Questions

Q1.

A non-singular matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 3 & k & 0 \\ k & 2 & 0 \\ k & 0 & 1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

(a) Find, in terms of *k*, the inverse of the matrix **M**.

(5)

The point *A* is mapped onto the point (–5, 10, 7) by the transformation represented by the matrix

$$\begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

(b) Find the coordinates of the point *A*.

(3)

(Total for question = 8 marks)

Q2.

$$\mathbf{M} = \begin{pmatrix} 0 & 1 & 9 \\ 1 & 4 & k \\ 1 & 0 & -3 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

Given that $\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix}$ is an eigenvector of the matrix **M**,

(a) find the eigenvalue of **M** corresponding to $\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix}$

(2)

(b) show that k −= 7,

(2)

(c) find the other two eignevalues of the matrix **M**.

(4)

The image of the vector $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$ under the transformation represented by **M** is $\begin{pmatrix} -6 \\ 21 \\ 5 \end{pmatrix}$.

(d) Find the values of the constants p, q and r.

(4)

(Total for question = 12 marks)

Q3.

$$\mathbf{M} = \begin{pmatrix} 4 & -5 & 0 \\ k & 2 & 0 \\ -3 & -5 & k \end{pmatrix},$$ where k is a real constant, $k \neq 0$, $k \neq -\frac{8}{5}$

(a) Find, in terms of k, the inverse of the matrix **M**.

(5)

A transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by the matrix

$$\begin{pmatrix} 4 & -5 & 0 \\ -1 & 2 & 0 \\ -3 & -5 & -1 \end{pmatrix}$$

The transformation T maps the plane Π_1 onto the plane Π_2

Given that the plane Π_2 has equation $2x - z = 4$

(b) find a cartesian equation of the plane Π_1

(6)

(Total for question = 11 marks)

Q4.

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & a \end{pmatrix} \quad a \neq 1$$

(a) Find $\mathbf{A}^{-1}$ in terms of a.

(4)

$$\mathbf{B} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix}$$

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix **B**.

The equation of l_2 is

$$(\mathbf{r} - (12\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})) \times (-6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = 0$$

(b) Find a vector equation for the line l_1

(4)

(Total for question = 8 marks)

Q5.

The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 3 & 0 & 1 \\ 1 & 2 & 2 \\ 4 & 0 & 3 \end{pmatrix}$$

(a) (i) Show that 5 is an eigenvalue of **M**.

(ii) Find the other two eigenvalues of **M**.

(5)

(b) Find an eigenvector corresponding to the eigenvalue 5

(3)

The transformation represented by the matrix **M** maps the straight line l_1 onto the straight line l_2

The equation of l_1 is $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$ where $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$

(c) Find an equation for the line l_2, giving your answer in the form $(\mathbf{r} - \mathbf{c}) \times \mathbf{d} = \mathbf{0}$ where **c** and **d** are constant vectors.

(5)

(Total for question = 13 marks)

Examiner's Report

Q1.

In part (a) most succeeded in finding the determinant correctly although the sign error of $6 + k^2$ was seen occasionally. The method of finding the inverse was clearly well rehearsed and it was pleasing to see so many correct inverses with no slips. There were very few candidates who did not know the full method for finding an inverse.
In part (b), most succeed by pre-multiplying the column vector by the inverse of the matrix, (having substituted $k = 1$) to give the correct answer. The main error was in multiplying the vector by the matrix itself. Some candidates took the more laborious approach of setting up three simultaneous equations to establish the coordinates of A. Given the location of the three zero elements of **M**, this essentially meant solving equations in two unknowns and candidates were largely successful with this approach.

Q2.

In part (a), the vast majority of students knew what an eigenvector was, and could easily establish the eigenvalue for the given eigenvector. Students could then easily show that k had the value given in part (b) by using the y component.

In part (c) most students made a sound attempt at the characteristic equation although there were some sign errors and occasionally some missing terms.

Part (d) was solved by one of two methods. Some students attempted $\mathbf{M}^{-1}$ and then used

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} -6 \\ 21 \\ 5 \end{pmatrix},$$

others set up 3 equations in 3 unknowns from

$$\mathbf{M}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} -6 \\ 21 \\ 5 \end{pmatrix}.$$

The simultaneous equations method was probably the more popular of the two but both were equally successful with any error generally being arithmetic slips. A small minority of students erroneously attempted

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \mathbf{M}\begin{pmatrix} -6 \\ 21 \\ 5 \end{pmatrix}.$$

Q3.

