Questions

Q1.

Given that

$$y = \arctan\left(\frac{\sin x}{\cos x - 1}\right) \qquad x \neq 2n\pi, \qquad n \in \mathbb{Z}$$

Show that

$$\frac{dy}{dx} = k$$

where k is a constant to be found.

(Total for question = 6 marks)

Q2.

An ellipse has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a > 0,\ b > 0$$

The line l has equation $y = mx + c$ where $m < 0$ and $c > 0$

Given that l is a tangent to the ellipse,

(a) show that $c^2 = b^2 + a^2m^2$

(4)

The tangent l meets the positive x-axis at the point A and the positive y-axis at the point B.
The origin is O.

Given that m varies,

(b) show that the minimum area of triangle OAB is ab.

(7)

(Total for question = 11 marks)

Q3.

The curve C has equation

$$y = 9 \cosh x + 3 \sinh x + 7x$$

Use differentiation to find the exact x coordinate of the stationary point of C, giving your answer as a natural logarithm.

(6)

(Total for question = 6 marks)

Q4.

Given that

$$y = x - \operatorname{artanh}\left(\frac{2x}{1+x^2}\right)$$

(a) show that

$$1 - \frac{dy}{dx} = \frac{k}{1-x^2}$$

where k is a constant to be found.

(4)

(b) Hence, or otherwise, show that

$$\frac{d^2y}{dx^2} + x\left(1 - \frac{dy}{dx}\right)^2 = 0$$

(4)

(Total for question = 8 marks)

Q5.

Given that $y = \arctan\left(\frac{2x}{3}\right)$,

(a) find $\frac{dy}{dx}$, giving your answer in its simplest form.

(2)

(b) Use integration by parts to find

$$\int \arctan\left(\frac{2x}{3}\right) dx$$

(4)

(Total for question = 6 marks)

Examiner's Report

Q1.

This was either very well answered or very poorly. By far the most common method was Way 1 with a small number using Way 2 and with only a very small minority using the quotient rule incorrectly. The most common error was to differentiate the arctan part and forget to multiply by the derivative of $\left(\frac{\sin x}{\cos x - 1}\right)$. Having completed the differentiation, the simplification of the fractions did cause problems at times.

Q2.

This question on an ellipse was fairly discriminating, particularly in part (b), but a substantial number of candidates scored most of the marks here.

In part (a), most eliminated y and expanded $(mx + c)^2$ correctly but then some did not realise the need to use the discriminant. Those that did, often went on to achieve the given answer with no errors although some got bogged down in the algebra. It was rare to see the alternative verification of the result using a parameter. Some weaker responses merely found $\frac{\mathrm{d}y}{\mathrm{d}x}$ or attempted the equation of the tangent but made no further progress.

Some candidates did not offer an attempt for part (b) but it was relatively common to see the correct coordinates for the intercepts and an appropriate expression for the area. Some could not progress from this point but many did use the result in part (a) to establish an expression for the area in one variable, usually m. Those who did further work usually differentiated their expression correctly, set it equal to 0 and solved for m. The last mark proved quite hard to score since many did not correctly handle the fact that m was negative and that the minimum area occurred when $m = -\frac{b}{a}$. A significant number of candidates were successful using parametric coordinates. Those who obtained the correct $A = \frac{ab}{\sin 2\theta}$ often went on to score all the marks provided they had a valid argument to show that this produced a minimum area of ab.

Q3.

This question was found to be accessible. Most students could differentiate correctly although there was the occasional student who thought that cosh x differentiated to − sinh x. There was also the occasional error in signs for the expression of sinh x and cosh x in terms of e^x and e^{-x}. Most then realised they needed to rearrange their expressions to obtain a quadratic in e^x and were able to solve their quadratic correctly although the occasional one had incorrect signs. Most realised that they should only chose the positive value for e^x and then got the correct answer although some still wrote $x = lm\left(\frac{-3}{2}\right)$ which does not exist as well as the correct answer. Very few used the alternate method of squaring and attempting the quadratic in cosh x but most who used this method got to the correct answer although again some had cosh x as negative which is not possible.

