in terms of x (for example), proved more challenging and careless errors were quite common. Some solutions stopped with just one equation though many students managed to reach either a Cartesian equation or a vector equation of the line, albeit with numerical errors.
Part (b) was possibly the most challenging question on the paper. Many students did not explain their strategy clearly. Many solutions only scored the first mark for calculating **PQ**. A diagram would have been helpful.
(i) The "easiest" method used a vector product of $(1\ 2\ -2)^T$ and $(-4\ 7\ 13)^T$ to calculate $\sin\theta$ (where θ is the angle between PQ and the line) followed by multiplication by the length of PQ. Not a popular method though students using this approach made few errors.
(ii) The most common method was to use a scalar product to calculate $\cos\theta$. Many used the two position vectors of P and Q rather than **PQ** and the direction of the line. Evaluation of a correct scalar product was often achieved. Many thought that multiplication by 3 would give the answer rather than the length of the projection of **PQ** on the line. Calculation of $3\sqrt{(1-\sin^2\theta)}$ to reach an exact answer proved tricky.
A relatively small number of candidates reached one of many equivalent exact values such as $5\sqrt{74}/234$, $5\sqrt{481}/39$, $(5\sqrt{37})/3\sqrt{13})$, $\sqrt{(925/117)}$ etc
(iii) A few solutions attempted **NP** (where N is the foot of the perpendicular from P to the line), calculated NP^2 as a function of the parameter and then aimed to minimise either by calculus or completing the square. Only careless arithmetic prevented a successful solution.
(iv) **NP** was also used in the calculation **NP**.$(-4\ 7\ 13)^T = 0$ to find the parameter and then distance.
A few solutions used the formula for the calculation of the distance from a point to a plane.

Q4.

Although many fully correct solutions were seen, this vector question discriminated as expected. The tasks involved were fairly standard ones but many candidates were clearly ill-prepared for them.

It was common to see a correct normal in part (a). However, many did not appreciate the need to find the direction of line *l* in part (b). A correct value for the scalar product divided by the product of the magnitudes was often achieved but it was common to see confusion about what trigonometric function and of what angle (*α* or 90 – *α*) this should be equated to. Occasionally an otherwise correct final answer was not given to the nearest degree. A vector product approach was rarely seen. A very small number used their answer to part (c) followed by right-angled trigonometry.

A wide range of methods were viable and all were seen to some extent in part (c). The parallel plane method was the most common, often in combination with use of the distance formula.

The arguably easier routes of calculating $\sqrt{38}$ sin *α* or using $\dfrac{\overrightarrow{AP}.\,\mathbf{n}}{|\mathbf{n}|}$ were not quite as common. Many confused responses were seen from candidates unsure of an appropriate strategy.

Q5.

There were quite a lot of students who did not attempt this question and there were few fully correct solutions. There were many successful attempts at part (a) but some did not find a second vector in the plane in order to find the vector product. Students frequently extracted 2**i** − **j** + 4**k** from the line but used 3**i** + 3**j** − 2**k** as the second vector. A small number of students attempted a verification approach, showing that both the point and the line satisfied the Cartesian equation of the plane.

Students who attempted part (b) seemed to know the method but there were errors finding the distance between the two planes. Some found the distance from each plane to the origin and subtracted to obtain one correct solution. However many then said that the two planes were on opposite sides of the origin and added the two distances obtaining a second, incorrect solution. Many initially used the modulus and got to $\left|\dfrac{3a-1}{5\sqrt{5}}\right| = \dfrac{1}{\sqrt{5}}$ but then abandoned the modulus signs and only obtained one solution and others never used the modulus, only considering the positive distance. It was relatively rare to see a solution that correctly obtained both values of α.

Q6.

In (a) students were unfamiliar with the method for finding an equation of a plane. The most popular approach was to find the normal vector to the plane using a vector product. A number of solutions proceeded using coordinates of two points rather than two vectors in the plane. The method for finding the constant in the plane equation was generally well known. There were many completely correct solutions but an answer of $\mathbf{r}.\begin{pmatrix}4\\7\\1\end{pmatrix}=21$ was seen on a number of occasions. A few solutions used their normal vector to write down the equation of a line and some students misunderstood the meaning of "Cartesian" and proceeded to write down an answer in the form **r** = **a** + λ**b** 4 *μ***c**

For (b) many attempts to find the volume of the tetrahedron were made using a vector **OD** rather than **AD** (or equivalent). Use of a triple scalar product was well recognised. A few students did not recognise they already had a normal vector and repeated their working. The need for a factor of $\frac{1}{6}$ was occasionally forgotten. Errors in finding a value of *k* included $\frac{1}{6} \times (4k + 21) = 6$ implies $4k + 21 = 6 \times \frac{1}{6}$ and a lack of recognition that the question said that *k* had to be positive.

Q7.

This was an accessible question with many students scoring full marks for parts (a) and (b), but part (c) was less successfully attempted.

For part (a) the most common error was to fail to use 3 edges with a common point. A few students used a vertex rather than an edge, often using two edges in their cross product but then the position vector of a vertex for the scalar product. There were occasional errors involving the 1/6, with either the 1/6 being omitted altogether or 1/3 being used instead.

The required method was well-known in part (b). Errors in finding a normal vector often followed from errors in part (a). A small number of students gave their answer in the wrong form, possibly not reading the question carefully enough or possibly indicating a lack of understanding of the difference between the vector equation, Cartesian equation and parametric equations for the plane.