Part (a) was very well answered with the vast majority of students scoring all 5 marks. Failing to score full marks was usually due to a sign error resulting in having a 5 in place of the 35. Part (b) on the other hand was poorly answered. Most solutions correctly substituted $k = -1$ into their inverse matrix. Very few students attempted to form a parametric form and so made no further progress as a result. Even those who were able to get the x and z coordinates expressed correctly in parametric form in terms of one variable often then wrote that the y coordinate was zero rather than expressing it in terms of another variable. Those choosing a correct parametric form of a point on Π_2 usually transformed it successfully but were then unable to convert their parametric form of a plane into a Cartesian one. The most common misconception seemed to be that applying the inverse transformation to the normal to the plane Π_2 would result in the normal to plane Π_1.

Q4.

Part (a) showed that some candidates seemed unfamiliar with basic matrix work and failed to apply a recognised method to calculate $\mathbf{A}^{-1}$. A correct expression for the determinant was generally seen though simplification to $2a$ or $2a - 1$ rather than $2a - 2$ was seen. A few attempts at $\mathbf{A}^{-1}$ calculated all the relevant determinants but then failed to use appropriate signs. There were occasional slips in the entries which meant the solution became messy in part (b)

(b) Four methods were seen for transforming l_2 to l_1, three of which used the inverse matrix.

(i) The most popular using the parametric form of l_2 $(6 - \lambda, -4 + 4\lambda, 2 - \lambda)^{\mathrm{T}}$ was generally done accurately but an incorrect value of the determinant lost three accuracy marks overall. The algebra was much simpler where the multiple of 1/6 was left outside the matrix until the end. The final mark was often lost when "line is" or "l =" was used rather than the vector form "r =". Occasionally the vector product form of the line was given which was fine.

(ii) Transforming a point on the line and its direction were quite common and generally presented few problems.

(iii) Transforming two points on the line was seen and was generally successful; the simpler the points chosen, the less likelihood of arithmetical slips.

(iv) A small number of students took a point (x, y, z) on l_1, transformed it with $\mathbf{A}$ and made the resulting point lie on line l_2. The algebra was complicated, and many gave up, though a few solutions did reach the required straight line.

Q5.

This matrix question was a good source of marks, particularly in parts (a) and (b). The methods for finding eigenvalues and an eigenvector were well known and few slips were seen. The most common source of errors was in the handling of the equations in part (b).

There were many correct solutions to part (c) but again despite this transformation being a task that has been seen many times in exams for this module, some were unsure about the method. A common pitfall was to calculate the cross product of **a** and **b** and to apply **M** to the resulting vector. Some others succumbed to confusion over the points and directions of the lines.

Mark Scheme

Q1.

Question Number	Scheme		Marks
(a)	$\det \mathbf{M} = 6 - k^2$	A correct (possibly un-simplified) determinant	B1
	$\mathbf{M}^T = \begin{pmatrix} 3 & k & k \\ k & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ or minors $\begin{pmatrix} 2 & k & -2k \\ k & 3 & -k^2 \\ 0 & 0 & 6-k^2 \end{pmatrix}$ or cofactors $\begin{pmatrix} 2 & -k & -2k \\ -k & 3 & k^2 \\ 0 & 0 & 6-k^2 \end{pmatrix}$		B1
	$\dfrac{1}{6-k^2}\begin{pmatrix} 2 & -k & 0 \\ -k & 3 & 0 \\ -2k & k^2 & 6-k^2 \end{pmatrix}$	M1: Identifiable full attempt at inverse **including reciprocal of determinant**. Could be indicated by at least 6 correct elements. A1: Two rows or two columns correct (ignoring determinant) **BUT M0A1A0 or M0A1A1 is not possible** A1: Fully correct inverse	M1A1A1
			(5)
(b)	$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \dfrac{1}{5}\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & 0 \\ -2 & 1 & 5 \end{pmatrix}\begin{pmatrix} -5 \\ 10 \\ 7 \end{pmatrix}$ $\Rightarrow a = \ldots \text{or } b = \ldots \text{or } c = \ldots$	Uses $k = 1$ in the inverse and attempts to multiply to obtain a numerical value for at least one of a, b or c	M1
	$x = -4,\ y = 7,\ z = 11$	M1: Obtains values for all three coordinates A1: Correct coordinates	M1A1cao
			(3)
			Total 8
	Alternative for (b)		
	$\begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 7 \end{pmatrix} \Rightarrow \begin{matrix} 3a+b=-5 \\ a+2b=10 \\ a+c=7 \end{matrix}$ $\Rightarrow a = \ldots \text{or } b = \ldots \text{or } c = \ldots$	Multiplies to give 3 equations and attempts to obtain a numerical value for at least one of a, b or c	M1
	$x = -4,\ y = 7,\ z = 11$	M1: Obtains values for all three coordinates A1: Correct coordinates	M1A1cao