Q4.

This question proved to be a challenging one for a lot of students. The need to use the chain rule and either the quotient or product rule in part (a) was usually understood though some students did forget to include one of the two elements. Manipulating the details, including keeping track of the signs, was often done badly. Very few students attempted the alternative methods.

Those who scored all the marks in part (a) usually went on to answer part (b) correctly. A small number who had made no progress in part (a) still attempted part (b) using *k* with some success. However, only 2 of the 4 marks for part (b) were available to students who did not use a numerical value of *k* which they had obtained in part (a).

Q5.

In part (a) most candidates managed to get the correct answer but many failed to put the answer in a simple enough form, as required in the question. It was at least expected that candidates would eliminate fractions over fractions. So whilst $\frac{6}{9+4x^2}$ was acceptable as a final answer, $\frac{\frac{2}{3}}{1+\frac{4}{9}x^2}$ was not. Some candidates began with tan *y* = $\frac{2x}{3}$ and then differentiated implicitly to obtain the required result. A minority of candidates did not use the chain rule and differentiated to obtain $\frac{1}{1+\frac{4}{9}x^2}$ a few candidates used the result for artanh rather than artctan.

Part (b) was usually approached correctly by the majority of candidates. This was helped by the instruction to use parts. Even those with an incorrect result in part (a)could integrate to obtain an expression of the required form involving a natural logarithm.

Mark Scheme

Q1.

Question Number	Scheme	Notes	Marks
Way 1	$\frac{\mathrm{d}\left(\frac{\sin x}{\cos x-1}\right)}{\mathrm{d}x}=\frac{\cos x(\cos x-1)+\sin^2 x}{(\cos x-1)^2}$	M1: Correct use of quotient (or product) rule A1: Correct expression	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\left(\frac{\sin x}{\cos x-1}\right)^2}\left(\frac{\cos x(\cos x-1)+\sin^2 x}{(\cos x-1)^2}\right)$	dM1: $\frac{1}{1+\left(\frac{\sin x}{\cos x-1}\right)^2}\times$ quotient (or product) rule must be a function of x A1: Correct expression	dM1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(\cos x-1)^2}{(\cos x-1)^2+\sin^2 x}\left(\frac{1-\cos x}{(\cos x-1)^2}\right)=\frac{1}{2}$	ddM1: Attempts to simplify to obtain a constant. Must reach a constant A1: cao	ddM1A1
	Special Case: Quotient rule used with numerator terms wrong way round and work otherwise correct: award M1A0 and M1A0ddM1A0 if rest of method correct		(6) **Total 6**
Way 2	$\frac{\mathrm{d}\left(\frac{\sin x}{\cos x-1}\right)}{\mathrm{d}x}=\frac{\cos x(\cos x-1)+\sin^2 x}{(\cos x-1)^2}$	M1: Correct use of quotient (or product) rule A1: Correct expression	M1A1
	$\tan y=\left(\frac{\sin x}{\cos x-1}\right)\Rightarrow \sec^2 y\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\cos x(\cos x-1)+\sin^2 x}{(\cos x-1)^2}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\left(\frac{\sin x}{\cos x-1}\right)^2}\left(\frac{\cos x(\cos x-1)+\sin^2 x}{(\cos x-1)^2}\right)$	dM1: $\frac{1}{1+\left(\frac{\sin x}{\cos x-1}\right)^2}\times$ quotient (or product) rule must be a function of x A1: Correct expression	dM1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(\cos x-1)^2}{(\cos x-1)^2+\sin^2 x}\left(\frac{1-\cos x}{(\cos x-1)^2}\right)=\frac{1}{2}$	ddM1: Attempts to simplify to obtain a constant. Must reach a constant. A1: cao	ddM1A1
Way 3	$\tan y=\left(\frac{\sin x}{\cos x-1}\right)\Rightarrow(\cos x-1)\tan y=\sin x$		
	$\Rightarrow -\sin x\tan y+(\cos x-1)\sec^2 y\frac{\mathrm{d}y}{\mathrm{d}x}=\cos x$	M1: Differentiates implicitly A1: Correct differentiation	M1A1
	$\Rightarrow\frac{-\sin^2 x}{\cos x-1}+(\cos x-1)\left(1+\frac{\sin^2 x}{(\cos x-1)^2}\right)\frac{\mathrm{d}y}{\mathrm{d}x}=\cos x$	dM1: Substitutes for y throughout A1: Correct equation in terms of x only (and $\mathrm{d}y/\mathrm{d}x$)	dM1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}$	ddM1: Attempts to simplify to obtain a constant. Must reach a constant. A1: cao	ddM1A1
Way 4	$\frac{\sin x}{\cos x-1}=\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{1-2\sin^2\frac{x}{2}-1}$	M1: Using the correct double angle formula A1: Correct expression	M1A1
	$=-\cot\frac{x}{2}=-\tan\left(\frac{\pi}{2}\pm\frac{x}{2}\right)=\tan\left(\frac{x}{2}\pm\frac{\pi}{2}\right)$	M1: Obtains tan in terms of x A1: $\tan\left(\frac{x}{2}\pm\frac{\pi}{2}\right)$	dM1A1
	So $y=\arctan\left(\tan\left(\frac{x}{2}\pm\frac{\pi}{2}\right)\right)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}$	ddM1: Attempts to simplify to obtain a constant. Must reach a constant. A1: cao	ddM1A1