In part (c) it was disappointing to see that many students attempted to use incorrect methods and scored no marks. Only a minority of students were able to produce a correct parametric expression for a point on *DT* to enable them to find the value of the parameter and hence the coordinates of the required point. Those who knew what they were doing generally scored full marks, with only occasional numerical slips along the way.

Mark Scheme

Q1.

Question Number	Scheme		Marks
(a) Mark (i) and (ii) together	$\begin{vmatrix} 3-5 & 0 & 1 \\ 1 & 2-5 & 2 \\ 4 & 0 & 3-5 \end{vmatrix} = -2(-3)(-2)-4(-3)=0 \Rightarrow 5$ is an eigenvalue This mark is for demonstrating that 5 is an eigenvalue as above and **requires a conclusion**. This may also be done by **substituting** $\lambda = 5$ into their CE to obtain 0 **with a conclusion** or by performing long division and obtaining a remainder of 0 **with conclusion**. However if the candidate forms and solves the CE and correctly obtains $\lambda = 5$ this is sufficient without a conclusion. The conclusion can be minimal e.g. "proven", "QED", tick etc.		B1 **M1 on ePEN**
	$\begin{vmatrix} 3-\lambda & 0 & 1 \\ 1 & 2-\lambda & 2 \\ 4 & 0 & 3-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda)(3-\lambda)-4(2-\lambda)$ M1: Attempts determinant of **A** - λ**I** Whichever method is chosen e.g. row/column/Sarrus, the expression should be similar to the above e.g. • Row 2: $(2-\lambda)\{(3-\lambda)(3-\lambda)-4\}$ • Row 3: $4(0-(2-\lambda))+(3-\lambda)(3-\lambda)(2-\lambda)$ • Column 1 = as Row 1 • Column 2 = as Row 2 • Column 3 = as Row 3 A1: Correct equation including " = 0" which may be implied by an attempt to solve. NB Correct expanded cubic is $\lambda^3 - 8\lambda^2 + 17\lambda - 10 = 0$		M1A1
	$(2-\lambda)(\lambda-5)(\lambda-1)=0 \Rightarrow \lambda = ...$ Attempts to solve cubic – **may just be seen from calculator**		M1
	$\lambda = 2,\ 1,\ \ (5)$	Other eigenvalues 2 and 1	A1
			(5)
(b)	$\begin{pmatrix} 3 & 0 & 1 \\ 1 & 2 & 2 \\ 4 & 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 5\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} 3-5 & 0 & 1 \\ 1 & 2-5 & 2 \\ 4 & 0 & 3-5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ Demonstrates the understanding that 5 is an eigenvalue Either of these statements is sufficient		M1
	$3x+z=5x$ $x+2y+2z=5y$ $4x+3z=5z$	Multiplies out to obtain at least 2 **correct** equations	M1
	$\therefore$ Eigenvector is $\begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix}$	Allow any multiple of this vector e.g. $\begin{pmatrix} 1 \\ \frac{5}{3} \\ 2 \end{pmatrix}$ will be common. Allow $x = \ldots$, $y = \ldots$, and $z = \ldots$ where x, y and z were seen in a vector earlier and isw.	A1
			(3)

<table>
<tr><td rowspan="9">(c)</td><td colspan="2">$$\begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}2\\1\\-3\end{pmatrix}=\ldots \quad \textbf{or} \quad \begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}1\\2\\-1\end{pmatrix}=\ldots$$
Attempts to multiply one of the given vectors by **M** to find at least one image</td><td>M1
B1 on ePEN</td></tr>
<tr><td colspan="2">$$\begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}2\\1\\-3\end{pmatrix}=\ldots \quad \textbf{and} \quad \begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}1\\2\\-1\end{pmatrix}=\ldots$$
Attempts to multiply both of the given vectors by **M** to find both images</td><td>M1</td></tr>
<tr><td colspan="2">Note that an attempt at $\begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}2+\mu\\1+2\mu\\-3-\mu\end{pmatrix}=\cdots$ scores both M's provided
this results in the parametric image</td><td></td></tr>
<tr><td>$\begin{pmatrix}3\\-2\\-1\end{pmatrix}$ **or** $\begin{pmatrix}2\\3\\1\end{pmatrix}$</td><td>One correct</td><td>A1
M1 on ePEN</td></tr>
<tr><td>$\begin{pmatrix}3\\-2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}2\\3\\1\end{pmatrix}$</td><td>Both correct</td><td>A1</td></tr>
<tr><td colspan="2">$\begin{pmatrix}3+2\mu\\-2+3\mu\\-1+\mu\end{pmatrix}$ scores both A marks</td><td></td></tr>
<tr><td colspan="2">$(\mathbf{r}-\mathbf{c})\times\mathbf{d}=\mathbf{0}$ where $\mathbf{c}=3\mathbf{i}-2\mathbf{j}-\mathbf{k}$ and $\mathbf{d}=2\mathbf{i}+3\mathbf{j}+\mathbf{k}$
Or
$(\mathbf{r}-(3\mathbf{i}-2\mathbf{j}-\mathbf{k}))\times(2\mathbf{i}+3\mathbf{j}+\mathbf{k})=\mathbf{0}$
Correct equation in the correct form.
Follow through their vectors but they must be correctly placed and **depends on both method marks.**</td><td>A1ft</td></tr>
<tr><td colspan="2"></td><td>**(5)**</td></tr>
<tr><td colspan="2"></td><td>**[Total 13]**</td></tr>
</table>

Note that candidates may transform 2 points on l_1 and then use the transformed points to find the direction of l_2. In this case the second M and the second A1 will only be scored when the direction of l_2 is found and then the final mark becomes available.