Q2.

Question Number	Scheme	Notes	Marks
(a)	$\begin{pmatrix} 0 & 1 & 9 \\ 1 & 4 & k \\ 1 & 0 & -3 \end{pmatrix}\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix} = \lambda\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix}$	Correct statement	M1
	$7-3=\lambda$ or $28=7\lambda \Rightarrow \lambda=4$	Correct eigenvalue	A1
			(2)
(b)	$7+4\times19+k=4\times19 \Rightarrow k=-7$ *	M1: Uses y component to establish an equation for k A1*: Correct k	M1A1*
			(2)
(c)	$\begin{vmatrix} 0-\lambda & 1 & 9 \\ 1 & 4-\lambda & -7 \\ 1 & 0 & -3-\lambda \end{vmatrix} = 0$		
	$\lambda(4-\lambda)(3+\lambda)+(3+\lambda)-7+9(\lambda-4)=0$ or $-7+9(\lambda-4)-(3+\lambda)[\lambda(\lambda-4)-1]$	M1: Correct characteristic equation method (allow sign errors only) A1: Correct equation in any form	M1A1
	$(4-\lambda)[\lambda(3+\lambda)-1-9]=0$	NB $\lambda^3-\lambda^2-22\lambda+40=0$	
	$(\lambda-2)(\lambda+5)=0 \Rightarrow \lambda=2,-5$	A1: $\lambda=2$ or $\lambda=-5$ A1: $\lambda=2$ and $\lambda=-5$	A1A1
			(4)
(d) Way 1	$\begin{pmatrix} 0 & 1 & 9 \\ 1 & 4 & -7 \\ 1 & 0 & -3 \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} q+9r \\ p+4q-7r \\ p-3r \end{pmatrix}$	Multiplies by **M** to obtain a vector in terms of p, q and r	M1
	$\begin{pmatrix} q+9r \\ p+4q-7r \\ p-3r \end{pmatrix} = \begin{pmatrix} -6 \\ 21 \\ 5 \end{pmatrix}$	Correct equations	A1
	$p=2, q=3, r=-1$	M1: Solves simultaneously to obtain at least one of p, q or r. **Dependent on the previous method mark.** A1: Correct answers	dM1A1
	Correct equations followed by correct answers scores full marks in part (d)		
			(4)

Q3.

Question Number	Scheme	Notes	Marks
	$\mathbf{M}=\begin{pmatrix}4 & -5 & 0\\ k & 2 & 0\\ -3 & -5 & k\end{pmatrix}$		
(a)	$\lvert\mathbf{M}\rvert=4(2k)+5(k^2)(+0)$	Correct determinant in any form (Quadratic may be unsimplified)	B1
	Minors : $\begin{pmatrix}2k & k^2 & -5k+6\\ -5k & 4k & -35\\ 0 & 0 & 8+5k\end{pmatrix}$ or cofactors : $\begin{pmatrix}2k & -k^2 & 6-5k\\ 5k & 4k & 35\\ 0 & 0 & 8+5k\end{pmatrix}$ B1: A correct first step of minors or cofactors		B1
	$\mathbf{M}^{-1}=\dfrac{1}{5k^2+8k}\begin{pmatrix}2k & 5k & 0\\ -k^2 & 4k & 0\\ 6-5k & 35 & 8+5k\end{pmatrix}$	M1: Fully recognisable attempt at the inverse including reciprocal of the determinant B1: Any 2 correct rows or columns ignoring determinant (may be missing) M mark not required A1: Fully correct inverse	M1B1A1
			(5)
(b)	$\mathbf{M}^{-1}=-\dfrac{1}{3}\begin{pmatrix}-2 & -5 & 0\\ -1 & -4 & 0\\ 11 & 35 & 3\end{pmatrix}$	Substitutes $k=-1$	M1
	$\Pi_2: x=s, y=t, z=2s-4$	Attempts parametric form ($s\neq 0, t\neq 0$) Any pair of letters (inc x and y) can be used as parameters	M1
	$-\dfrac{1}{3}\begin{pmatrix}-2 & -5 & 0\\ -1 & -4 & 0\\ 11 & 35 & 3\end{pmatrix}\begin{pmatrix}s\\ t\\ 2s-4\end{pmatrix}$	Attempts $\mathbf{M}^{-1}\times$ their parametric form Depends on both M marks above	ddM1
	$-\dfrac{1}{3}\begin{pmatrix}-2s-5t\\ -s-4t\\ 11s+35t+6s-12\end{pmatrix}$	Correct parametric form for Π_1 with s, t	A1
	$11x-5y+z=4$	dddM1:Eliminates s and t to obtain a cartesian equation All 3 previous M marks needed $x=-2x-5y$ gets M0 here (unless the parameters are now changed) A1:Correct equation (oe)	dddM1A1
			(6)
			Total 11