Q2.

Question Number	Scheme		Marks
(a) Way 1	$\frac{x^2}{a^2}+\frac{(mx+c)^2}{b^2}=1=\ldots$ Eliminates y from the equation of the ellipse and attempts to expand $(mx+c)^2$		M1
	$\frac{x^2}{a^2}+\frac{m^2x^2+2cmx+c^2}{b^2}=1$ Correct equation with the $(mx+c)^2$ expanded correctly		A1
	$4a^4m^2c^2=4a^2(b^2+a^2m^2)(c^2-b^2)$ Uses discriminant = 0 or equivalent **Dependent on the first method mark**		dM1
	$a^2m^2c^2=b^2c^2-b^4+a^2m^2c^2-a^2m^2\times b^2$ $c^2=b^2+a^2m^2$ * Complete to obtain the GIVEN result with no errors seen. At least one intermediate step should be shown.		A1 cso
			(4)
(a) Way 2	$m=\frac{b\cos\theta}{-a\sin\theta}\Rightarrow y-b\sin\theta=\frac{b\cos\theta}{-a\sin\theta}(x-a\cos\theta)$ $\Rightarrow y=\frac{b\cos\theta}{-a\sin\theta}x+\frac{b}{\sin\theta}\Rightarrow m=\frac{b\cos\theta}{-a\sin\theta},\ \ c=\frac{b}{\sin\theta}$ M1: Forms the equation of a general tangent and "extracts" c and m A1: Correct c and m		M1A1
	$b^2+a^2m^2=b^2+a^2\left(\frac{b\cos\theta}{-a\sin\theta}\right)^2$	Substitutes their m into $b^2+a^2m^2$ or equivalent work. **Dependent on the first method mark**	dM1
	$b^2+a^2m^2=\frac{b^2\cos^2\theta+b^2\sin^2\theta}{\sin^2\theta}=\frac{b^2}{\sin^2\theta}=c^2$ $b^2+a^2m^2=c^2$ * Completes correctly with conclusion e.g. tick, QED etc.		A1