Q2.

Question Number	Scheme	Notes	Marks
	Π_1: $x+y+z=3$, Π_2: $2x+3y-z=4$		
(a) Way 1	$x=\lambda \Rightarrow y=\frac{7}{4}-\frac{3}{4}\lambda$ or $\lambda=\frac{4y-7}{-3}$	M1: Obtains 2 equations connecting x, y or z with λ A1: Correct equations	M1A1
	$z=\frac{5}{4}-\frac{1}{4}\lambda$ or $\lambda=5-4z$	M1: Obtains 3 equations connecting x, y or z with λ A1: Correct equations	M1A1
	$\frac{x}{1}=\frac{7-4y}{3}=\frac{5-4z}{1}(=\lambda)$	M1: Correct use of cartesian form A1: Correct equation (allow equivalents)	M1A1
	$y=\lambda \Rightarrow \frac{7-3x}{4}=\frac{y}{1}=\frac{3z-2}{1}$ $\left(\text{or}\,\frac{7-3x}{4}=y=3z-2\right)$ $z=\lambda \Rightarrow \frac{5-x}{4}=\frac{y+2}{3}=\frac{z}{1}$ $(\text{or }=z)$		
			(6)

(a) Way 2	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & 3 & -1 \end{vmatrix}=\begin{pmatrix}-4\\3\\1\end{pmatrix}$	M1: Attempt vector product of normals A1: Correct vector	M1A1
	$x=0 \Rightarrow y+z=3,\ 3y-z=4$ $\Rightarrow y=\frac{7}{4},\ z=\frac{5}{4} \to \left(0,\ \frac{7}{4},\ \frac{5}{4}\right)$ NB $y=0$ gives $x=\frac{7}{3}$, $z=\frac{2}{3}$ $z=0$ gives $x=5$, $y=-2$	M1: Attempt a point on the line A1: Correct point (1, 1, 1) seen frequently	M1A1
	$\frac{x}{-4}=\frac{y-\frac{7}{4}}{3}=\frac{z-\frac{5}{4}}{1}(=\lambda)$	M1: Correct use of cartesian form A1: Correct equation (allow equivalents)	M1A1
	or $\frac{x-1}{-4}=\frac{y-1}{3}=\frac{z-1}{1}(=\lambda)$	Equation seen if (1, 1, 1) used	**(6)**

(a) Way 3	$x=-\frac{4}{3}y+\frac{7}{3}$	M1: Eliminates 1 variable A1: Correct equation	M1A1
	$x=5-4z$	M1: Eliminates 2nd variable A1: Correct equation	M1A1
	$\frac{x}{1}=-\frac{4}{3}y+\frac{7}{3}=5-4z$	M1: Correct use of cartesian form A1: Correct equation (allow equivalents)	M1A1
			(6)

(b)	$5(-4\lambda)-4\left(\frac{7}{4}+3\lambda\right)+4\left(\frac{5}{4}+\lambda\right)=12$	Substitutes parametric form of L into Π_3	M1
	$\lambda=-\frac{1}{2}\Rightarrow x=...,y=...,z=...$	Solves for λ and attempts coordinates	dM1
	$\left(2,\frac{1}{4},\frac{3}{4}\right)$ or $x=2,y=\frac{1}{4},z=\frac{3}{4}$ or $\begin{pmatrix}2\\1/4\\3/4\end{pmatrix}$	Correct coordinates	A1
			(3)

(b) Way 2	$5x-4.\frac{3}{4}\left(\frac{7}{3}-x\right)+4.\frac{1}{4}(5-x)=12$	Substitutes for y and z in terms of x into Π_3	M1
	$x=2\Rightarrow y=...,z=...$	Solves for x and attempts other coordinates	dM1
	$\left(2,\frac{1}{4},\frac{3}{4}\right)$ or $x=2,y=\frac{1}{4},z=\frac{3}{4}$ or $\begin{pmatrix}2\\1/4\\3/4\end{pmatrix}$	Correct coordinates	A1

(c)	$\begin{pmatrix}-2\\-\frac{1}{4}\\-\frac{3}{4}\end{pmatrix}\bullet\begin{pmatrix}-4\\3\\1\end{pmatrix}=\sqrt{\frac{37}{8}}\sqrt{26}\cos\theta$	Use scalar product between $\pm$ their $\overrightarrow{OA}$ and direction of their L	M1
	$\frac{13}{2}=\sqrt{\frac{37}{8}}\sqrt{26}\cos\theta\Rightarrow\theta=...$	Evaluate the scalar product and complete to $\theta=$ … (or the supplementary angle) (Check the product if the vectors are incorrect)	dM1
	$\theta=53.6^{\circ}$	cao	A1
			(3)
			Total 12

Q3.