(b) Way 2	$\mathbf{M} = \begin{pmatrix} 4 & -5 & 0 \\ -1 & 2 & 0 \\ -3 & -5 & -1 \end{pmatrix}$		
	$\Pi_2 : x = s, y = t, z = 2s - 4$	Attempts parametric form	M1
	$\begin{pmatrix} 4 & -5 & 0 \\ -1 & 2 & 0 \\ -3 & -5 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4x-5y \\ -x+2y \\ -3x-5y-z \end{pmatrix}$	Attempts $\mathbf{M}x$	M1
	$\begin{pmatrix} 4x-5y \\ -x+2y \\ -3x-5y-z \end{pmatrix} = \begin{pmatrix} s \\ t \\ 2s-4 \end{pmatrix}$	ddM1: Sets $\mathbf{M}x =$ their parametric form A1: Correct equations	ddM1 A1
	$11x - 5y + z = 4$	M1:Eliminates s and t to obtain a cartesian equation A1:Correct equation (oe)	dddM1 A1

Way 3	$\begin{pmatrix} 4 & -5 & 0 \\ -1 & 2 & 0 \\ -3 & -5 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$	M1: General point (a, b, c) on first plane M1: Setting up the transformation equation (as left)	M1 M1
	$4a - 5b = x$ $-a + 2b = y$ $-3a - 5b - c = z$	M1: Multiply the matrices on the lhs and equate to rhs A1: correct equations	ddM1A1
	$2x - z = 4 \Rightarrow 2(4a - 5b) - (-3a - 5b - c) = 4$	M1: Using $2x - z = 4$	dddM1
	$11a - 5b + c = 4$		
	$11x - 5y + z = 4$	A1: Correct equation of the plane. Must have x, y, z	A1

Q4.

Question Number	Scheme	Notes	Marks
(a)	$\mathbf{A}=\begin{pmatrix}1&-1&1\\1&1&1\\1&2&a\end{pmatrix}$		
	$\lvert\mathbf{A}\rvert=a-2+a-1+2-1(=2a-2)$	Correct determinant in any form	B1
	$\begin{pmatrix}1&-1&1\\1&1&1\\1&2&a\end{pmatrix}\to\begin{pmatrix}a-2&a-1&1\\-a-2&a-1&3\\-2&0&2\end{pmatrix}\to\begin{pmatrix}a-2&1-a&1\\a+2&a-1&-3\\-2&0&2\end{pmatrix}$ Applies the correct method to reach at least a matrix of cofactors 2 correct rows or 2 correct columns needed		M1
	$\begin{pmatrix}a-2&1-a&1\\a+2&a-1&-3\\-2&0&2\end{pmatrix}\to\begin{pmatrix}a-2&a+2&-2\\1-a&a-1&0\\1&-3&2\end{pmatrix}$ Correct transpose of cofactors		A1
	$\mathbf{A}^{-1}=\dfrac{1}{2a-2}\begin{pmatrix}a-2&a+2&-2\\1-a&a-1&0\\1&-3&2\end{pmatrix}$	Correct inverse	A1
			(4)
(b)	$a=4\Rightarrow\mathbf{A}^{-1}=\dfrac{1}{6}\begin{pmatrix}2&6&-2\\-3&3&0\\1&-3&2\end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$=\dfrac{1}{6}\begin{pmatrix}2&6&-2\\-3&3&0\\1&-3&2\end{pmatrix}\begin{pmatrix}12-6\lambda\\4+2\lambda\\6+3\lambda\end{pmatrix}=\ldots$	Attempt to multiply the parametric form of l_2 by their inverse	M1
	$=\begin{pmatrix}6-\lambda\\-4+4\lambda\\2-\lambda\end{pmatrix}$	Correct parametric form	A1
	$\mathbf{r}=\begin{pmatrix}6\\-4\\2\end{pmatrix}+\lambda\begin{pmatrix}-1\\4\\-1\end{pmatrix}$	Correct equation (allow equivalent forms) but if given as $l=\ldots$ award A0	A1
			(4)
			Total 8