(b) Way 1	$x=0\Rightarrow y=c,\quad y=0\Rightarrow x=-\frac{c}{m}$ *or* $(A \text{ is })\left(-\frac{c}{m},\ 0\right)\ (B\text{ is })(0,\ c)$ Correct values for the intercepts or correct coordinates.		B1
	Area $\Delta OAB=-\frac{c^2}{2m}\left(\text{or } \frac{c^2}{2m}\right)$	Correct expression for the area (allow + or − here)	B1
	$=-\frac{b^2+a^2m^2}{2m}\left(\text{or } \frac{b^2+a^2m^2}{2m}\right)$	Uses $c^2=b^2+a^2m^2$ in the area expression to eliminate c.	M1
		Correct expression (may be unsimplified) (allow + or − here)	A1
	$\frac{\mathrm{d}A}{\mathrm{d}m}=\frac{b^2}{2}m^{-2}-\frac{a^2}{2}$ or $\frac{\mathrm{d}A}{\mathrm{d}m}=-\frac{2m\times 2a^2m-2\left(b^2+a^2m^2\right)}{4m^2}$ Differentiates wrt m (must be correct differentiation for their A). **Dependent on the previous M**		dM1
	At min $m^2=\frac{b^2}{a^2}\quad m=(\pm)\frac{b}{a}$	Equate their derivative to 0 and solves for m^2 or m. **Dependent on all the previous M's**	ddM1
	Min area $=-\frac{b^2+b^2}{-\frac{2b}{a}}=ab\ \left(\text{units}^2\right)$	Correct completion with no errors.	A1
			(7)
			[Total 11]
(b) Way 2	$x=0\Rightarrow y=c,\quad y=0\Rightarrow x=-\frac{c}{m}$ *or* $(A \text{ is })\left(-\frac{c}{m},\ 0\right)\ (B\text{ is })(0,\ c)$ Correct values for the intercepts or correct coordinates.		B1
	Area $\Delta OAB=-\frac{c^2}{2m}\left(\text{or } \frac{c^2}{2m}\right)$	Correct expression for the area (allow + or − here)	B1
	$=-\frac{b^2+a^2m^2}{2m}\left(\text{or } \frac{b^2+a^2m^2}{2m}\right)$	Uses $c^2=b^2+a^2m^2$ in the area expression to eliminate c.	M1
		Correct expression (may be unsimplified) (allow + or − here)	A1
	$A=-\frac{(am+b)^2-2amb}{2m}$	Correct completion of the square in the numerator. **Dependent on the previous M**	dM1
	$=ab-\frac{(am+b)^2}{2m}$ is minimum when $am+b=0$ Correct argument for establishing the minimum **Dependent on all the previous M's**		ddM1
	Min area $=ab\ \left(\text{units}^2\right)$	Correct completion with no errors.	A1

<table>
<tr><td rowspan="12">(b)
Way 3</td><td colspan="2">$$x=0\Rightarrow y=\frac{ab\cos^2\theta}{a\sin\theta}+b\sin\theta\left(=\frac{b}{\sin\theta}\right)$$
or
$$y=0\Rightarrow y=\frac{ab\sin^2\theta}{b\cos\theta}+a\cos\theta\left(=\frac{a}{\cos\theta}\right)$$
Correct value for one of the intercepts or correct coordinates.</td><td>B1</td></tr>
<tr><td colspan="2">$$x=0\Rightarrow y=\frac{ab\cos^2\theta}{a\sin\theta}+b\sin\theta\left(=\frac{b}{\sin\theta}\right)$$
and
$$y=0\Rightarrow y=\frac{ab\sin^2\theta}{b\cos\theta}+a\cos\theta\left(=\frac{a}{\cos\theta}\right)$$
Correct values for both intercepts or correct coordinates.</td><td>B1</td></tr>
<tr><td rowspan="2">Area ΔOAB $A=\frac{1}{2}\frac{a}{\cos\theta}\frac{b}{\sin\theta}$</td><td>Correct method for the area using their intercepts</td><td>M1</td></tr>
<tr><td>Correct area (may be unsimplified)</td><td>A1</td></tr>
<tr><td colspan="2">$$\frac{dA}{d\theta}=\frac{-2ab\left(\cos^2\theta-\sin^2\theta\right)}{4\sin^2\theta\cos^2\theta}=0\Rightarrow\theta=\frac{\pi}{4}$$
Adopts a correct strategy for finding θ at the minimum
Dependent on the previous M</td><td>dM1</td></tr>
<tr><td>$A_{\min}=\dfrac{ab}{2\sin\frac{\pi}{4}\cos\frac{\pi}{4}}$</td><td>Uses their value for θ to find the minimum value. **Dependent on all the previous M's**</td><td>ddM1</td></tr>
<tr><td>$=ab\ \left(\text{units}^2\right)$</td><td>Cso</td><td>A1</td></tr>
<tr><td colspan="2">**Alternative for last 3 marks:**</td><td></td></tr>
<tr><td colspan="2">Area ΔOAB $A=\dfrac{ab}{2\sin\theta\cos\theta}=\dfrac{ab}{\sin 2\theta}$
And the minimum will occur when sin 2θ is maximum i.e. when sin 2θ = 1
Score this mark for a valid argument for determining the minimum
Dependent on the previous M</td><td>dM1</td></tr>
<tr><td>$A_{\min}=\dfrac{ab}{1}$</td><td>Completes the process of finding the minimum. **Dependent on all the previous M's**</td><td>ddM1</td></tr>
<tr><td>$=ab\ \left(\text{units}^2\right)$</td><td>Correct completion with no errors.</td><td>A1</td></tr>
</table>