Question Number	Scheme	Notes	Marks
(a)	$\mathbf{n}=\begin{pmatrix}1\\-5\\3\end{pmatrix}\times\begin{pmatrix}3\\-2\\2\end{pmatrix}=\begin{pmatrix}-10+6\\-(2-9)\\-2+15\end{pmatrix}$	Attempt vector product between normal vectors	M1
	$=\begin{pmatrix}-4\\7\\13\end{pmatrix}$	Correct vector	A1
	$x=0\Rightarrow -5y+3z=11,\quad -2y+2z=7$ $\Rightarrow y=-\frac{1}{4}, z=\frac{13}{4}$ or $y=0\Rightarrow x+3z=11,\quad 3x+2z=7$	Correct strategy to find a point on l	M1
	$\Rightarrow x=-\frac{1}{7}, z=\frac{26}{7}$ or $z=0\Rightarrow x-5y=11, 3x-2y=7$ $\Rightarrow x=1, y=-2$	Correct position vector of point on l	A1
	$\mathbf{r}=\mathbf{i}-2\mathbf{j}+\lambda(-4\mathbf{i}+7\mathbf{j}+13\mathbf{k})$	Correct equation. (follow through their position and direction vectors but must be "$\mathbf{r}$ =")	A1ft
			(5)
ALT	$x=11+5y-3z$		
	$3x-2y+2z=7\Rightarrow 3(11+5y-3z)-2y+2z=7$ $\Rightarrow y-\frac{7z}{13}=-\frac{26}{13}\quad\left(z=\frac{13y+26}{7}\right)$ Eliminate one variable		M1
	$x=11+5\left(-\frac{26}{13}+\frac{7z}{13}\right)\Rightarrow z=\frac{13-13x}{4}$	Obtain 2 correct expressions for one of the variables	A1
	$\frac{x-1}{-\frac{4}{13}}=\frac{y+2}{\frac{7}{13}}=z$	M1 Obtain a Cartesian equation for l A1 Correct equation	M1A1
	$\mathbf{r}=(\mathbf{i}-2\mathbf{j})+\lambda\left(-\frac{4}{13}\mathbf{i}+\frac{7}{13}\mathbf{j}+\mathbf{k}\right)$ oe	Deduce a vector equation for l Follow through their Cartesian equation	A1ft
			(5)

(b)	$\begin{pmatrix}3\\2\\1\end{pmatrix}-\begin{pmatrix}2\\0\\3\end{pmatrix}=\begin{pmatrix}1\\2\\-2\end{pmatrix}$	Correct vector joining P to Q	B1
	$\begin{pmatrix}-4\\7\\13\end{pmatrix}\times\begin{pmatrix}1\\2\\-2\end{pmatrix}=\begin{pmatrix}-40\\5\\-15\end{pmatrix}$	Attempt vector product between the direction of l and their $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$	M1
		Correct vector	A1
	$\sin\theta=\frac{\lvert-40\mathbf{i}+5\mathbf{j}-15\mathbf{k}\rvert}{\lvert-4\mathbf{i}+7\mathbf{j}+13\mathbf{k}\rvert\lvert\mathbf{i}+2\mathbf{j}-2\mathbf{k}\rvert}$	Angle between PQ and line n	
	$d=\lvert PQ\rvert\sin\theta$		
	$d=\frac{\lvert-40\mathbf{i}+5\mathbf{j}-15\mathbf{k}\rvert}{\lvert-4\mathbf{i}+7\mathbf{j}+13\mathbf{k}\rvert}=\frac{1}{\sqrt{234}}\sqrt{40^2+5^2+15^2}$	Fully correct method for the distance	M1
	$d=\frac{5\sqrt{481}}{39}$	Cao Allow equivalent exact forms e.g. $d=\frac{5\sqrt{74}}{\sqrt{234}}$	A1
			(5)