	Alternatives for (b)		
(i)	$a=4 \Rightarrow \mathbf{A}^{-1}=\frac{1}{6}\begin{pmatrix}2 & 6 & -2\\ -3 & 3 & 0\\ 1 & -3 & 2\end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$\frac{1}{6}\begin{pmatrix}2 & 6 & -2\\ -3 & 3 & 0\\ 1 & -3 & 2\end{pmatrix}\begin{pmatrix}12\\ 4\\ 6\end{pmatrix}=\frac{1}{6}\begin{pmatrix}36\\ -24\\ 12\end{pmatrix}$ $\frac{1}{6}\begin{pmatrix}2 & 6 & -2\\ -3 & 3 & 0\\ 1 & -3 & 2\end{pmatrix}\begin{pmatrix}-6\\ 2\\ 3\end{pmatrix}=\frac{1}{6}\begin{pmatrix}-6\\ 24\\ -6\end{pmatrix}$	Attempt $\mathbf{A}^{-1}$(point on l_2) **and** $\mathbf{A}^{-1}$(direction of l_2) **Both** correct (NB No ft)	M1 A1
	$\mathbf{r}=\begin{pmatrix}6\\ -4\\ 2\end{pmatrix}+\lambda\begin{pmatrix}-1\\ 4\\ -1\end{pmatrix}$	Correct equation (allow equivalent forms) but if given as $l = \ldots$ award A0	A1
			(4)
(ii)	$a=4 \Rightarrow \mathbf{A}^{-1}=\frac{1}{6}\begin{pmatrix}2 & 6 & -2\\ -3 & 3 & 0\\ 1 & -3 & 2\end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$\frac{1}{6}\begin{pmatrix}2 & 6 & -2\\ -3 & 3 & 0\\ 1 & -3 & 2\end{pmatrix}\begin{pmatrix}12\\ 4\\ 6\end{pmatrix}=\frac{1}{6}\begin{pmatrix}36\\ -24\\ 12\end{pmatrix}$ $\frac{1}{6}\begin{pmatrix}2 & 6 & -2\\ -3 & 3 & 0\\ 1 & -3 & 2\end{pmatrix}\begin{pmatrix}6\\ 6\\ 9\end{pmatrix}=\frac{1}{6}\begin{pmatrix}30\\ 0\\ 6\end{pmatrix}$	Attempt $\mathbf{A}^{-1}$(point on l_2) for 2 points **Both** correct (NB No ft)	M1 A1
	$\mathbf{r}=\begin{pmatrix}6\\ -4\\ 2\end{pmatrix}+\lambda\begin{pmatrix}-1\\ 4\\ -1\end{pmatrix}$	Obtain the direction vector and deduce correct equation (allow equivalent forms) but if given as $l = \ldots$ award A0	A1
			(4)

Q5.