(b) **Way 4**	$x=0 \Rightarrow y=c, \quad y=0 \Rightarrow x=-\dfrac{c}{m}$ *or* $(A \text{ is })\left(-\dfrac{c}{m},\ 0\right)\ (B \text{ is })(0,\ c)$ Correct values for the intercepts or correct coordinates.		B1
	Area $\Delta OAB = -\dfrac{c^2}{2m}\left(\text{or } \dfrac{c^2}{2m}\right)$	Correct expression for the area (allow + or − here)	B1
	$=\dfrac{ac^2}{2\sqrt{c^2-b^2}}\left(=-\dfrac{ac^2}{2\sqrt{c^2-b^2}}\right)$	Uses $c^2=b^2+a^2m^2$ in the area expression to eliminate m.	M1
		Correct expression. (allow + or − here)	A1
	$\dfrac{\mathrm{d}A}{\mathrm{d}c}=\dfrac{2ac\times 2\sqrt{c^2-b^2}-2ac^3\left(c^2-b^2\right)^{-\frac{1}{2}}}{4\left(c^2-b^2\right)}$ Differentiates wrt c. **Dependent on the previous M**		dM1
	At min $2c^2-2b^2-c^2=0 \Rightarrow c^2=2b^2$ Equate their derivative to 0 and solves to obtain c in terms of b **Dependent on all the previous M's**		dM1
	Min area $=\dfrac{2ab^2}{2\sqrt{2b^2-b^2}}=ab\ \left(\text{units}^2\right)$	Correct completion with no errors.	A1

There will be other valid methods in part (b). Generally, the first 4 marks are for obtaining an expression for the area of AOB and then applying the result in part (a) to enable progress to be made in establishing the minimum or for using the general tangent in terms of θ to find the intercepts and hence the area of AOB. The final 3 marks are for selecting and implementing a correct strategy for proving that the minimum area is ab.

There may also be other valid methods in part (a).

If you are in any doubt whether a particular method is valid then please seek advice from your Team Leader.

Q3.