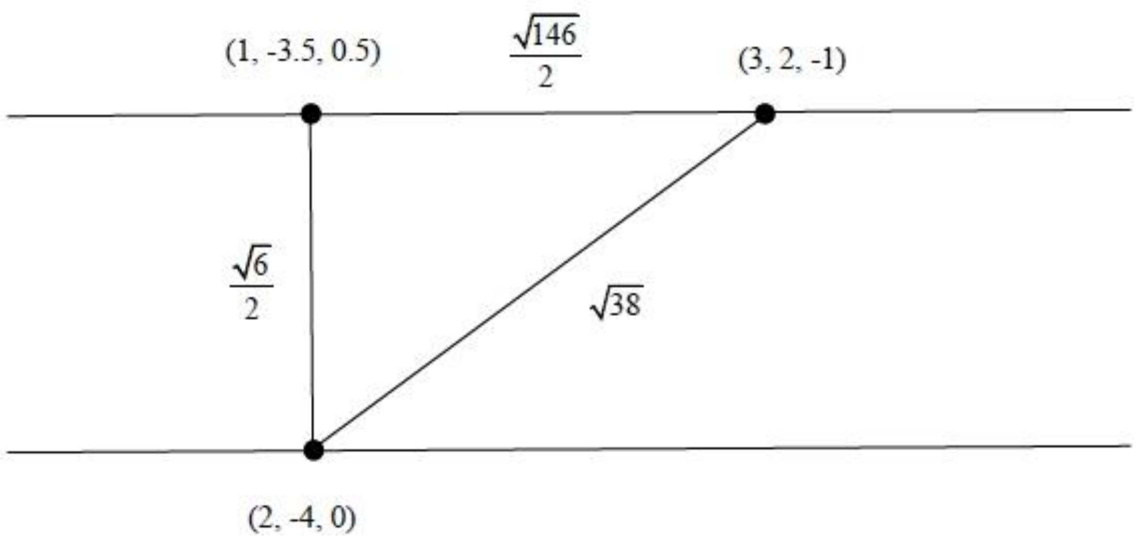
ALT 1	$\mathbf{r}_m = \begin{pmatrix}2\\0\\3\end{pmatrix} + \lambda\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}$ or $\mathbf{r}_n = \begin{pmatrix}3\\2\\1\end{pmatrix} + \mu\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}$	Vector equation for either line with their direction vector from (a)	B1ft
	$\overrightarrow{OP} = \begin{pmatrix}2\\0\\3\end{pmatrix}$ $\overrightarrow{ON} = \begin{pmatrix}3-\frac{4}{7}\mu\\2+\mu\\1+\frac{13}{7}\mu\end{pmatrix}$ $\overrightarrow{NP} = \begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}$	Uses either P and the parametric form of a point on n OR Q and the parametric form of a point on m	
	$\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}.\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix} = 0$	M1: Forms scalar product of vector NP and direction vector of l and equates to zero A1: Correct vectors	M1A1
	$\Rightarrow \mu = \frac{56}{117}$	Solves	M1
	$\Rightarrow d = \sqrt{\left(-\frac{85}{117}\right)^2 + \left(-\frac{290}{117}\right)^2 + \left(\frac{10}{9}\right)^2} = \frac{5\sqrt{481}}{39}$	Obtains the correct distance	A1
			(5)
	Alternative for M1A1M1		
	$\overrightarrow{NP} = \begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix} \Rightarrow d = \sqrt{\left(-1+\frac{4}{7}\mu\right)^2 + (-2-\mu)^2 + \left(2-\frac{13}{7}\mu\right)^2} \Rightarrow d$ is min when $\Rightarrow \mu = \frac{56}{117}$ M1: Find d in terms of a parameter A1: correct expression M1: use calculus (or simplify and complete the square) to find the parameter corresponding to the min d		

ALT 2	Correct vector PQ		B1
	$\begin{pmatrix}1\\2\\-2\end{pmatrix}.\begin{pmatrix}-4\\7\\13\end{pmatrix} = \begin{vmatrix}1\\2\\-2\end{vmatrix}\begin{vmatrix}-4\\7\\13\end{vmatrix}\cos\theta$	Forms the scalar product and attempts to evaluate the LHS	M1
	$\cos\theta = \frac{-16}{3\sqrt{234}}$	Correct value for $\cos\theta$ exact or decimal	A1
	$d = \lvert PQ\rvert \sin\theta = 3\sqrt{1-\left(\frac{-16}{3\sqrt{234}}\right)^2} = \frac{5\sqrt{74}}{\sqrt{234}}$	M1: Correct method for the distance. A1: Correct **EXACT** distance	M1A1
			(5)
			Total 10

Q4.

Question Number	Scheme		Marks
(a)	$(\mathbf{i}+2\mathbf{k})\times(2\mathbf{i}+\mathbf{j}+3\mathbf{k})=\mathbf{i}(0-2)-\mathbf{j}(3-4)+\mathbf{k}(1-0)$ Attempt the correct vector product between the direction vectors If no method is shown at least 2 "components" should be correct		M1
	$-2\mathbf{i}+\mathbf{j}+\mathbf{k}$	Any multiple of this vector	A1
			(2)
(a) Way 2	$\mathbf{n}=\begin{pmatrix}1\\a\\b\end{pmatrix}\Rightarrow\begin{pmatrix}1\\a\\b\end{pmatrix}\cdot\begin{pmatrix}1\\0\\2\end{pmatrix}=0,\begin{pmatrix}1\\a\\b\end{pmatrix}\cdot\begin{pmatrix}2\\1\\3\end{pmatrix}=0\Rightarrow a=-\frac{1}{2},b=-\frac{1}{2}$ Correct method leading to values for a and b		M1
	$-2\mathbf{i}+\mathbf{j}+\mathbf{k}$	Any multiple of this vector	A1

Useful Diagram:



<u>**Mark (b) and (c) together**</u>

(b)	l has direction $\mathbf{i}+6\mathbf{j}-\mathbf{k}$	Any multiple of this vector	B1
	$(\mathbf{i}+6\mathbf{j}-\mathbf{k})\cdot(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=-2+6-1$	Attempts scalar product between the direction of l and the normal to the plane.	M1
	$\sin\alpha$ or $\cos(90°-\alpha)=\dfrac{-2+6-1}{\sqrt{6}\times\sqrt{38}}$ $\left(\text{NB } \sqrt{6}\times\sqrt{38}=2\sqrt{57}\right)$	$\sin\ldots$ or $\cos\ldots=\pm\left(\dfrac{-2+6-1}{\sqrt{6}\times\sqrt{38}}\right)$	A1
	$\alpha=(11.45\ldots=)11°$	For 11 (degrees symbol not required). **Do not isw and mark their final answer.**	A1
			(4)
(b) Way 2	l has direction $\mathbf{i}+6\mathbf{j}-\mathbf{k}$	Any multiple of this vector	B1
	$\lvert\mathbf{i}+6\mathbf{j}-\mathbf{k}\rvert=\sqrt{1^2+6^2+1^2}\left(=\sqrt{38}\right)$ Followed by a complete method to find the perpendicular distance from A to the plane i.e. as in part (c) $\left(\dfrac{3}{\sqrt{6}}\right)$ or finds the distance from (3, 2, -1) to the intersection of the perpendicular from A with the plane (1, -3.5, 0.5) $\left(\dfrac{\sqrt{146}}{2}\right)$ and then uses correct trigonometry to find the sin or cos or tan (would need both distances) to find the required angle.		M1
	$\sin\alpha=\dfrac{\frac{3}{\sqrt{6}}}{\sqrt{38}},\quad \cos\alpha=\dfrac{\frac{\sqrt{146}}{2}}{\sqrt{38}},\quad \tan\alpha=\dfrac{\frac{3}{\sqrt{6}}}{\frac{\sqrt{146}}{2}}$		A1
	$\alpha=(11.45\ldots=)11°$	For 11 (degrees symbol not required). **Do not isw and mark their final answer.**	A1
(b) Way 3	l has direction $\mathbf{i}+6\mathbf{j}-\mathbf{k}$	Any multiple of this vector	B1
	$(\mathbf{i}+6\mathbf{j}-\mathbf{k})\times(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=7\mathbf{i}+\mathbf{j}+13\mathbf{k}$ Attempts vector product between the direction of l and the normal to the plane. If no method is shown at least 2 "components" should be correct		M1
	$\sin\alpha=\dfrac{\sqrt{169+49+1}}{\sqrt{6}\times\sqrt{38}}$	Correct value for sin…	A1
	$\alpha=(11.45\ldots=)11°$	For 11 (degrees symbol not required). **Do not isw and mark their final answer.**	A1

(c) **Way 1**	Π has equation: $\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=(4\mathbf{i}+2\mathbf{j}+\mathbf{k}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=-5$ or $\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=(3\mathbf{i}+2\mathbf{j}-\mathbf{k}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=-5$ M1: Forms scalar product of a point in the plane with their normal vector A1: Correct equation. Allow this mark if -5 (**or the equivalent multiple of -5 for their normal**) is obtained i.e. do not need to see the equation of the plane explicitly		M1A1
	$\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=(2\mathbf{i}-4\mathbf{j}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=-8$ $\Rightarrow d=\left\|\dfrac{-8-(-5)}{\sqrt{2^2+1^2+1^2}}\right\|=\ldots$		M1
	$d=\dfrac{\sqrt{6}}{2}$	Correct distance in any equivalent exact form e.g. $\dfrac{3}{\sqrt{6}}$	A1
			(4)
(c) **Way 2**	Dist (2, -4, 0) to $(3,2,-1)=\sqrt{(3-2)^2+(2+4)^2+1^2}=\ldots$ Correct attempt to find the distance between (2, - 4, 0) and (3, 2, 1)		M1
	$=\sqrt{38}$	Correct distance	A1
	d $="\sqrt{38}"\sin\alpha="\sqrt{38}"\dfrac{3}{\sqrt{6}\times\sqrt{38}}=\ldots$	Uses correct trigonometry to find the required distance	M1
	$d=\dfrac{\sqrt{6}}{2}$	Correct distance in any equivalent exact form e.g. $\dfrac{3}{\sqrt{6}}$	A1
(c) **Way 3**	Π has equation: $\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=(4\mathbf{i}+2\mathbf{j}+\mathbf{k}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=-5$ or $\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=(3\mathbf{i}+2\mathbf{j}-\mathbf{k}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=-5$ M1: Forms scalar product of a point in the plane with their normal vector A1: Correct equation. Allow this mark if -5 (**or the equivalent multiple of -5 for their normal**) is obtained i.e. do not need to see the equation of the plane explicitly		M1A1
	$\Rightarrow d=\left\|\dfrac{-2\times2+1(-4)+(0)+5}{\sqrt{2^2+1^2+1^2}}\right\|=\ldots$	Uses a correct formula for the distance. (Allow $\pm$ their -5)	M1
	$d=\dfrac{\sqrt{6}}{2}$	Correct distance in any equivalent exact form e.g. $\dfrac{3}{\sqrt{6}}$	A1

(c) **Way 4**	Let P be (3, 2, -1) so $\mathbf{AP} = \mathbf{i} + 6\mathbf{j} - \mathbf{k}$ $(\mathbf{i}+6\mathbf{j}-\mathbf{k})\bullet(-2\mathbf{i}+\mathbf{j}+\mathbf{k}) = -2+6-1$ M1: Forms the vector AP and calculates scalar product with normal vector A1: Correct scalar product for their normal i.e. 3 or a multiple of 3 depending on their normal (may be unsimplified)		M1A1
	$\Rightarrow d = \left\lvert\dfrac{-2+6-1}{\sqrt{2^2+1^2+1^2}}\right\rvert = \ldots$	Uses a correct formula for the distance.	M1
	$d = \dfrac{\sqrt{6}}{2}$	Correct distance in any equivalent exact form e.g. $\dfrac{3}{\sqrt{6}}$	A1
(c) **Way 5**	Π has equation: $\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k}) = (4\mathbf{i}+2\mathbf{j}+\mathbf{k}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k}) = -5$ or $\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k}) = (3\mathbf{i}+2\mathbf{j}-\mathbf{k}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k}) = -5$ M1: Forms scalar product of a point in the plane with their normal vector A1: Correct equation Allow this mark if -5 (**or the equivalent multiple of -5 for their normal**) is obtained i.e. do not need to see the equation of the plane explicitly		M1A1
	$(2\mathbf{i}-4\mathbf{j})+\lambda(-2\mathbf{i}+\mathbf{j}+\mathbf{k}),\ 2x-y-z=5$ $\Rightarrow 4-4\lambda+4-\lambda-\lambda=5 \Rightarrow \lambda=\dfrac{1}{2}$ $\Rightarrow d=\dfrac{1}{2}\lvert -2\mathbf{i}+\mathbf{j}+\mathbf{k}\rvert = \dfrac{\sqrt{2^2+1^1+1^2}}{2}$ Requires a complete method: Uses the parametric form of the line through (-2, 4, 0) perpendicular to the plane and substitutes into the equation of the plane to find the value of the parameter and uses this correctly to find the required distance.		M1
	$d = \dfrac{\sqrt{6}}{2}$	Correct distance in any equivalent exact form e.g. $\dfrac{3}{\sqrt{6}}$	A1
			[10]