Question Number	Scheme		Marks
(a) **Mark (i) and (ii) together**	$\begin{vmatrix} 3-5 & 0 & 1 \\ 1 & 2-5 & 2 \\ 4 & 0 & 3-5 \end{vmatrix} = -2(-3)(-2)-4(-3)=0 \Rightarrow 5$ is an eigenvalue This mark is for demonstrating that 5 is an eigenvalue as above and **requires a conclusion**. This may also be done by **substituting** $\lambda = 5$ into their CE to obtain 0 **with a conclusion** or by performing long division and obtaining a remainder of 0 **with conclusion**. However if the candidate forms and solves the CE and correctly obtains $\lambda = 5$ this is sufficient without a conclusion. The conclusion can be minimal e.g. "proven", "QED", tick etc.		B1 **M1 on ePEN**
	$\begin{vmatrix} 3-\lambda & 0 & 1 \\ 1 & 2-\lambda & 2 \\ 4 & 0 & 3-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda)(3-\lambda)-4(2-\lambda)$ M1: Attempts determinant of $\mathbf{A} - \lambda\mathbf{I}$ Whichever method is chosen e.g. row/column/Sarrus, the expression should be similar to the above e.g. • Row 2: $(2-\lambda)\{(3-\lambda)(3-\lambda)-4\}$ • Row 3: $4(0-(2-\lambda))+(3-\lambda)(3-\lambda)(2-\lambda)$ • Column 1 = as Row 1 • Column 2 = as Row 2 • Column 3 = as Row 3 A1: Correct equation including " = 0" which may be implied by an attempt to solve. NB Correct expanded cubic is $\lambda^3 - 8\lambda^2 + 17\lambda - 10 = 0$		M1A1
	$(2-\lambda)(\lambda-5)(\lambda-1)=0 \Rightarrow \lambda = \ldots$ Attempts to solve cubic – **may just be seen from calculator**		M1
	$\lambda = 2,\ 1,\ \ (5)$	Other eigenvalues 2 and 1	A1
			(5)

(b)	$\begin{pmatrix} 3 & 0 & 1 \\ 1 & 2 & 2 \\ 4 & 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 5\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} 3-5 & 0 & 1 \\ 1 & 2-5 & 2 \\ 4 & 0 & 3-5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ Demonstrates the understanding that 5 is an eigenvalue Either of these statements is sufficient		M1
	$3x + z = 5x$ $x + 2y + 2z = 5y$ $4x + 3z = 5z$	Multiplies out to obtain at least 2 **correct** equations	M1
	$\therefore$ Eigenvector is $\begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix}$	Allow any multiple of this vector e.g. $\begin{pmatrix} 1 \\ \frac{5}{3} \\ 2 \end{pmatrix}$ will be common. Allow $x = \ldots$, $y = \ldots$, and $z = \ldots$ where x, y and z were seen in a vector earlier and isw.	A1
			(3)

(c)	$\begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}2\\1\\-3\end{pmatrix}=\ldots$ **or** $\begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}1\\2\\-1\end{pmatrix}=\ldots$ Attempts to multiply one of the given vectors by **M** to find at least one image	M1 **B1 on ePEN**
	$\begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}2\\1\\-3\end{pmatrix}=\ldots$ **and** $\begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}1\\2\\-1\end{pmatrix}=\ldots$ Attempts to multiply both of the given vectors by **M** to find both images	M1
	Note that an attempt at $\begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}2+\mu\\1+2\mu\\-3-\mu\end{pmatrix}=\cdots$ scores both M's provided this results in the parametric image	
	$\begin{pmatrix}3\\-2\\-1\end{pmatrix}$ **or** $\begin{pmatrix}2\\3\\1\end{pmatrix}$ — One correct	A1 **M1 on ePEN**
	$\begin{pmatrix}3\\-2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}2\\3\\1\end{pmatrix}$ — Both correct	A1
	$\begin{pmatrix}3+2\mu\\-2+3\mu\\-1+\mu\end{pmatrix}$ scores both A marks	
	$(\mathbf{r}-\mathbf{c})\times\mathbf{d}=\mathbf{0}$ where $\mathbf{c}=3\mathbf{i}-2\mathbf{j}-\mathbf{k}$ and $\mathbf{d}=2\mathbf{i}+3\mathbf{j}+\mathbf{k}$ Or $(\mathbf{r}-(3\mathbf{i}-2\mathbf{j}-\mathbf{k}))\times(2\mathbf{i}+3\mathbf{j}+\mathbf{k})=\mathbf{0}$ Correct equation in the correct form. Follow through their vectors but they must be correctly placed and **depends on both method marks.**	A1ft
		(5)
		[Total 13]

Note that candidates may transform 2 points on l_1 and then use the transformed points to find the direction of l_2. In this case the second M and the second A1 will only be scored when the direction of l_2 is found and then the final mark becomes available.