Question Number	Scheme	Notes	Marks
	$y = 9\cosh x + 3\sinh x + 7x$		
	$\frac{dy}{dx} = 9\sinh x + 3\cosh x + 7$	Correct derivative	B1
	$9\frac{(e^x - e^{-x})}{2} + 3\frac{(e^x + e^{-x})}{2} + 7 = 0$	Replaces sinh$x$ and cosh$x$ by the correct exponential forms	M1
	Note that the first 2 marks can score the other way round: M1: $y = 9\frac{(e^x + e^{-x})}{2} + 3\frac{(e^x - e^{-x})}{2} + 7x$ B1: $\frac{dy}{dx} = 9\frac{(e^x - e^{-x})}{2} + 3\frac{(e^x + e^{-x})}{2} + 7$		
	$12e^{2x} + 14e^x - 6 = 0$ oe	M1: Obtains a quadratic in e^x A1: Correct quadratic	M1A1
	$(3e^x - 1)(2e^x + 3) = 0 \Rightarrow e^x = ...$	Solves their quadratic as far as $e^x = ...$	M1
	$x = \ln\left(\frac{1}{3}\right)$	cso (Allow $-\ln 3$) $e^x = -\frac{3}{2}$ need not be seen. Extra answers, award A0	A1
			(6)
	Alternative		
	$\frac{dy}{dx} = 9\sinh x + 3\cosh x + 7$	Correct derivative	B1
	$9\sinh x = -3\cosh x - 7 \Rightarrow 81\sinh^2 x = 9\cosh^2 x + 42\cosh x + 49$		
	$72\cosh^2 x - 42\cosh x - 130 = 0$	Squares and attempts quadratic in cosh$x$	M1
	$(3\cosh x - 5)(12\cosh x + 13) = 0 \Rightarrow \cosh x = \frac{5}{3}$	M1: Solves quadratic A1: Correct value	M1A1
	$x = \ln\left(\frac{5}{3} \pm \sqrt{\left(\frac{5}{3}\right)^2 - 1}\right)$	Use of ln form of arcosh	M1
	$x = \ln\left(\frac{1}{3}\right)$	cso (Allow $-\ln 3$)	A1

NB: Ignore any attempts to find the y coordinate

Q4.

Question Number	Scheme	Notes	Marks
	$y = x - \text{artanh}\left(\dfrac{2x}{1+x^2}\right)$		
(a)	$\dfrac{\mathrm{d}\left\{\text{artanh}\left(\dfrac{2x}{1+x^2}\right)\right\}}{\mathrm{d}x} = \dfrac{1}{1-\left(\dfrac{2x}{1+x^2}\right)^2} \times \left(\dfrac{2(1+x^2)-4x^2}{(1+x^2)^2}\right)$ M1: Correct attempt to differentiate artanh using the chain and quotient (or product) rule, obtaining an expression in terms of x A1: Correct expression in any form ie no simplification required Quotient rule must have denominator $(1+x^2)^2$ and numerator to be $\lambda(1+x^2) - \mu x^2,\ \lambda, \mu > 0$ Product rule must reach $\dfrac{\lambda}{(1+x^2)} - \dfrac{\mu x^2}{(1+x^2)^2}\quad \lambda, \mu > 0$		M1A1
	$1 - \dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left(1 + \dfrac{2}{x^2-1}\right) = \dfrac{2}{1-x^2}$	dM1: Attempt $1 - \dfrac{\mathrm{d}y}{\mathrm{d}x}$ A1: cao and cso	dM1 A1cso
			(4)

	(a) Way 2		
	$y = x - \text{artanh}\left(\frac{2x}{1+x^2}\right) = x - \frac{1}{2}\ln\left(\dfrac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$		
	$= x - \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)^2 = x - \ln(x+1) + \ln(x-1)$		
	$\dfrac{\mathrm{d}\{-\ln(x+1)+\ln(x-1)\}}{\mathrm{d}x} = \dfrac{-1}{x+1} + \dfrac{1}{x-1}$	M1: Attempt to differentiate A1: Correct derivative	M1A1
	$1 - \dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left(1 + \dfrac{2}{x^2-1}\right) = \dfrac{2}{1-x^2}$	dM1: Attempt $1 - \frac{\mathrm{d}y}{\mathrm{d}x}$ A1: cao and cso	dM1 A1cso