Q5.

Question Number	Scheme	Notes	Marks
(a)	$(3,\ 3,\ -2),\quad \frac{x-1}{2}=\frac{y-2}{-1}=\frac{z+1}{4}$		
	$2\mathbf{i}-\mathbf{j}+4\mathbf{k},\ 2\mathbf{i}+\mathbf{j}-\mathbf{k}$	2 correct vectors lying in Π_1	B1
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & -1 & 4\\ 2 & 1 & -1\end{vmatrix}=\begin{pmatrix}-3\\10\\4\end{pmatrix}$	M1: Attempt normal vector using 2 vectors lying in Π_1. If the method is unclear, at least 2 components should be correct. A1: Correct normal (any multiple)	M1A1
	$\begin{pmatrix}-3\\10\\4\end{pmatrix}\bullet\begin{pmatrix}3\\3\\-2\end{pmatrix}=-9+30-8=13$	Attempt scalar product with a point lying in the plane . **Dependent on the previous method mark.**	dM1
	$3x-10y-4z=-13$*	Correct equation	A1*
			(5)
(b)	$\begin{pmatrix}-3\\10\\4\end{pmatrix}\bullet\begin{pmatrix}\alpha\\1\\1\end{pmatrix}=14-3\alpha$ or $\left(\begin{pmatrix}\alpha\\1\\1\end{pmatrix}-\begin{pmatrix}3\\3\\-2\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-10\\-4\end{pmatrix}=3\alpha-1$	Attempt scalar product between $\begin{pmatrix}\alpha\\1\\1\end{pmatrix}$ and their $\begin{pmatrix}-3\\10\\4\end{pmatrix}$ or $\begin{pmatrix}\alpha\\1\\1\end{pmatrix}$ – (a point in the plane Π_1) and their $\begin{pmatrix}-3\\10\\4\end{pmatrix}$	M1
	$\therefore d=\left\lvert\frac{3\alpha-1}{\sqrt{3^2+10^2+4^2}}\right\rvert$ or $\therefore d=\left\lvert\frac{13}{\sqrt{3^2+10^2+4^2}}-\frac{14-3\alpha}{\sqrt{3^2+10^2+4^2}}\right\rvert$	Use of correct distance method. **Dependent on the previous method mark.** (Modulus not needed here)	dM1
	Note: $d=\left\lvert\frac{3\alpha-10-4\pm13}{\sqrt{3^2+10^2+4^2}}\right\rvert$ could score the first 2 method marks		
	$\left\lvert\frac{3\alpha-1}{5\sqrt{5}}\right\rvert=\frac{1}{\sqrt{5}}$	Set their distance $=\frac{1}{\sqrt{5}}$. **Dependent on both previous method marks**	ddM1
	$3\alpha-1=\pm5$	Correct equations (must see $\pm$ for this mark but may still score one of the final A marks if $\pm$ is missing). May be implied and allow un-simplified.	A1
	$\alpha=2,\ -\frac{4}{3}$	cso	A1, A1
			(6)
			Total 11

Q6.

Question Number	Scheme	Notes	Marks
(a)	$\overrightarrow{AB}=\begin{pmatrix}-2\\1\\1\end{pmatrix}$, $\overrightarrow{AC}=\begin{pmatrix}1\\-1\\3\end{pmatrix}$, $\overrightarrow{BC}=\begin{pmatrix}3\\-2\\2\end{pmatrix}$	Two correct vectors in Π Can be negatives of those shown	B1
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & 1 & 1\\1 & -1 & 3\end{vmatrix}=\begin{pmatrix}4\\7\\1\end{pmatrix}$	M1: Attempt cross product of two vectors lying in Π (At least one no. to be correct.) A1: Correct normal vector	M1A1
	$\begin{pmatrix}4\\7\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\3\end{pmatrix}=4+14+3$	Attempt scalar product with their normal and a point in the plane	dM1
	$4x+7y+z=21$	Cao (oe)	A1

	(a) Alternative		
	$a+2b+3c=d$ $-a+3b+4c=d$ $2a+b+6c=d$	Correct equations	B1
	$a=\frac{4}{21}d,\ b=\frac{1}{3}d,\ c=\frac{1}{21}d$	M1: Solve for a, b and c in terms of d A1: Correct equations	M1A1
	$d=21\Rightarrow a=...,\ b=...,\ c=...$	Obtains values for a, b, c and d	M1
	$4x+7y+z=21$	Cao (oe)	A1

			(5)
	Alternative: Using $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$ where **b** and **c** are vectors in Π		
	Two correct vectors in the plane	See main scheme	B1
	Eg $\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + s\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$		M1
	$x = 1 - 2s + t$ $y = 2 + s - t$ $z = 3 + s + 3t$	Deduce 3 correct equations	A1
	$4x + 7y + z = 21$	M1: Eliminate s, t A1: Cao	M1A1
(b)	AD□AB × AC	Attempt suitable triple product	M1
	$= \begin{pmatrix} 4 \\ 7 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} k-1 \\ 2 \\ 11 \end{pmatrix} = 4k - 4 + 14 + 11$		
	$\therefore \frac{1}{6}(4k + 21) = 6$	M1: Set $\frac{1}{6}$(their triple product) = 6 A1: Correct equation	dM1A1
	$k = \frac{15}{4}$	Cao (oe)	A1

	(b) Alternative		
	Area ABC $= \frac{1}{2}\left\|\overrightarrow{AB}\right\|\left\|\overrightarrow{AC}\right\| = \frac{1}{2}\sqrt{6}\sqrt{11}$ D to Π is $\frac{4k + 28 + 14 - 21}{\sqrt{16 + 49 + 1}}$	Attempt area *ABC* and distance between *D*and*Π*	M1
	$\frac{1}{6}\sqrt{6}\sqrt{11}\,\frac{4k + 28 + 14 - 21}{\sqrt{16 + 49 + 1}} = 6$	M1: Set $\frac{1}{3}$(their area x their distance) = 6 A1: Correct equation	dM1A1
	$k = \frac{15}{4}$	Cao (oe)	A1
			(4)
			Total 9

Q7.

Question Number	Scheme	Notes	Marks
	$A(-1, 5, 1),\ B(1, 0, 3),\ C(2, -1, 2),\ D(3, 6, -1)$		
(a)	$\mathbf{AB}=\begin{pmatrix}2\\-5\\2\end{pmatrix},\ \mathbf{AC}=\begin{pmatrix}3\\-6\\1\end{pmatrix},\ \mathbf{AD}=\begin{pmatrix}4\\1\\-2\end{pmatrix}$ $\mathbf{DB}=\begin{pmatrix}2\\6\\-4\end{pmatrix},\ \mathbf{DC}=\begin{pmatrix}1\\7\\-3\end{pmatrix},\ \mathbf{BC}=\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	Attempts 3 edges of the tetrahedron Any triple with a common vertex Method to be shown or at least 1 correct	M1
	$\begin{vmatrix}2 & -5 & 2\\3 & -6 & 1\\4 & 1 & -2\end{vmatrix}$ or $\begin{pmatrix}2\\-5\\2\end{pmatrix}\cdot\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\3 & -6 & 1\\4 & 1 & -2\end{vmatrix}$	Attempt appropriate triple product with their edges. (M0 if a vector is obtained)	dM1
	$=\frac{1}{6}(22-50+54)=\frac{13}{3}\ \left(4\frac{1}{3}\text{ or } 4.3\text{rec}\right)$	dM1: Completes including $\frac{1}{6}$ (depends on both M marks above) A1: Correct volume (allow equivalents)	ddM1A1
			(4)
	Cartesian method: Find the equation of a plane containing a face of the tetrahedron		M1
	Then find area of triangle and perp height		dM1
	Complete by using Vol $=\frac{1}{3}\times\text{base area}\times\text{height}=\frac{13}{3}$		ddM1A1 (4)
(b)	$\mathbf{AB}\times\mathbf{AC}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\2 & -5 & 2\\3 & -6 & 1\end{vmatrix}=\begin{pmatrix}7\\4\\3\end{pmatrix}$	M1: Attempt cross product between two sides of ABC Min one element correct. A1: Correct normal vector (any multiple)	M1A1
	$\begin{pmatrix}7\\4\\3\end{pmatrix}\cdot\begin{pmatrix}-1\\5\\1\end{pmatrix}(=16)$	Attempt scalar product using their normal vector Answer correct for their vectors or method shown.	dM1
	$7x+4y+3z=16$	Correct equation (any multiple)	A1 **(4)**

	Alternative to (b)		
	$-a+5b+c=d,\ a+3c=d,\ 2a-b+2c=d$	Uses A, B and C to form 3 equations	M1
	$a=7,\ b=4,\ c=3$	Correct values	A1
	$\begin{pmatrix}7\\4\\3\end{pmatrix}\cdot\begin{pmatrix}-1\\5\\1\end{pmatrix}(=16)$		dM1
	$7x+4y+3z=16$	Correct equation (any multiple)	A1

(c)	$\mathbf{DT}=\begin{pmatrix}3\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}7\\4\\3\end{pmatrix}$	Attempt parametric form of **DT** using their normal vector	M1
	$7(3+7\lambda)+4(6+4\lambda)+3(-1+3\lambda)=16$ $\Rightarrow \lambda=\ldots$	Substitutes parametric form of **DT** into their plane equation and solves for λ	dM1
	$\lambda=-\frac{13}{37}\Rightarrow T$ is $\left(\frac{20}{37}, \frac{170}{37}, -\frac{76}{37}\right)$	M1: Uses their value of λ in their **DT** equation. Can be indicated by any coordinate correct for their **DT** and λ A1: Correct exact coordinates Or correct vector $\overrightarrow{OT}$	ddM1A1
			(4)
			Total 12