	(a) Way 3		
	$y = x - \ln\left(\dfrac{x+1}{x-1}\right)$	Obtained as Way 2	
	$\dfrac{\mathrm{d}\left(\ln\left(\dfrac{x+1}{x-1}\right)\right)}{\mathrm{d}x} = \left(\dfrac{x-1}{x+1}\right) \times \dfrac{(x-1)-(x+1)}{(x-1)^2}$	M1: Attempt to differentiate A1: Correct derivative	M1A1
	$1 - \dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 - \left(1 + \dfrac{2}{x^2-1}\right) = \dfrac{2}{1-x^2}$	dM1: Attempt $1 - \frac{\mathrm{d}y}{\mathrm{d}x}$ A1: cao and cso	dM1 A1 cso

	(a) Way 4		
	$u = \text{artanh}\left(\dfrac{2x}{1+x^2}\right) \Rightarrow \tanh u = \dfrac{2x}{1+x^2}$		
	$\left(1-\tanh^2 u\right)\dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{2\left(1-x^2\right)}{\left(1+x^2\right)^2}$		
	$\dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{2\left(1-x^2\right)}{\left(1+x^2\right)^2} \times \dfrac{1}{1-\dfrac{4x^2}{\left(1+x^2\right)^2}}$	M1: Attempt to differentiate to obtain d(artanh(...))/dx as a function of x A1: Correct (unsimplified) derivative	
	Reduces to $\dfrac{2}{1-x^2}$		
	Then as main scheme	dM1A1cso	
(b)	$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2\left(1-x^2\right)^{-2} \times -2x \left(= \dfrac{-4x}{\left(1-x^2\right)^2}\right)$	M1: Attempt second derivative (quotient/product rule as in (a))	M1A1ft
		A1ft: Follow through their k or leave as k	
	$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + x\left(1-\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \dfrac{-4x}{\left(1-x^2\right)^2} + x\left(\dfrac{2}{1-x^2}\right)^2 = 0$	M1: Attempt $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + x\left(1-\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ with their value for k from (a)	M1A1cso
		A1: cso	
			(4)
			Total 8

Q5.

Question Number	Scheme		Marks
(a)	$\left(\frac{dy}{dx}=\right)\left(\frac{2}{3}\right)\frac{1}{1+\frac{4x^2}{9}} = \frac{6}{9+4x^2}$	M1: Use formula for derivative of arctan: $\left(\frac{dy}{dx}=\right)\frac{p}{1+(qx)^2},\ q\neq 1$ Condone missing brackets around qx but must be $1+(qx)^2$ not $1-(qx)^2$ and p may be 1	M1A1
		A1: Answer as shown	
	Allow correct answer only		
			(2)
	Alternative		
	$y=\arctan\left(\frac{2x}{3}\right)\Rightarrow \tan y=\frac{2x}{3}\Rightarrow \sec^2 y\frac{dy}{dx}=\frac{2}{3}$		
	$\frac{dy}{dx}=\frac{2}{3\sec^2 y}=\frac{2}{3(1+\tan^2 y)}$		
	$=\frac{2}{3\left(1+\left(\frac{2}{3}x\right)^2\right)}$	$\left(\frac{dy}{dx}=\right)\frac{p}{1+(qx)^2},\ q\neq 1$ Condone missing brackets around qx but must be $1+(qx)^2$ not $1-(qx)^2$ and p may be 1	M1
	$=\frac{6}{9+4x^2}$	Answer as shown	A1
(b)	$\therefore \int \arctan\left(\frac{2x}{3}\right)dx=\left[x\arctan\left(\frac{2x}{3}\right)\right]-\int \frac{6x}{9+4x^2}dx$		M1A1ft
	M1: Use of parts in correct direction Allow e.g. $x\arctan\left(\frac{2x}{3}\right)-\int x\,d\left(\arctan\left(\frac{2x}{3}\right)\right)$ **for M1** A1ft: Follow through their answer from part (a)		
	$=\left[x\arctan\left(\frac{2x}{3}\right)\right]-\left[\frac{3}{4}\ln(9+4x^2)\right](+c)$		M1A1
	M1: Use of ln correctly for their fraction A1: Cao (+ c not required) Allow $x\arctan\left(\frac{2x}{3}\right)\times x$ and $-\frac{3}{4}\ln k(9+4x^2)$		
			(4)
			Total 6